

# A group-theoretic characterization of the direct product of a ball and punctured planes

Akio Kodama

kodama@kenroku.kanazawa-u.ac.jp

Division of Mathematical and Physical Sciences  
Graduate School of Natural Science and Technology  
Kanazawa University, Kanazawa Japan

**Abstract:** Let  $M$  be a connected complex manifold and let  $\text{Aut}(M)$  be the group of all holomorphic automorphisms of  $M$  equipped with the compact-open topology. Then one of the fundamental problems in complex geometric analysis is to determine the complex analytic structure of  $M$  by the topological group structure of  $\text{Aut}(M)$ .

In this talk, concerning this problem, I would like to announce that the following group-theoretic characterization of the space  $B^k \times (C^*)^l$  can be established. The details can be found in a joint paper of the author with professors J. Byun and S. Shimizu: A group-theoretic characterization of the direct product of a ball and punctured planes (preprint):

**Theorem.** Let  $M$  be a connected Stein manifold of dimension  $n$ . Assume that  $\text{Aut}(M)$  is isomorphic to  $\text{Aut}(B^k \times (C^*)^{n-k})$  as topological groups for some integer  $k$  with  $0 \leq k \leq n$ . Then  $M$  is biholomorphically equivalent to  $B^k \times (C^*)^{n-k}$ .

Consequently, this combined with Riemann's extension theorem yields the following fundamental fact on the topological group structure of  $\text{Aut}(B^k \times (C^*)^l)$ :

**Corollary.** If two pairs  $(k, l)$  and  $(k', l')$  of non-negative integers do not coincide, then the groups  $\text{Aut}(B^k \times (C^*)^l)$  and  $\text{Aut}(B^{k'} \times (C^*)^{l'})$  are not isomorphic as topological groups.

## First order techniques in the solution of second order boundary value problems

Alan McIntosh

alan.mcintosh@anu.edu.au

Australian National University, Canberra, ACT, Australia

**Abstract:** The first order Cauchy Riemann equations have long been used in the study of harmonic boundary value problems in the plane. The Dirac operator can sometimes be employed in higher dimensions. Perturbed Dirac

operators provided insight into the solution of the Kato square-root problem for elliptic operators. In recent joint work with Andreas Axelsson and Pascal Auscher, we show how they can also be used to study the solvability of elliptic equations with square integrable boundary conditions. I shall survey this chain of ideas.

## Zero sets of Clifford polynomials and slice regular

Alessandro Perotti  
perotti@science.unitn.it  
Dept. Mathematics University of Trento Via Sommarive  
14 I-38123 Trento Italy

**Abstract:** In this talk we present a Fundamental Theorem of Algebra with multiplicities for Clifford polynomials and some properties of the zero sets of slice regular functions of a Clifford variable.

## Zolotarev fraction and its higher analogues

Andrei Bogatyrev  
gourmet@inm.ras.ru ab.bogatyrev@gmail.com  
Russian Federation, 119991 Moscow, ul. Gubkina, 8, INM RAS

**Abstract:** E.I.Zolotarev around year 1870 has solved the problem of the best uniform rational approximation of *sign* on two given segments, zero between them. This solution – known today as Zolotarev fraction – has a parametric representation in terms of elliptic functions. Later in 1930-ies this solution was used by W.Cauer for the design of the so-called elliptic low-frequency filter, which since then is extensively used in the electronic industry because of its unsurpassable properties.

In this talk we present an approach to the solution of the higher analogs of Zolotarev problem which arise in the multiband filter design. The answer to those problems is given in terms of abelian integrals on suitable Riemann surfaces.

# Contributions to the Theory of Generalized Hypergeometric Series with Applications in Ramanujan's, Bailey's, Preece's, Kummer's and Fox's Work

Arjun K. Rathie  
akrathie@gmail.com

Mathematics Vedant College Of Engineering, Rajasthan State, INDIA

**Abstract:** It is well known that the classical summation theorem such as of Gauss, Gauss second, Kummer and Bailey for the series  ${}_2F_1$  and Watson, Dixon, Whipple and Saalschutz for the series  ${}_3F_2$  play an important role in the theory of hypergeometric and generalized hypergeometric series. Applications of these summation theorems are well known now. Some of the applications can be seen in the well known Bailey's tract.

Berdt has pointed out that the very interesting summations due to Ramanujan can be obtained quite simply by employing the above mentioned classical summation theorems.

Evidently, if the product of two hypergeometric series can be expressed as a hypergeometric series with argument  $x$ , the coefficient  $x_n$  in the product must be expressible in terms of gamma functions.

Bailey in his well known and popular research article [Products of generalized hypergeometric series, Proc. London Math. Soc., 28(2), 242-254, (1928)] had obtained a large number of very interesting results involving products of generalized hypergeometric series by employing the above mentioned classical summation theorems.

Recently good progress has been done in generalizing the above mentioned classical summation theorems for the series  ${}_2F_1$  and  ${}_3F_2$  in a series of the following three research papers by Lavoie, Grondin and Rathie:

1. Generalizations of Watson's theorem on the sum of a  ${}_3F_2$ , Indian J. Math., 34, 23- 32,(1992).
2. Generalizations of Dixon's theorem on the sum of a  ${}_3F_2$ , Math. Comp.,62, 267-276, (1994).
3. Generalizations of Whipple's theorem on the sum of a  ${}_3F_2$ , J. Comput. Appl.Math., 72, 293-300, (1996)

It is not out of place to mention here that these results have been obtained, checked and verified with the help of MATHEMATICA and MAPLE, the general systems for doing mathematics by computers.

In the present invited talk, I aim at presenting the generalizations of the results due to Ramanujan, Bailey, Preece, Kummer and Fox (including summations, products of generalized hypergeometric series).

# A Study of $pl(m,n)$ -Kummer Matrix Function of Two Complex Variables under Differential Operator

Ayman Shehata Mohammed Ahmed El-Shazly  
drshehata2006@yahoo.com

Department of Mathematics, Faculty of Science, Assiut University,  
71516-Assiut, Egypt.

**Abstract:** The main aim of this paper is to define and study of a new matrix functions, say, the  $pl(m, n)$ -Kummer matrix function of two complex variables. The radius of regularity, recurrence relation and several new results on this function are established. We study the operating of differential operator of  $pl(m, n)$ -Kummer matrix function that is solutions of differential equation.

## Dirichlet problem for linear complex partial differential equations in annular domains

A. Okay Celebi

acelebi@yeditepe.edu.tr

Yeditepe University, Department of Mathematics, Kayisdagi Cad. 34755  
Kadikoy - Istanbul, TURKEY

**Abstract:** Using the Green function for an annular domain

$$R = \{z \in \mathbb{C} : 0 < r < |z| < 1\},$$

a class of integral operators are introduced with their properties.the Dirichlet problem for linear elliptic second order differential equations with main parts as the harmonic operator is discussed in  $R$ .The problem is transformed into a singular integral equation and the solvability is obtained via Fredholm theory.

**Key Words:**Dirichlet boundary condition,Poisson euqation,ring domain, singular integral equation.

## Towards Schmidt's Theorem for Algebraic Points of Bounded Degree

Aaron Levin  
adlevin@math.msu.edu  
Michigan State University

**Abstract:** The Schmidt subspace theorem is a deep generalization of Roth's theorem in Diophantine approximation to the setting of hyperplanes in projective space. Another well-known generalization of Roth's theorem is the theorem of Wirsing, which extends Roth's theorem from rational points to algebraic points of bounded degree. In a similar way, I will discuss some results giving a version of Schmidt's theorem for algebraic points of bounded degree.

## Boundary value problem and some applications in several complex variables

Chen Lvping  
lpchen@xmu.edu.cn  
School of Mathematics Sciences, Xiamen University, Xiamen, Fujian,  
P.R.China, 361005

**Abstract:**In this talk, we will focus on the boundary properties of the singular integral and the solution of the singular integral equations with variable and constant coefficients. We will study some relations between the higher order singular integral and the usual Cauchy integral. Especially, by means of the definition of Hadamard principal value permutation formula and composition formula for higher order singular integral, we will develop the technique of the regularization method for the higher order singular integral-differential equation.

## Fixed points and factorization of meromorphic functions of one or several variables

Chung-Chun Yang  
chungchun.yang@gmail.com  
Hong Kong University of Science and Technology, Hong Kong, China

**Abstract:**Fixed points and factorization of meromorphic functions of one or several variables Abstract (Attached / Later):Let  $E(C)$  denote all the non-constant entire functions on the complex-plane  $C$ . Let  $F, f, g,$  and  $h$  denote some functions in  $E$ . A point  $c$  ( in  $C$ ) is called a fixed point of  $f$  if  $f(c)=c$ . P.C. Rosenbloom appeared to be the first one to study the quantitative estimate of the fixed points of a composite transcendental function, by utilizing Nevanlinna's value distribution theory.He (in his 1956's paper ) defined an expression of the form :  $F(z) =f(g(z))$  or  $F=f(g(z))$  a factorization of  $F$ , and  $F$  is called a prime function iff whenever a factorization  $F=f(g)$  holds, then it follows that either  $f$  or  $g$  must be linear. In that paper he asserted , without giving a proof ,that the function  $F(z)=\exp(z) +z$  is prime. Note that  $F$  has two interesting properties : 1.  $F$  has no fixed points and 2.  $F$  satisfies a linear differential equation , with polynomials as the coefficients. Later on and naturally, the above topics and related studies were extended to meromorphic functions of several variables or meromorphic mappings, by peers throughout the world. Here we shall give a brief account on some of the interesting and significant results and open problems relating to the two topics that were obtained or derived by the present speaker and his co-workers. The main tool of our studies has been the value distribution theory of meromorphic functions of one or several variables.

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## Boundedness of oscillating hyper-singular integral operators along curves

Cheng Jiecheng

jcchen@zju.edu.cn

Department of Mathematics, Zhejiang University

P.R.China

**Abstract:**In this talk, we mainly show some recent joint work, with D. S. Fan, M. Wang and X. R. Zhu, on the following oscillating hyper-singular Hilbert transform

$$H_{n,\alpha,\beta}(f)(x) = \int_{R^1} f(x - \Gamma(t)) e^{i|t|^{-\beta}} |t|^{-1-\alpha} dt \quad (1)$$

along the curves  $\Gamma(t) = (t^{p_1}, \dots, t^{p_n})$  where  $p_1, \dots, p_n, \alpha, \beta$  are positive real numbers.

