

University of Macau

Faculty of Science and Technology

Department of Mathematics

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Heat kernels of non-symmetric Levy operators

By

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Time: 11:00 a.m. - 12:00 p.m.

Venue: E11- 1036

Abstract

Let $d \geq 1$ and $\alpha \in (0, 2)$. Consider the following non-local and non-symmetric Lévy-type operator on \mathbb{R}^d :

$$L_\alpha^\kappa f(x) := \text{p.v.} \int_{\mathbb{R}^d} (f(x+z) - f(x)) \frac{\kappa(x, z)}{|z|^{d+\alpha}} dz,$$

where $0 < \kappa_0 \leq \kappa(x, z) \leq \kappa_1$, $\kappa(x, z) = \kappa(x, -z)$, and $|\kappa(x, z) - \kappa(y, z)| \leq \kappa_2 |x - y|^\beta$ for some $\beta \in (0, 1)$. Using Levi's method, we construct the fundamental solution (also called heat kernel) $p_\alpha^\kappa(t, x, y)$ of L_α^κ , and establish its sharp two-sided estimates as well as its fractional derivative and gradient estimates. We also show that $p_\alpha^\kappa(t, x, y)$ is jointly Hölder continuous in (t, x) . The lower bound heat kernel estimate is obtained by using a probabilistic argument. The fundamental solution of L_α^κ gives rise a Feller process $\{X, P_x, x \in \mathbb{R}^d\}$ on \mathbb{R}^d . We determine the Lévy system of X and show that P_x solves the martingale problem for $(L_\alpha^\kappa, C_b^2(\mathbb{R}^d))$. Furthermore, we show that the C_0 -semigroup associated with L_α^κ is analytic in $L^p(\mathbb{R}^d)$ for every $p \in [1, \infty)$. A maximum principle for solutions of the parabolic equation $\partial_t u = L_\alpha^\kappa u$ is also established. As an application of the main result of this paper, sharp two-sided estimates for the transition density of the solution of $dX_t = A(X_{t-})dY_t$ is derived, where Y is a (rotationally) symmetric stable process on \mathbb{R}^d and $A(x)$ is a Hölder continuous $d \times d$ matrix-valued function on \mathbb{R}^d that is uniformly elliptic and bounded.

Biography

Prof. Zhang obtained his PhD. at University of Science and Technology of Central China in 2000, and National Outstanding Young Person Award in 2013. Prof. Zhang has published more than 90 high quality papers on international journals, including Annals of Probability, Communication in Mathematical Physics, Probability Theory and Related Fields, Journal of Functional Analysis, SIAM series.

All are Welcome!

FST Seminar - MAT - " Heat kernels of non-symmetric Levy operators " at 11:00am on 10 April 2015 (Friday),
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