Theorem 1 For  $p_1, \dots, p_n, \alpha$  and  $\beta > 0$ , we have (1) For  $\beta \geq (n+1)\alpha$ ,  $H_{n,\alpha,\beta}$  is  $L^2$ -bounded; (2) For  $\beta > (n+1)\alpha$ ,  $H_{n,\alpha,\beta}$  is  $L^p$ -bounded if  $\frac{2\beta}{2\beta-(n+1)\alpha} < p < \frac{(n+1)\alpha}{2\beta}$ .

Theorem 2 For  $p_1, \dots, p_n, \alpha$  and  $\beta > 0$ , if  $p_i \neq p_j$  for  $i \neq j$  and  $\beta > \alpha$ , then  $H_{n,\alpha,\beta}$  is  $L^2$ -bounded iff  $\beta \geq (n+1)\alpha$ .

## Schwarz-Pick inequality for harmonic quasiconformal mappings and its applications

Chen Xingdi

chxtt@hqu.edu.cn

Department of Mathematics, Huaqiao University

**Abstract:**The main result of this talk is the sharp generalized Schwarz-Pick inequality for euclidean harmonic quasiconformal mappings with convex ranges, which generalizes a result given by Mateljevic. As its applications, we obtain the property of quasiisometry with respect to the Poincare distance and an analogue of Koebe theorem for this class of mappings.

## Quasisymmetrically minimal Moran sets

Dai Meifeng

daimf@ujs.edu.cn

Nonlinear Scientific Research Center, Faculty of Science, Jiangsu University,  
212013, Zhenjiang, Jiangsu, P.R. China

**Abstract:**We call a set  $E \subset \mathbb{R}^n$  quasisymmetrically minimal, if

$$\dim_{\mathcal{H}} f(E) \geq \dim_{\mathcal{H}} E$$

for any  $n$ -dimensional quasisymmetric map  $f$ . According to Tukia, a set in  $\mathbb{R}$  of Hausdorff dimension 1 may be not minimal for 1-dimensional quasisymmetric maps. Recently, Hakobyan and Hu proved that middle interval Cantor sets and uniform Cantor sets of Hausdorff dimension 1 are all minimal. These are the known examples of minimal sets in  $\mathbb{R}$  of Hausdorff dimension 1. We now consider a class of Moran sets  $E(\{n_k\}, \{\delta_k\}, \{c_k\})$  of Hausdorff dimension 1. These sets include the middle interval Cantor sets and uniform Cantor sets. We prove that they are also minimal for 1-dimensional quasisymmetric maps.

## The Scasimir operator in higher spin Clifford Analysis

David Eelbode

(joint work with Peter Von Lancker)

david.eelbode@ua.ac.be

Department of Mathematics and Computer Science, University of Antwerp  
Campus Middelheim, G-Building, Middelheimlaan 1, 2020 Antwerpen  
(Belgium)

**Abstract:** Clifford analysis is often described as a refinement of both classical complex analysis and harmonic analysis in  $R^m$ . From a purely algebraic point of view, this is encoded in the fact that the basic operators studied in Clifford analysis (both in one or several vector variables) generate the orthosymplectic Lie superalgebra, whose even part is the classical Lie algebra underlying harmonic analysis (in one or several variables). It is therefore obvious that algebraic phenomena have their counterpart in Clifford analysis. In this lecture, we will consider the so-called Scasimir operator  $S_{c_k}$  and show how it can be used to define so-called higher spin Dirac operators, which generalize the classical Dirac operator in a broad sense.

## Fueter's theorem revisited

Dixan PeñaPeña

dixanpena@gmail.com; dixanpena@ua.pt

Department of Mathematics, Aveiro University, 3810-193 Aveiro, Portugal



**Abstract:** Among the techniques to generate monogenic functions in Clifford analysis Fueter's theorem is a well-known method. Let  $f(z) = u + iv$  be a  $\mathbb{C}$ -valued holomorphic function in some open subset  $\Xi$  of the upper half of  $\mathbb{C}$  and assume that  $P_k(\underline{x})$  is a homogeneous monogenic polynomial of degree  $k$  in  $R^m$ . If  $m$  is odd, then Fueter's theorem asserts that the function

$$\Delta_x^{k+\frac{m-1}{2}} [(u(x_0), |\underline{x}|) + \frac{x}{|\underline{x}|} v(x_0, |\underline{x}|) P_k(\underline{x})]$$

is monogenic in  $\Omega = \{x = (x_0, \underline{x}) \in R^{m+1} : (x_0, \underline{x}) \in \Xi\}$ .

The main goal of this talk is to show that this theorem is still valid if  $P_k(\underline{x})$  is replaced by a homogeneous monogenic polynomial  $P_k(x_0, \underline{x})$  of degree  $k$  in  $R^{m+1}$  (see *D. PeñaPeña* and *F. Sommen*, Fueter's theorem: the saga continues, *J. Math. Anal. Appl.* 365(2010), no. 1, 29-35).

## On boundary value problems for regular functions in hypercomplex analysis

Du Jinyuan

jydu@whu.edu.cn

School of Mathematics and Statistics, Wuhan University

**Abstract:** In this paper, we survey the researches of boundary value problems for regular functions in hypercomplex analysis. Some component parts of basic theory in works on the boundary value problems for regular functions are introduced, which includes the generalized Cauchy's theorem, Painlevé's theorem, Liouville's theorem, Laurent series development, Plemelj-Sochocki formula, Privalov-Muskhelishvili theorem, etc. Then some boundary value problems are also introduced.

## Some hypergeometric inequalities with applications in multivariate statistics

Dmitry Karp

dimkrp@gmail.com

Institute of Applied Mathematics, Far Eastern Branch of the Russian Academy of Sciences, 7 Radio Street, Vladivostok, 690041, Russia

**Abstract:** We discuss various inequalities for hypergeometric functions and their ratios. In particular, we find sufficient conditions for log-convexity and log-concavity in parameters for generalized hypergeometric function. We demonstrate how this sort of results for the Kummer function can be used to obtain tight two-sided bounds for the solution of maximum likelihood equations for multivariate Watson distribution in directional statistics. Besides we present several open questions and conjectures.

The work has been supported by Russian Foundation for Basic Research under grant 08-01-00028-a.

## On Nonlinear Boundary Value Problems for Holomorphic Functions

Elias Wegert  
wegert@math.tu-freiberg.de  
TU Bergakademie Freiberg, Germany

**Abstract:** We present classical and new results on nonlinear boundary value problems for holomorphic functions with emphasis on their geometric aspects. At the beginning we give an introduction to nonlinear Riemann-Hilbert problems, which originate from Bernhard Riemann thesis. In particular we summarize results on the existence and uniqueness of solutions for different classes of problems and characterize the solution by extremal properties. The second part is devoted to nonlinear transmission problems, which include a hyperbolic version of the Riesz decomposition of functions on the circle into an analytic and an anti-analytic part. We then briefly discuss generalizations of a (free) boundary value problem posed and studied by Arne Beurling. The translation of some these boundary value problems into the context of circle packing is the consequent continuation of their metamorphosis back to geometry. Parts of the talk are based on joint work with David Bauer (Leipzig) Daniela Kraus, Oliver Roth (Würzburg), Gunter Semmler (Freiberg), Ken Stephenson (Knoxville), and Lothar von Wolfersdorf (Freiberg). The author is supported by the Deutsche Forschungsgemeinschaft.

## An introduction to the quaternionic evolution operator

Fabrizio Colombo  
fabrizio.colombo@polimi.it  
Dipartimento di Matematica, Politecnico di Milano

**Abstract:** We introduce the quaternionic functional calculus based on slice regular functions and applied to right or left linear quaternionic operators. Then we study the quaternionic evolution operator based on this quaternionic functional calculus. We show that the classical Hille-Phillips-Yosida results can be extended to the quaternionic setting. We also show that the Laplace transform of the quaternionic semigroup turns out to be the S-resolvent operator. Such an operator is the key tool to construct the quaternionic functional calculus and it plays the same role that the classical resolvent plays for Riesz-Dunford functional calculus.

## Micro-localization from Clifford Analysis

Franciscus C. Sommen

fs@cage.ugent.be

Clifford Research Group, Ghent University, Belgium

**Abstract:** The theory of formal boundary values of holomorphic functions in the upper and lower half plane leads to the definition of the spaces of hyperfunctions and microfunctions on the real line. This can be thought of as singular analytic signals. In the higher-dimensional case there are two approaches to the theory of hyperfunctions and of microfunctions: The classical one using functions of several complex variables and an approach from Clifford analysis, which is related to the Riesz transform and makes use of the monogenic Cauchy kernel, so that it is related to monogenic signals. In the process of micro-localization one starts from the Radon transform which gives the delta function as an integral over the sphere of one-dimensional delta functions on hyperplanes. The microlocal decomposition of the delta function is obtained as a deformation of the Radon inversion formula, where the kernel is singular in one point and in one direction. In our presentation we derive this formula from the Radon decomposition of the Cauchy kernel restricted on a parabolic surface. In this way complicated techniques from several complex variables can be bypassed.

## On the functional equation $A(B)=C(D)$

Fedor Pakovich

pakovich@math.bgu.ac.il

33, Aharon Katzir, Beer Sheva, Israel

**Abstract:** In the talk we will discuss the recent progress in the study of the functional equation  $A(B)=C(D)$  for different classes of functions of one complex variable and its connections with Algebraic Geometry, Number Theory, Dynamics, and Approximation Theory.

## Clifford Analysis for Finite Reflection Groups

Minggang Fei  
fei@uestc.edu.cn

School of Mathematical Sciences, University of Electronic Science and Technology of  
China, Chengdu, 610054, P. R. China

**Abstract:**In this talk we first present a spherical representation of Dunkl-Dirac operator which is invariant under finite reflection groups. Then for applications, we give a version of Fueter's Theorem in Dunkl-Clifford analysis. Based on the spherical representation of Dunkl-Dirac operator, we will finally introduce the definition of spherical Dunkl-harmonics and spherical Dunkl-monogenics so that we can consider and study the factorization of Dunkl Laplace operator in spherical coordinates and a direct sum decomposition of spherical Dunkl-harmonics.

## Non-linear and linear operators and the zeros of entire functions

George Csordas  
george@math.hawaii.edu

Department of Mathematics, University of Hawaii, Honolulu, Hawaii, USA

**Abstract:**The analysis of linear and non-linear operators, preserving hyperbolicity or Hurwitz stability, plays a fundamental role in the study of the distribution of zeros of entire functions. In the first part of this talk, we will focus on some non-linear operators and cite, in particular, P. Branden's solution of the outstanding conjecture due to S. Fisk, P. R. W. McNamara, B. E. Sagan and R. P. Stanley. Some related questions and infinitely logconcave sequences (cf. the Boros-Moll conjecture) will also be discussed. In the second part of the lecture, we will consider linear operators and the distribution of zeros of finite and infinite Fourier transforms of the Jacobi theta function and related kernels (cf. G. Csordas and C. C. Yang). The recently established intriguing result of H. Ki., Y-O Kim and J. Lee, shows that under the action of certain infinite-order differential operators (cf. G. Csordas, W. Smith and R. S. Varga) the Riemann  $\xi$ -function has at most a finite number of non-real zeros. In addition, we will demonstrate, with the aid of some simple linear operators, that certain interpolation problems involving functions in the Laguerre-Polya class, yield necessary and sufficient conditions for the validity of the Riemann Hypothesis.

# The theory of Loewner chains in several complex variables. Applications

Gabriela Kohr

gkohl@math.ubbcluj.ro

Babes-Bolyai University, Faculty of Mathematics and Computer Science, 1 M.  
Kogalniceanu Str., 400084 Cluj-Napoca, Romania

**Abstract:** In this talk we shall present recent and update results in the theory of Loewner chains and the Loewner differential equation on the unit ball in  $C^n$ . Various applications to univalence criteria, quasiconformal extension results, geometric characterizations of Loewner chains, and parametric representation, will be also presented. In addition, we will focus on a recent abstract construction of Loewner chains in several complex variables and hyperbolic complex manifolds.

## A physical interpretation of Clifford Analysis

Ghislain Franssens

ghislain.franssens@aeronomy.be

Belgian Institute for Space Aeronomy (BISA) Ringlaan 3, B-1180 Brussels, Belgium,  
EUROPE

**Abstract:** The aim of this talk is twofold. First, to introduce mathematicians, working in Clifford analysis, to a natural interpretation of their topic in the field of physics. Secondly, it also aims to introduce physicists and engineers to an elegant and powerful mathematical tool, called Clifford analysis, which lies behind many familiar physics models. The presentation will rather focus on conveying basic ideas, with the mathematical details kept to a minimum.

Clifford analysis is a part of mathematical analysis where one studies a selected subset of functions, which take values in a particular hypercomplex algebra, called a Clifford algebra. Both Euclidean and pseudo-Euclidean Clifford analysis, involving a first order vector derivation (Dirac) operator, will be discussed and their relevance for physics applications emphasized. The latter discipline is still an area of active research and some of the intricacies of its distributional foundation will be reviewed

Mathematical formulations will be supplemented with interesting physical interpretations, revealing the naturalness and potential of Clifford analysis for use in physics applications. Especially, the particular Clifford analysis, based on the Clifford algebra of signature (1,3), will emerge as a tailor-made function theory describing electromagnetic and quantum fields in Minkowski space.

# Convergent Interpolation to Cauchy Integrals

L. Baratchart , M. Yattselev  
laurent.baratchart@sophia.inria.fr  
INRIA, BP 93, 06902 Sophia-Antipolis Cedex, FRANCE,)

**Abstract:**We present convergent interpolation schemes for multipoint Pade approximants to functions defined as Cauchy integrals of complex densities over analytic arcs. Admissible densities consist of a Jacobi weight  $W(z) := (a - z)^\alpha (b - z)^\beta$ , where  $-1 < \alpha, \beta < \infty$  and  $a, b$  are the endpoints of the arc (any branch of the roots may be chosen), times a nonvanishing function which has to be all the more smooth than  $\alpha, \beta$  are large. The interpolation points have to be chosen in close connection with the geometry of the arc. The convergence is locally uniform outside of the arc. The proof elaborates on the Riemann-Hilbert approach to asymptotics of orthogonal polynomials, which is here carried over to non-Hermitian orthogonality and non-analytic densities. The result more or less settles the convergence issue of multipoint Pade approximants to functions whose singular set has dimension 1.

# Theorems and Problems of Value Distribution of Meromorphic Functions

Le Yang  
yanglo@math.ac.cn  
Chinese Academy of Sciences, P.R.China)

**Abstract:**Some important results and problems of modular and angular distribution, as well as those in normal families of meromorphic functions and their derivatives will be discussed.

# Von Neumann Inequalities and Toeplitz $C^*$ -Algebras

H. Turgay Kaptanoglu  
kaptan@fen.bilkent.edu.tr  
Bilkent University, Ankara, Turkey (06800, Dept. of Mathematics)

**Abstract:**The weighted symmetric Fock spaces considered are realized as reproducing kernel Hilbert spaces  $\mathcal{D}_q$  ( $q \in \mathbb{R}$ ) of holomorphic functions on the unit ball  $\mathbb{B}$  of  $\mathbb{C}^N$ . Their kernels are  $K_q(z, w) = (1 - \langle z, w \rangle)^{-(1+N+q)}$  for  $q > -(1+N)$  and hypergeometric functions for  $q \leq -(1+N)$ . We also call them Dirichlet spaces and they include the Drury-Arveson space ( $q = -N$ ), the Dirichlet space ( $q = -(1+N)$ ),

the Hardy space ( $q = -1$ ), and Bergman spaces ( $q > -1$ ). Our results are known in some of these special cases.

Our results concern the  $N$ -tuple of operators of multiplication by the coordinate functions on each  $\mathcal{D}_q$ ; we call it the  $q$ -shift  $S_q$ . A von Neumann inequality is an upper bound on the norm of a polynomial of a row contraction of operators on an arbitrary Hilbert space. We first obtain a von Neumann inequality in terms of the norm of a polynomial of  $S_q$  for any  $q$ .

We also consider the  $C^*$ -algebra  $\mathcal{T}_q$  of operators on  $\mathcal{D}_q$  generated by  $S_q$ . We show that  $\mathcal{T}_q$  contains all compact operators and also certain generalized Toeplitz operators with continuous symbols. We show further that a quotient map sends  $\mathcal{T}_q$  onto the continuous functions on the boundary of  $\mathbb{B}$  and thus obtain the related short exact sequence of  $C^*$ -algebras for any  $q$ .

## Complex Analysis Methods Related Optimization Problems with Complex Variables

Hang-Chin Lai

hclai@cycu.edu.tw

Chung Yuan Christian University and National Tsing Hua University, Taiwan

**Abstract:** Consider a nonsmooth minimax fractional programming with complex variables as the following form:

$$(P) \quad \max_{\zeta \in X} \min_{\eta \in Y} \frac{\operatorname{Re}[f(\zeta, \eta) + (z^H A z)^1 / 2]}{\operatorname{Re}[g(\zeta, \eta) + (z^H A z)^1 / 2]}$$

$$s.t. \quad X = \{\zeta(z, \bar{z}) \in \mathbb{C}^{2n} \mid -h(\zeta) \in S, \}$$

$$Y = \{\eta(w, \bar{w}) \in \mathbb{C}^{2m}\}$$

where  $Y$  is a specified compact subset of  $\mathbb{C}^{2m}$ ,  $A$  and  $B$  are positive semidefinite Hermitian matrices in  $\mathbb{C}^{n \times n}$ ,  $S$  is a polyhedral cone in  $\mathbb{C}^p$ ,  $f(\cdot, \cdot)$  and  $g(\cdot, \cdot)$  are continuous functions in  $\mathbb{C}^n \times \mathbb{C}^m$ .

In this talk, two duality models of (P) are established and the duality theorems related to problem (P) are proved with nonduality gap under some reasonable conditions.

## A Complex Number Type Representation for Higher Dimensional spaces With Applications in Harmonic Functions

H. M. NASIR

nasirh@pdn.ac.lk

Department of Mathematics, University of Peradeniya, SRI LANKA

**Abstract:**The search for higher dimensional numbers that behave like real numbers has a very long history. The only other number system available to us, besides real, is the complex field. Complex numbers have been proved useful in solving numerous mathematical problems describing almost all the fields of science.

Many complex-like extensions for higher dimensional spaces have been proposed in the past. For example, Quaternions [1], Octonions [2], hyper-complex numbers [3] and Clifford Algebra are the early attempts in this direction. Recently, Fleury et al [4] have defined a multi-complex number system which is an n-dimensional -algebra generated by a fundamental unit  $e$  fulfilling  $e^n = 1$ . All these extensions were defined on finite dimensional spaces. In order to formulate these extended complex spaces, some relaxation of some field properties had to be made. For example, the commutative property is not satisfied on Quaternions.

We propose a complex type number system on a space of sequences of real numbers as follows: Let  $t$  be a symbolic variable and  $\mathcal{L}(t)$  be the space of infinitely differentiable functions in  $t$  with real coefficients.

**Definition** The complex type space  $\mathbb{I}(t)$  is defined as the set  $X(t) = \{(a(t), b(t)) | a(t), b(t) \in P(t)\}$  with operations defined by

$$1.(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2)$$

$$2.(a_1, b_1)(a_2, b_2) = (a_1 a_2 - (1 + t^2)b_1 b_2, a_1 b_2 + a_2 b_1)$$

where  $a_i, b_i \in P(t), i = 1, 2$ .

In this talk, we show that the complex type space is a field and is an extension of the complex number system. We express the complex type numbers in exponential and trigonometric forms. We also construct Cauchy-Riemann type conditions for complex type valued holomorphic functions in three dimensions and show that the components of a holomorphic function as a function of the coordinate variables  $x, y$  and  $z$  satisfy the three dimensional Laplace's equation.

## Fourier transforms related to radially deformed Dirac operators

Hendrik De Bie

Hendrik.DeBie@Ugent.be

Department of Mathematical Analysis, Ghent University Krijgslaan 281, Building S22, 9000 Ghent (BELGIUM)

**Abstract:**In recent work, it was established that the fundamental osp(1—2) relations of Clifford analysis are also fulfilled by a family of radially deformed Dirac operators, depending on 3 numerical parameters.

In this talk, I will first summarize our previous results on these radial deformations. Then I will introduce the associated Fourier transforms. I will explain the



action of intertwining operators in this theory and I will discuss how a useful series representation for the kernels (related to these Fourier transforms) can be obtained..

## The Properties of the Meromorphic Solutions of Some Difference Equations

Huang Zhi-Bo

hzbo20019@sina.com

School of Mathematical Sciences, South China Normal University, Guangzhou,  
510631, P.R. China

**Abstract:**In this paper, by considering the difference equation of the form  $A_n(z)f(z+n) + \cdots + A_1(z)f(z+1) + A_0(z)f(z) = A_{n+1}(z)$ , (\*) where  $A_j(z)$  ( $j = 0, 1, \cdots, n, n+1$ ) are meromorphic functions, we give answers to the growth estimates of the meromorphic solutions of difference equations (\*). Moreover, We firstly show that the deficiency and fixed points of the meromorphic solutions of difference equations (\*). We also list by giving a number of examples that the result about the fixed points of the meromorphic solutions of the difference equations is the best possible in certain senses.

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## The growth of the solutions of differential equations

Huang Bin

huangbincscu@163.com

College of Mathematics and Computing Sciences, Changsha University of Science and Technology, No. 960, 2nd Section, South Wanjiali RD, Changsha, Hunan 410114, P.R.China

**Abstract:**None.

## Harmonic quasiconformal mappings in the upper half-plane

Huang Xinzong

huangxz@hqu.edu.cn

School of Mathematical Science, Huaqiao University

**Abstract:**We study the analytic characteristic property for the sense preserving univalent harmonic mappings on the upper half plane onto itself.Using the representing formula for positive harmonic mapping on the upper half plane,one necessary and

sufficient condition for sense preserving univalent harmonic mapping on the upper half-plane onto itself to be harmonic quasiconformal mappings is obtained. As an application, by the property of the two-sided harmonic mappings on the upper half plane onto itself, we prove that the set of univalent harmonic quasiconformal mappings on the upper half plane onto itself with respect to composition is not a group. The results generalize the one made by *D.Kalaj* and *M.Pavlović*.

## Special monogenic polynomials via operational calculus

Isabel Cacao

isabel.cacao@ua.pt

Department of Mathematics, University of Aveiro, 3810-193 Aveiro

M.I.Falcão

mif@math.uminho.pt

Department of Mathematics and Applications, University of Minho, Braga, Portugal.

H.R.Malonek

hmalon@ua.pt

Department of Mathematics, University of Aveiro, Aveiro, Portugal.

**Abstract:** Classical polynomials of a real or complex variable and their generalizations to the case of several real or complex variables are well-known and have widely been studied. We propose a different approach to the higher dimensional case for the construction of analogues of Hermite and Laguerre polynomials in  $Rn + 1$  by using hypercomplex function theoretic tools. As part of Clifford analysis, hypercomplex function theory generalizes the theory of holomorphic functions of one complex variable by using Clifford algebras. In this framework the analogue of holomorphic functions is obtained as the set of null-solutions to a generalized Cauchy-Riemann system and they are usually called monogenic. Contrary to the complex case, the construction of monogenic polynomials is not a trivial task mainly because neither the canonical hypercomplex variable nor its powers are monogenic. However, a special linear combination involving only products of the canonical hypercomplex variable and its hypercomplex conjugate provides a sequence of monogenic polynomials that form an Appell set with respect to the hypercomplex derivative. Taking that sequence as a basic polynomial sequence associated to some lowering operators, we construct monogenic Hermite and Laguerre polynomials in arbitrary dimensions.

## On Rarita-Schwinger type operators

John Ryan

jryan@uark.edu

University of Arkansas

**Abstract:**This is joint work with Junxia Li and is done with considerable help from Peter VanLancker. In representation theory for the group  $SO(n)$  it is usual to consider spaces of harmonic polynomials. If one refines to the covering group  $Spin(n)$  of  $SO(n)$  one now considers functions defined on a domain  $U$  in  $R^n$  and taking values in the space of polynomials, homogeneous of degree  $k$  and annihilated by the euclidean Dirac operator. Associated to those spaces is a first order differential operator  $R_k$  known as the Rarita-Schwinger operator. Here we shall build up on results published by Bures, Sommen, Soucek and VanLancker (Journal of Functional Analysis, 2001). We shall provide some proofs. We shall focus on the role played by the group of Moebius transformations and introduce intertwining operators for both  $R_k$  and  $R_k^2$ . We also provide a Green's Formula.

## Saito free divisors, systems of uniformization equations and related topics

Jiro Sekiguchi  
sekiguti@cc.tuat.ac.jp

Tokyo University of Agriculture and Technology

**Abstract:**I start my talk with giving a survey on the theory of integrable connections along Saito free divisors and systems of uniformization equations. Then I focus the attention on the case of such hypersurfaces in a three dimensional affine space. I show examples of Saito free divisors containing discriminant sets of real and complex irreducible reflection groups of rank three. The main subject of this talk is to show many systems of uniformization equations with respect to such hypersurfaces. Last I mention solutions of the systems of uniformization equations.

## Explicit description of automorphism group of Cartan-Hartogs Domains

Jisoo Byun  
jisoobyun@kyungnam.ac.kr  
KyungNam University

**Abstract:**In this talk, I introduce the Cartan Hartogs domain which is a general domain of Thullen domain. We find all holomorphic 1-1, and onto map of these domain. It can be applied to the computation of Bergman kernel of domains.

## Series associated with the Zeta Functions and their Applications

Junesang Choi

junesang@mail.dongguk.ac.kr

Department of Mathematics, Dongguk University Gyeongju 780-714,

Republic of Korea

**Abstract:** The main purpose of this talk is to introduce the remarkably widely investigated subject of closed-form summation of series involving the Zeta functions and to show how many of these summation formulas will find their applications such as in the evaluations of the determinants of the Laplacians for the  $n$ -dimensional sphere  $\mathbf{S}^n$  with the standard metric, in the theory of multiple Gamma functions, and in the theory of Mathieu Series.

## Emergent Conformal Structure Via Circling Packing

Ken Stephenson

kens@math.utk.edu

University of Tennessee, Knoxville, TN, USA

**Abstract:** The subject of the talk is "discrete conformal geometry", a topic that started with William Thurston's 1985 conjecture on the use of circle packing in the approximation of conformal mappings. The discretization effort that his talk sparked has made various conformal phenomena both computationally and visually accessible, often for the first time. As a result, conformality is playing an increasing role in several areas of mathematics, computer science, engineering and their applications.

In the first part of the talk, I will illustrate the discretization process visually, with examples of discrete conformal maps, discrete extremal length, discrete Riemann surfaces, and other notions familiar from the classical continuous theory. The emphasis will be on the faithful way in which circle packing captures conformality –this is "discrete" conformality, not just "approximate" conformality.

In the second part of the talk, I will argue that conformality is in fact an "emergent" phenomenon. When geometry is imposed on random triangulations via circle packing, that geometry consistently displays conformal characteristics. This suggests a sort of "central limit" behavior which would make conformality a natural ingredient to consider in almost any topic involving meshed surfaces. The talk will end with a few applications.

# An estimate of the maximal dilatations of quasiconformal automorphisms of annuli

Katsuhiko Matsuzaki  
matsuzak@waseda.jp

Department of Mathematics, School of Education, Waseda University Nishi-Waseda  
1-6-1, Shinjuku, Tokyo169-8050, Japan

**Abstract:**In this previous work, we have estimated how much dilatation is at least necessary when a quasiconformal homeomorphism of a Riemann surface yields a given large amount of twist along a simple closed geodesic on it. The purpose of this note is to give a result which can be applied to this kind of estimate more generally, especially in the case where the amount of twist is relatively small. After some reduction and simplification, our problem turns out to be the following estimate.

**Problem 1.** let  $A = \{z \in \mathbb{C} \mid 1 < |z| < R\}$  be an annulus and  $f : A \rightarrow A$  a quasiconformal automorphism satisfying  $f(1) = 1$ . Set

$$\tau(f) := \max_{0 \leq \theta < 2\pi} |\arg f(Re^{i\theta}) - \theta|,$$

where the branch of the argument is determined in a continuous way from  $\arg(1) = 0$ . Then, give an estimate of the maximal dilatation  $K(f)$  of  $f$  from below in terms of  $R$  and  $\tau = \tau(f)$ , namely,

$$K_f \geq c_R(\tau)$$

Also, find the best possible estimate.

It is possible to obtain a constant  $c_R(\tau)$  such that  $c_R(\tau) > 1$  for any  $\tau > 0$  and  $c_R(\tau) \rightarrow \infty$  with  $R$  fixed. This will be demonstrated in this note. Then a remaining problem is to find a better estimate.

One may think that an extremal quasiconformal map for this problem is

$$f_1(z) = z \exp\left(i \frac{\log|z|}{\log R} \tau\right)$$

and the best possible constant for  $c_R(\tau)$  is

$$K(f_1) = \left[1 + \left(\frac{\tau}{2\log R}\right)^2\right]^{1/2} + \frac{\tau}{2\log R}$$

However, this is not true when  $R$  is sufficiently close to 1.

# Windowed Linear Canonical Transform and its Applications

KitIan Kou and Ruihui Xu  
kikou@umac.mo

Department of Mathematics, FST, University of Macau

**Abstract:**In this paper the analogue of the windowed Fourier transform under the linear canonical transform is introduced. A new modulation operator is given and the corresponding properties are studied. Some useful properties of the windowed linear transform are derived, such as covariance property, orthogonality property and inversion formulas. The analogue of the Paley-Wiener theorem and uncertainty principle are proved.

# Half Dirichlet problem for the Holder continuous matrix functions in Hermitean Clifford analysis

Min Ku  
kumin0844@126.com  
Daoshun Wang

Department of Computer Science and Technology, Tsinghua University, Beijing, 100084, People's Republic of China

**Abstract:**The simultaneous null solutions of the two complex Hermitean Dirac operators are focused on in Hermitean Clifford analysis, where the Hermitean Cauchy integral was constructed and will play an important role in the framework of circulant  $(2 \times 2)$  matrix functions. Under this setting we will present the half Dirichlet problem with boundary spaces of Holder continuous circulant  $(2 \times 2)$  matrix functions on the sphere of even dimensional Euclidean space. We will give the unique solution to it merely by using the Hermitean Cauchy transformation, get the solutions to the Dirichlet problem on the unit ball for Holder continuous circulant  $(2 \times 2)$  matrix functions as the boundaries and the solutions to the classical Dirichlet problem as the special case, and derive a decomposition of the Poisson kernel for matrix Laplace operator.

**Key Words:**Hermitean Clifford analysis, Cauchy transformation, Dirichlet problem, Holder continuous, Matrix functions.

# Results on difference Riccati equations and differences of meromorphic functions

Kwang Ho Shon  
khshon@pusan.ac.kr  
Pusan National University

**Abstract:**We obtain some properties of solutions for the difference Riccati equations. We research the existence and forms of rational solutions, the Borel exceptional value, zeros, poles and fixed points of transcendental solutions. Also we research the numbers of results concerning zeros and fixed points of the difference equations and the divided difference equations for transcendental meromorphic functions.

# The Initial Value Problem for Generalized Potential Vector

Le Cuong  
lecuong-fami@mail.hut.edu.vn  
Hanoi University of Technology

**Abstract:**The Initial Value Problem (IVP) problem of the type

$$\partial_t u = L(t, x, u, \partial_{x_i} u), u(0, x) = \phi(x) \quad (2)$$

can be solved by contraction mapping principle in the case that the initial function  $\phi$  belongs to an associated space, whose elements satisfy an interior estimate. The present paper proves such interior estimate in the sup-norm for the generalized potential vectors. This proof is based on representation of generalized potential vectors by potential vectors. In that way the paper is another example for the technique of transforming solutions of simpler partial differential equations into solutions of more general ones. .

# The Associated Pairs of Differential Operators and Initial Value Problem in Clifford Analysis

Le Hung Son  
sonlh-fami@mail.hut.edu.vn  
Hanoi University of Technology

**Abstract:**Initial Value Problems of Cauchy-Kovalevskaya type (IVP) can be solved not only in the space of holomorphic functions but also in a more general space such



as a space of regular functions taking value in a Clifford Analysis. In order to solve the IVP it is assumed that the initial data is the solution of an elliptic differential operator  $l$  which associates to the operator  $L$ , this means that  $L$  transforms  $\ker(l)$  into itself. The most interesting problem is to describe all partial differential operators  $L$  associate to the given elliptic partial operator  $l$ . The goal of this talk is to study this problem and its applications to solve the IVP in Clifford Analysis.

## A Characterization of the Hardy space $H_\lambda^1(R)$ Associated with the Dunkl Transform

Jianquan Liao  
lmath@163.com

School of Mathematical Sciences, Capital Normal University

**Abstract:**In this talk, we shall introduce the theory of the Hardy spaces  $H_\lambda^p(R_+^2)$  associated with the Dunkl transform, which was obtained in our previous paper. A new result to be presented is to give a characterization of the Hardy space  $H_\lambda^1(R)$  (the collection of the real part of the boundary value of all  $F \in H_\lambda^1(R_+^2)$ ) in terms of weak derivatives in the Dunkl sense.

## Differential and Difference Equations,sharing values

Liangwen Liao  
maliao@nju.edu.cn

Department of Mathematics,Nanjing University,Nanjing China

**Abstract:**In this talk,we will introduce Nevanlinna theory and its applications to complex differential and difference equations.We will present some research problems in complex differential and difference equations and their progresses.

# Stability of Cauchy type integral applied to the fundamental problem in plane elasticity

Lin Juan

lj7862124@yahoo.com.cn

Department of Foundation, Fujian Commercial College

**Abstract:** Let the elastic domain be a disk, and its boundary the unit circle. By dint of the stability of Cauchy-type integral with respect to the perturbation of integral curve, the stability of the first fundamental problem and the second fundamental problem in plane elasticity will be discussed under the smooth perturbation for the boundary curve.

**Key Words:** elastic domain, Cauchy-type integral, complex stress functions, normal stress, smooth perturbation.

# Value sharing results for shifts of meromorphic functions

Wei-chuan Lin Xudan Luo

sxlwc936@fjnu.edu.cn

Concord University College Fujian Normal University

**Abstract:** In this paper, we deal with the value distribution of difference products of entire functions, and present some result on two difference products of entire functions sharing one value with the same multiplicities. The research findings also include an analogue for shift of a well-known conjecture by Brück. Our theorems improve the results of I. Laine and C. C. Yang [Proc. Japan Acad. Ser. A, 83(2007), 148-151], K. Liu and L. Z. Yang [Arch. Math. 92(2009), 270-278], and J. Heittokangas, R. Korhonen, I. Laine, J. Rieppo and J. L. Zhang [J. Math. Anal. Appl. 355(2009), 352-363]. Moreover, we show by illustrating a number of examples that our results are best possible in certain senses.

# The Sufficient Conditions and the Radius of Convexity for Starlike Mappings

Ming-Sheng Liu

liumsh@scnu.edu.cn

School of Mathematical Sciences, South China Normal University, Guangzhou

**Abstract:**Some sufficient conditions are provided for starlike mappings, which give the  $n$ - dimensional versions of the corresponding results of one complex variable. Furthermore, an estimate is provided for the radius of convexity of starlike mappings in the Euclidean unit ball.

## Hardy-Sobolev derivatives of phase and amplitude of signals

Pei Dang  
ya77408@umac.mo

Department of Mathematics, FST, University of Macau

**Abstract:**In time-frequency analysis there exist fundamental formulas that express the means of Fourier frequency and the bandwidth of a signal  $s$ , etc., originally defined in the Fourier frequency domain, in terms of integrals against the density  $|s(t)|^2$  in the time domain. In the literature the obtained formulas, however, are restricted to smooth signals, because it is the classical derivatives of the phase and amplitude of a signal that are involved in the formulas. The definition and the equivalence of the two forms of the covariance also rely on the classical derivatives and thus are restrictive, too. In the present study, by introducing new types of derivatives, we extend the formulas to signals in the Sobolev space that do not necessarily have classical derivatives. We also investigate these formulas for periodic signal and discrete signal cases.

## A class of hyperbolic type Fourier multipliers on starlike Lipschitz surfaces

Pengtao Li  
li-ptao@163.com

Department of Mathematics, FST, University of Macau

**Abstract:**In this paper, we consider a class of hyperbolic type Fourier multipliers whose symbols are controlled by a polynomial on starlike Lipschitz surfaces and get the  $L^2$  boundedness of these operators on Sobolev spaces and their endpoint estimates.

# Bohr's Radius for the Four Classical Domains

Taishun Liu  
tsliu@hutc.zj.cn  
Huzhou Teachers College  
Jianfei Wang  
Zhejiang Normal University

**Abstract:** We show that if  $f$  is a holomorphic mapping from  $\Omega$  to  $\Omega$ , where  $\Omega$  is one of the four classical domains, then it holds

$$\sum_{k=0}^{\infty} \frac{\|D\varphi_P(P)[D^k f(0)(Z^k)]\|}{k! \|D\varphi_P(P)\|} < 1$$

over the scaled domain  $\frac{1}{3}\Omega$ . Our result is a generalization of the classical result of Bohr in higher dimensions. Moreover, the constant  $\frac{1}{3}$  is the best possible. Notice that when  $\Omega$  is the unit disc in  $\mathbb{C}$ , our theorem reduces to that of Bohr.

**Key Words:** holomorphic mapping, homogeneous expansions, classical domains, Bohr's theorem.

## Theorems of Denjoy-Wolff type

Tadeusz Kuczumow, Wiesława Kaczor  
wieslawa.kaczor@poczta.umcs.lublin.pl  
Department of Mathematics, Maria Curie-Skłodowska University

**Abstract:** We consider a some boundary problem for differential operator of the fourth order with summable coefficients. This problem is very actual now. If the potential is equal to zero, the equation for eigenvalues (the equation in the main) has three-tuple roots. . If the potential is not equal to zero, the boundary problem has three simple series of eigenvalues (this is the effect of splitting). For operators with summable coefficients such effect has studied in the first time.

## Adaptive Takenaka-Malmquist system and analytic approximation

Tao Qian  
fsttq@umac.mo  
Department of Mathematics, FST, University of Macau

**Abstract:**As a generalization of the Fourier system, each entry in the TM system is the boundary value of an analytic function in the unit disc, as well possesses positive phase derivative on the boundary (or that after a constant phase- modulation). Those properties are of interest or necessary from theory and practice of signal analysis and physics. The work concerns adaptive decomposition of any given signal using TM system to get fast convergence in energy sense. The formulation has connections with adaptive decomposition into linear combinations of shifted Cauchy kernels and of best approximation by rational functions of given orders.

## Area Operators from $H_p$ Spaces to $L_p$ Spaces

Zengjian Lou  
zjlou@stu.edu.cn

College of Science , Shantou University, Guangdong 515063, P. R. China

**Abstract:**We characterize non-negative measures  $\mu$  on the unit disk  $D$  for which the area operator  $A_\mu$  is bounded or compact from Hardy space  $H_p$  to  $L_q(\partial D)$  spaces this is a joint work with Mingqing. Gong and Zhijian Wu.

## A new class of non-linear stability preserving operators

Lukasz Grabarek  
lukasz@math.hawaii.edu

P.O.Box 89259 Honolulu, Hawaii 96830-7259

**Abstract:**The Laguerre-Polya class of entire functions consists of precisely those functions that can be approximated, uniformly on compacta, by real polynomials all of whose zeros are real. Necessary and sufficient conditions for membership in the Laguerre-Polya class are of particular interest.

The conjecture of R.P.Stanley that if the zeros of the real polynomial  $\sum_{k=0}^n a_k x^k$  are all real and negative, then the zeros of the polynomial  $\sum_{k=0}^n (a_k^2 - a_{k-1}a_{k+1})x^k$  ( $a_{-1} = 0 = a_{n+1}$ ) remain real and negative has been recently confirmed by P.Branden.

We extend the non-linear operator  $a_k \mapsto a_k^2 - a_{k-1}a_{k+1}$  to a new class of non-linear operators that map real polynomials with all real negative zeros to polynomials of the same type and extend this action to transcendental entire functions. We show that these non-linear operators preserve weak Hurwitz stability and the Laguerre-Polya class.

# Open sets which satisfy the Oka-Grauert principle in a Stein orbifold

Makoto Abe

mabe@kumamoto-u.ac.jp

Faculty of Life Sciences, Kumamoto University, Kumamoto 862-0976, Japan

**Abstract:** A complex space  $X$  is said to be an orbifold if every  $x \in \text{Sing}(X)$  is a quotient singular point. Let  $X$  be a Stein orbifold of pure dimension  $n$ . Let  $D$  be an open set of  $X$  such that  $H^k(D, \mathcal{O}) = 0$  for  $2 \leq k \leq n - 1$ . Assume that there exists a complex Lie group  $G$  of positive dimension such that the canonical map  $H^1(D, \mathcal{O}^G) \rightarrow H^1(D, (\mathcal{E}^\infty)^G)$  is quasi-injective, that is, the inverse image of the neutral element in  $H^1(D, (\mathcal{E}^\infty)^G)$  consists only of the neutral element in  $H^1(D, \mathcal{O}^G)$ . Then we can prove that  $D$  is locally Stein point  $x \in \partial D$ . If moreover  $\text{Sing}(X)$  is discrete in  $X$ , then  $D$  is Stein by the theorem of Andreotti-Narasimhan(1964). This generalizes the theorem of Kajiwara-Nishihara(1979) which characterizes Stein open sets of a Stein manifold of dimension 2 by the validity of Oka-Grauert principle.

# Holomorphic functions in Riemann domains over infinite dimensional Grassmann manifolds

Masaru Nishihara

mr-nisi@fit.ac.jp

Fukuoka Institute of Technology, Fukuoka, 811-0295, JAPAN

**Abstract:** Oka proved that a pseudoconvex Riemann domain over  $\mathcal{C}^n$  is a domain of holomorphy, that is, he solved affirmatively the Levi problem in a Riemann domain over  $\mathcal{C}^n$ . The result of Oka has been extended to Riemann domains over a complex manifold. Moreover his result has been extended to Riemann domains over various infinite dimensional topological vector spaces.

Let  $E$  be a complex Banach space with a Schauder basis and let  $G(E;r)$  be the Grassmann manifold of all  $r$ -dimensional complex linear subspaces in  $E$ . Let  $(\omega, \varphi)$  be a Riemann domain over  $G(E;r)$  with  $\omega \neq G(E;r)$ . In this talk, on the base of the above results, we will investigate the Levi problem and the problem of holomorphic approximation etc. in the Riemann domain  $(\omega, \varphi)$ .

## Singular perturbation problems: a functional analytic approach

Massimo Lanza de Cristoforis  
mldc@math.unipd.it

Dipartimento di Matematica Pura ed Applicata, University of Padua, Via Trieste 63,  
Italy 35121

**Abstract:** This talk is devoted to the analysis of boundary value problems on singularly perturbed domains by an approach which is alternative to those of asymptotic analysis and of homogenization theory.

Such approach has been applied to linear and nonlinear boundary value problems and to linear eigenvalue problems and is based on potential theory, on functional analysis and for certain aspects on harmonic analysis.

We consider boundary value problems depending on a positive parameter  $\epsilon$  which describes the singular perturbation and we assume that the problem degenerates when  $\epsilon$  tends to zero.

Then the goal is to describe the behavior of the solutions when  $\epsilon$  is close to 0. We do so in terms of analytic functions and of singular but known functions of the parameter  $\epsilon$  such as  $\epsilon^{-1}$  or  $\log \epsilon$ .

## Boundary value problems for general Brinkman operators on Lipschitz domains in compact Riemannian manifolds. Applications.

Mirela Kohr  
mkohr@math.ubbcluj.ro

Babes-Bolyai University, Faculty of Mathematics and Computer Science, 1 M.  
Kogalniceanu Str., 400084 Cluj-Napoca, Romania

**Abstract:** In this talk we shall present recent and update results in the area interfacing potential theory and boundary value problems for general Brinkman operators on Lipschitz domains in compact Riemannian manifolds. Various applications related to fluid mechanics will be also considered.

# Effect of splitting multiple in the main eigenvalues for differential operators with summable coefficients

Mitrokhin Sergey  
sergey@yandex.ru  
SRCC Moscow State University

**Abstract:** We consider a some boundary problem for differential operator of the fourth order with summable coefficients. This problem is very actual now. If the potential is equal to zero, the equation for eigenvalues (the equation in the main) has three-tuple roots. . If the potential is not equal to zero, the boundary problem has three simple series of eigenvalues (this is the effect of splitting). For operators with summable coefficients such effect has studied in the first time.

# Differential Operators Associated to the Cauchy-Riemann Operator in Quaternion Analysis and Applications

Nguyen Thanh Van  
thanhvanao@yahoo.com  
Hanoi University of Science, VNU

**Abstract:**This paper deals with the initial value problem of the type

$$\frac{\partial w}{\partial t} = L(t, x, w, \frac{\partial w}{\partial x_i}) \quad (1)$$

$$w(0, x) = \varphi(x) \quad (2)$$

where  $t$  is the time,  $L$  is a linear first order operator (matri-type) in Quaternionic Analysis and  $\varphi$  is a regular function taking values in the Quaternionic Algebra. The article proves necessary and sufficient conditions on the coefficients of operator  $L$  under which  $L$  is associated to the Cauchy-Fueter operator of Quaternionic Analysis.

This criterion makes it possible to construct the operator  $L$  for which the initial problem(1),(2) is solvable for an arbitrary initial regular function  $\varphi$  and the solution is also regular for each  $t$ .

# Conformal monogenic signals

Uwe Kähler  
ukaehler@ua.pt  
University of Aveiro, Portugal



**Abstract:**In the last ten years higher-dimensional analogues of classic analytic signals have received a wide-ranging interest. The possibility of using phase-space information for identifying textures in images and movies has given rise to a multitude of such signals. But up to now most of these approaches allowed only the identification of intrinsically 1D-signals. Recently, G. Sommer and L. Wietzke suggested a new definition, called conformal monogenic signal, by mapping the original image on the sphere. After a short review of hypercomplex analytic signals we are going to take a look into such signals from the point of view of Clifford analysis.

## Inversion of the noisy Radon transform on $SO(3)$

Paula Cerejeiras

pceres@ua.pt

University of Aveiro, Portugal

Joint work with: M. Ferreira, U. Kähler, G. Teschke

**Abstract:**In this talk we present a new method to numerically approximate the inverse of the 1-D Radon transform on  $SO(3)$ . This is an ill-posed inverse problem and the proposed approach is composed by basic building blocks of the co-orbit theory on homogeneous spaces, Gabor frame constructions and variational principles for sparse reconstructions that yield iterative approximation to the solution of the inverse problem.

## Hyperbolicity preservers and majorization

*Petter Brändén*

pbranden@math.su.se

Stockholm University

**Abstract:**The majorization order on  $R^n$  induces a natural partial ordering on the space of all univariate hyperbolic polynomials of degree  $n$ . There has recently been interest in the question of which linear transformation on polynomials preserve majorization. We characterize such operator. This is joint work with Julius Borcea.

# Multiple Integral Transforms Involving A-Function Of r Variables

Prathima J

pamrutharaj@yahoo.co.in

Selection grade lecturer, Dept.of Mathematics, Manipal institute of technology,  
Manipal University, Manipal, Udupi District, Karnataka, India

**Abstract:**None.

## About Robin Radii Of Nonverlapping Domains

Prilepkina Elena

pril-elena@yandex.ru

E.G.Prilepkina, Institute of Applied Mathematics,Vladivostok

**Abstract:**Some inequalities for Robin radii of nonoverlapping domains were obtained in the paper[1]. For example, let nonoverlapping domains  $G_1, G_2$  are in the unit disk  $U$  and  $(\partial G_j \cap U) \subset \Gamma_j \subset \partial G_j, j = 1, 2$ . Then

$$r(z_1, \Gamma_1, G_1)r(z_2, \Gamma_2, G_2) \leq \frac{|z_1 - z_2|^2 |1 - \bar{z}_1 z_2|^2}{(1 - |z_1|^2)(1 - |z_2|^2)} \quad (3)$$

We discuss similar inequalities for an arbitrary finitely-connected domain  $G$ . In particular, if nonoverlapping domain  $G_1, G_2$  are in the domain  $G$  and  $(\partial G_j \cap G) \subset \Gamma_j \subset \partial G_j, j = 1, 2$ , then

$$r(z_1, \Gamma_1, G_1)r(z_2, \Gamma_2, G_2) \leq |f'_G(z_1; z_1, z_2)|^{-1} \quad (4)$$

where  $f_G(z; z_1, z_2)$  is univalent conformal mapping of  $G$  onto plane with radial cuts,  $f_G(z_1; z_1, z_2) = 0, f_G(z_2; z_1, z_2) = \infty$  and in a neighborhood of  $z_2$  expansion

$$f_G(z; z_1, z_2) = \frac{1}{z - z_2} + a_0 + a_1(z - z_1) + \dots$$

is true Equality in the (2) holds iff  $\overline{G_1 \cup G_2} = \bar{G}, \text{Int}(\Gamma_j) = \partial G_j \cap G, j = 1, 2$  and

$$|f_G(z; z_1, z_2)| = \text{const}, \quad z \in \Gamma_1 \cup \Gamma_2.$$

As corollary the equality in the (1) is true iff  $\overline{G_1 \cup G_2} = \bar{U}, \text{Int}(\Gamma_j) = \partial G_j \cap U, j = 1, 2$  and

$$\frac{|z - z_1| |1 - \bar{z}_1 z|}{|z - z_2| |1 - \bar{z}_2 z|} = \text{const}, \quad z \in \Gamma_1 \cup \Gamma_2.$$

This research was supported by Russian Foundation for Basic Research (grant no.08-01-00028).

I.V.N.Dubinin, D.A.Kirillova On extremal decomposition problems Journal of Mathematical Sciences, 157(4),573-583(2009).

## Recent Results on Hyperbolic Function Theory

Sirkka-Liisa Eriksson

sirkka-liisa.eriksson@tut.fi

Department of Mathematics Tampere University of Technology

P.O.Box 553 FI-33101 Tampere Finland

**Abstract:**The aim of this talk is to consider the hyperbolic version of the standard Clifford analysis. The need for such a modification arises when one wants to included to the theory the usual power function  $x^m$ . H.Leutwiler noticed in 1990 that the power function is the conjugate gradient of a harmonic function, defined with respect to the hyperbolic metric of the Poincare upper half space. The theory was extended to the total Clifford algebra valued functions called hypermonogenic in 2000 by H.Leutwiler and S.-L. Eriksson. Hypermonogenic functions have integral formulas, proved in 2004 and 2005. We present the kernel functions in term of hyperbolic metric. Using this new interpretation of the kernels we obtain power series presentations of the kernels. We also state a version of the Liouville's theorem and Maximum Modulus Theorem for hypermonogenic functions.

## Sampling of Non-bandlimited signals and Bedrosian Identity

Qiuhui Chen

chenqiuhui@hotmail.com

University of Aveiro, Aveiro Portugal

**Abstract:**We construct a generalized Sinc function based on ladder shaped filter, establish its corresponding sampling theorem for non-bandlimited signals and discuss the relationship between sampling and Bedrosian identity.

## On some questions of S. Fisk and P. Brändén

Rintaro Yoshida

yoshi@math.hawaii.edu

2463 Kuhio Ave. 504 Honolulu, Hawaii 96815

**Abstract:**P. Brändén recently proved a conjecture due to S. Fisk, R. P. Stanley, P. R. W. McNamara and B. E. Sagan. In addition, P. Brändén gave a partial answer to a question posed by S. Fisk regarding the distribution of zeros of polynomials under the action of certain non-linear operators. In this paper, we give an extension to a result of P. Brändén, and we also answer a question posed by S. Fisk.

## Twisted Rarita-Schwinger operators

Raeymaekers Tim

tr@cage.ugent.be

Ghent University, Galglaan 2, B-9000 Ghent, Belgium

**Abstract:**In this talk, we will define the twisted Rarita-Schwinger operator  $\mathcal{R}_k$  and explain how this invariant differential operator can be used to define general higher spin Dirac operators (HSD) acting on functions  $f(\underline{x})$  on  $R^m$  which take values in more complicated representations for the spin group. The classical approach to construct these operators, is defining them as being of the form  $\pi_\lambda \underline{\partial}_x : \mathcal{C}^\infty(R, \mathcal{S}_\lambda) \rightarrow \mathcal{C}^\infty(R, \mathcal{S}_\lambda)$ . Here,  $\pi_\lambda$  is a projection operator,  $\underline{\partial}_x$  is the Dirac operator,  $\lambda = (l_1, \dots, l_k)$  and  $\mathcal{S}$  is an irreducible representation of the spin group with highest weight  $\lambda + (\frac{1}{2}, \dots, \frac{1}{2})$ . We will make a comparison between this classical approach and our new approach based on the twisted Rarita-Schwinger operator. The latter has the advantage that one can define general higher spin operators through an inductive procedure, which will then also be reflected in the construction of homogeneous polynomial null solutions.

## Counter examples on non- $\alpha$ -normal functions with good integrability

Rauno Aulaskari

rauno.aulaskari@uef.fi

University of Eastern Finland, Department of Physics and Mathematics, P.O.Box 111, FI-80101 Joensuu, Finland

**Abstract:**Blaschke products are used to construct concrete examples of analytic functions with good integrability and bad behavior of spherical derivative. Moreover, it is shown that none of the classes  $M_p^\#$ ,  $0 < p < \infty$ , is contained in the  $\alpha$ -normal class  $\mathcal{N}^\alpha$  when  $0 < \alpha < 2$ . This implies that  $M_p^\#$  is in a sense a much larger class than  $Q_p^\#$ .

# Shannon Sampling Theorem: Non-bandlimited

Guangbin Ren  
rengb@ustc.edu.cn

University of Science and Technology of China

**Abstract:** We obtain a type of Shannon sampling theorem with parameters, i.e., for any real valued  $f(t) \in L^2(\mathbb{R}) \cap C(\mathbb{R})$ ,  $\Omega > 0$ , and  $a$  in the unit disc of the complex plane,

$$H(f(t)e^{i\theta_a(\Omega t)}) = -if(t)e^{i\theta_a(\Omega t)}$$

if and only if

$$f(t) = \sum_{k \in \mathbb{Z}} f(k\frac{\pi}{\Omega}) \text{Sinc}_{(-1)^k a}(\Omega t - k\pi).$$

Here  $H$  is the Hilbert transform in  $\mathbb{R}$ ,  $d\theta_a$  is the harmonic measure of the unit disc, and  $\text{Sinc}_a$  is the product of the classical sinc function with the Poisson kernel of the unit disc, normalized so that  $\text{Sinc}_a(0) = 1$ . When  $a = 0$ , the necessary and sufficient condition is also equivalent to the Fourier transformation of  $f$  being of compact support, so that the result recovers the classical Shannon sampling theorem.

# Analytic continuation and rigidity of germs of holomorphic isometries and measure-preserving maps

Ngaiming Mok  
nmok@hkucc.hku.hk

The University of Hong Kong, Hong Kong, China

**Abstract:** Motivated by problems in Arithmetic Dynamics, we prove two types of extension and rigidity results concerning germs of holomorphic maps between bounded symmetric domains. The first type of results concerns germs of holomorphic isometries  $f : (\Omega, \lambda ds_{\Omega}^2; 0) \rightarrow (\Omega', ds_{\Omega'}^2; 0)$ , where  $\Omega$  and  $\Omega'$  stand for bounded symmetric domains and  $\Omega$  is irreducible,  $ds_{\Omega}^2$  resp.  $ds_{\Omega'}^2$  stands for the Bergman metric, and  $\lambda$  is an arbitrary normalizing constant. The second type of results concerns germs of measure-preserving holomorphic maps  $g : (\Omega, \lambda d\mu_{\Omega}; 0) \rightarrow (\Omega, d\mu_{\Omega}; 0) \times \cdots \times (\Omega, d\mu_{\Omega}; 0)$ , where  $d\mu_{\Omega}$  stands for the volume form of the Bergman metric. Results of the first type on analytic continuation are actually proved in a very general setting for germs of holomorphic isometries up to normalizing constants between bounded domains for which the respective Bergman kernels extend analytically across the boundary in some precise sense.

Results of the second type on analytic continuation are proven whenever the domain is the Poincaré disk  $\Delta$  by Clozel-Ullmo in 2003. In a joint work with S.-C. Ng we

prove rigidity for germs  $g : (\Omega, \lambda d\mu_\Omega; 0) \rightarrow (\Omega, d\mu_\Omega; 0) \times \cdots \times (\Omega, d\mu_\Omega; 0)$  of measure-preserving holomorphic maps whenever  $\Omega$  is (irreducible and) of complex dimension  $> 1$ . The proofs of the two types of results are very different, but they share the common feature that they are applications of methods in Several Complex Variables on analytic continuation. Results of the first type rely on the use of potential functions for Kähler metrics starting with functional identities on *diastases* introduced by Calabi. To deal with extension problems beyond the boundary, we make use of the functional identities together with regularity properties of the Bergman kernel by resorting to projective geometry of the infinite-dimensional projective space. The strategy of proof is reminiscent of the use of Segre varieties in the proof of various boundary extension theorems in Several Complex Variables. Results of the second type rely on extension results of Webster-Huang on strictly pseudoconvex Cauchy-Riemann manifolds, and results on analytic continuation related to geometric structures defined by irreducible bounded symmetric domains. In the rank-1 and higher-dimensional case we make use of Alexander's Theorem characterizing automorphisms of the complex unit ball. In the higher-rank case we prove a new Alexander-type extension result for an irreducible bounded symmetric domain  $\Omega$  of rank  $\geq 2$  concerning smooth boundary points of  $\Omega$  with respect to the Harish-Chandra realization.

## On a problem with two non-local boundary conditions for mixed type equation with singular coefficient in a domain, elliptic part of which is half-band

Ruziev Menglibay Kholtojibayevich  
mruziev@mail.ru

Institute of Mathematics and Information Technologies,  
Durman yuli 29, 100125, Tashkent, Uzbekistan

**Abstract:** Series of problems of sub and supersonic aerodynamics lead to the Tricomi boundary problem in a domain with half-band as an elliptic part of considered domain. A non-local problem for generalized Tricomi equation was considered in the work [1]. The Tricomi problem for mixed type equation with singular coefficient was studied in [2]. In the work [3] the Dirichlet problem for degenerated elliptic equation with singular coefficient in vertical half-plane. Boundary value problems for the degenerated elliptic equation were studied in [4]. Consider an equation

$$\text{sign } y |y|^m u_{xx} + u_{yy} + \frac{\beta_0}{y} u_y = 0,$$

$m > 0$ ,  $-(m/2) < \beta_0 < 1$  in a domain  $D$ , bounded by straight lines  $J_0 = \{(x, y) | x = 0, y > 0\}$ ,  $J_1 = \{(x, y) | x = 1, y > 0\}$  and characteristics

$$OC : x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 0, \quad BC : x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 1$$

of the equation (1), outgoing from the points  $O(0,0)$  and  $B(1,0)$ . Let  $\bar{D} = D \cup J_0 \cup OC \cup BC \cup J_1$ ,  $D^+ = D \cap \{(x, y) : y > 0\}$ ,  $D^- = D \cap \{(x, y) : y < 0\}$ .

**Problem.** To find a function  $u(x, y)$  with properties

- 1)  $u(x, y) \in C(\bar{D}) \cap C^1(D^+ \cup J_0 \cup J_1 \cup D^-) \cap C^2(D^+ \cup D^-)$  and satisfies equation (1) in  $D^+ \cup D^-$ ;
- 2)  $\lim_{y \rightarrow \infty} u(x, y) = 0$  is uniform by  $x \in [0, 1]$ ;
- 3)  $u(x, y)$  satisfies boundary conditions

$$u(0, y) - u(1, y) = \varphi_1(y), \quad y \geq 0;$$

$$u_x(0, y) - u_x(1, y) = \varphi_2(y), \quad y > 0;$$

$$u(x, y)|_{OC} = \psi(x), \quad x \in \left[0, \frac{1}{2}\right]$$

And gluing condition  $\lim_{y \rightarrow +0} y^{\beta_0} u_y = \lim_{y \rightarrow -0} (-y)^{\beta_0} u_y$ ,  $x \in (0, 1)$ , where  $\varphi_1(y)$ ,  $\varphi_2(y)$  and  $\psi(x)$  are given functions such as  $\psi(x) \in C^2\left[0, \frac{1}{2}\right]$ ,  $\varphi_i(y) \in C[0, \infty) \cap C^1(0, \infty)$ ,  $\lim_{y \rightarrow +\infty} \varphi_i(y) = 0$ ,  $y^{\frac{3m+2\beta_0}{4}} \varphi_i(y) \in L(0, \infty)$ ,  $i = 1, 2$ ,  $\varphi_1(0) = \varphi_2(0) = 0$ ,  $\psi(0) = 0$ .

The uniqueness of solution of the problem is proved by an extremum principle. At proving the existence of solution of the problem, the method of separation of variables, Hankel transformation and method of integral equations are used.

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## On some inequalities for rational functions

Sergey Kalmykov

sergeykalmykov@inbox.ru

Institute for Applied Mathematics of Far Eastern Branch of Russian Academy of Sciences, Lab. Of Mathematical Analysis, Vladivostok, Russia, 690041.

**Abstract:** In my report new modulus estimates and differential inequalities for polynomials and rational functions will be presented. For instance the following statement is true (see [1]).

**Theorem.** If a rational function

$$r(z) = \frac{c_m z^m + \dots + c_0}{\prod_{k=1}^n (z - a_k)}, \quad c_j, a_k \in \mathbb{C}, \quad |a_k| > 1, k = 1, \dots, n, j = 1, \dots, m,$$

satisfies the condition  $\max\{r(z) : |z| = 1\} = 1$ , then for all

$$\rho > R = \frac{\prod_{k=1}^n |a_k|}{c_m} + \sqrt{\frac{\prod_{k=1}^n |a_k|^2}{|c_m|^2} - 1} \geq 1$$

and any point  $z$  on the circle  $|z| = \rho$  the following inequalities

$$t_0 \rho^{1+m-n} |B(z)| \leq |r(z)| \leq t_1 \rho^{1+m-n} |B(z)|$$

hold, where  $0 < t_0, t_1 < 1$ , are roots of the equations

$$|c_m|(1+t)^2 \rho = (\rho+R)^2 t \prod_{k=1}^n |a_k|,$$

$$|c_m|(1-t)^2 \rho = (\rho-R)^2 t \prod_{k=1}^n |a_k|,$$

respectively, and

$$B(z) = \prod_{k=1}^n \frac{1 - \bar{a}_k z}{z - a_k}.$$

If  $r(z) = z^k B(z)$ ,  $k \in \mathbb{N}_0$ , then  $R=1$  and these inequalities become equalities for any  $\rho > 1$  and for any point  $z$  on the circle  $|z| = \rho$ .

This statement supplements and if

$$\rho > \frac{R - \lambda + (R-1)\sqrt{\lambda}}{1 - \lambda}, \quad \lambda = |c_m| / \prod_{k=1}^n |a_k|,$$

improves some result of N.K.Govil and R.N.Mohapatra [2].

This research was supported by Russian Foundation for Basic Research (grant no. 0801-00028) and the Far-Eastern Branch of RAS (grant no. 09-I-P4-02).

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## Area Nevanlinna analytic spaces in the unit disk polydisk and related problems

Shamoyan Romi

shamoyan@tu-bryansk.ru

241050, Bryansk, Russian, Fokina 90 ap18



**Abstract:**I will introduce certain new analytic Nevanlinna type spaces in the unit disk and polydisk and study the following questions on them:the action of differentiation and integration operators,zero sets,the structure of closed ideals in such spaces,parametric representations of such spaces. Partially our sharp results will be extended to the case of several complex variables.

## Analysis on local fields

Su Weiyi  
suqiu@nju.edu.cn  
Nanjing University

**Abstract:**None.

## Diffusive wavelets

Swanhild Bernstein  
swanhild.bernstein@math.tu-freiberg.de  
Freiberg University of Mining and Technology, Institute of Applied  
Analysis,Prüferstr. 9, D-09596 Freiberg, Germany

**Abstract:**We want to explain the basic ideas behind the concept of diffusive wavelets in the language of representation theory of Lie groups and within the framework of the group Fourier transform given by Peter-Weyl decomposition. Diffusive wavelets were introduced by *W. Freeden, T. Gervens, M. Schreiner (1998)* for the case of the sphere  $S^2$ . The constructions of wavelet transforms can be entirely based on an abstract group theoretic and representation theoretic approach. This approach has several disadvantages, which becomes apparent in particular for spheres. Because  $L^2(\mathcal{M})$  is infinite dimensional, no compact group admits an irreducible unitary representation of this form. However, compact groups seem natural at least in the situation where  $M$  itself is a homogeneous space of a compact group. Furthermore, for the particular situation of spheres irreducible representations of that form are not square integrable and hence one can not find an admissible wavelet. Nevertheless, a related approach was incorporated for spheres by *J.-P. Antoine and P. Vandergheynst (1999)*. An alternative approach to wavelet transforms on spheres was followed by *R. Coifman and M. Maggioni (2006)*

We present a representation theory based approach to understand diffusive wavelet transforms on compact Lie groups and their homogeneous spaces. The wavelet transforms itself are constructed in terms of kernels of an approximate convolution identity in  $L^2(\mathcal{G})$ ,  $\mathcal{G}$  a compact Lie group, or in  $L^2(\mathcal{G}/\mathcal{H})$  with  $\mathcal{H} < \mathcal{G}$  a closed subgroup of  $\mathcal{G}$ .

Starting point is the notion of diffusive approximate identities, for which the heat kernel provides one interesting and simple example. The concept of diffusive approximate identity is closely related to the scaling function within the classical discrete wavelet approach. The constructed wavelet transform of  $f \in L^2(\mathcal{G})$  has the usual structure of an inner product between  $f$  and a translated wavelet  $\psi_\rho$ ,  $\rho$  being a scaling parameter, and also the corresponding inversion formula has the familiar form of a superposition of translations of  $\psi_\rho$  integrated over all scales. A projection methods transfers these wavelet transforms to homogeneous spaces of  $\mathcal{G}$  by identifying them as quotients  $\mathcal{G}/\mathcal{H}$  for a suitable subgroup  $\mathcal{H} < \mathcal{G}$ . On the other hand, the wavelet transform can be intrinsically defined on the homogeneous space itself. We provide two approaches to wavelet transforms on homogeneous spaces, one giving functions parametrised by points in the space and one parametrised by elements of the group.

## On the Caratheodory Inequality

Tahir Aliyev Azeroğlu  
 aliyev@gyte.edu.tr  
 Gebze Institute of Technology

**Abstract:**In this study, we obtain a sharp boundary version of a Caratheodory type inequality.

## Some generalizations of Bohr's theorem

Tatsuhiko Honda  
 thonda@cc.it-hiroshima.ac.jp  
 2-1-1 Miyake, Saeki-ku, Hiroshima 731-5193 JAPAN

**Abstract:**Let  $U$  be the unit disc in  $\mathbb{C}$  and let  $f : U \rightarrow U$  be a holomorphic function with Taylor expansion  $f(z) = \sum_{k=0}^{\infty} a_k z^k$ . Then

$$\sum_{k=0}^{\infty} |a_k z^k| < 1 \quad \text{for } |z| < \frac{1}{3}.$$

This result, known as Bohr's theorem, was originally obtained for  $|z| < 1/6$ . In fact, the inequality is actually true for  $|z| < 1/3$  and the constant  $1/3$  is best possible, it was obtained independently by Riesz, Schur and Wiener. Other proofs were given by Sidon, Tomic. In recent years, a lot of attention has been paid to multidimensional generalizations of Bohr's theorem for holomorphic mappings on the unit ball

B, e.g.[1],[2],[3],[4],[5]. In [1, Theorem 8], he gave another generalization of Bohr's theorem using homogeneous expansions of holomorphic functions. Also, in [6], they gave a generalization of Bohr's theorem to holomorphic mappings of B into itself using homogeneous expansions.

The purpose of the present study is to consider some generalization of Bohr's theorem to holomorphic mappings of B into itself.

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## Boundary behavior of bounded holomorphic mappings

Toshio Matsushima

matsush@ishikawa-nct.ac.jp

Ishikawa National College of Technology Kita-Chewjoe,  
Tsubata, Ishikawa, 929-0392 JAPAN

**Abstract:** We have already obtained the following result:

**Theorem 0.1.** Let  $\{\zeta_k\}_{k=1}^m$  be an arbitrary sequence of points in the boundary of the unit ball of  $\mathbb{C}^n$ , where  $1 \leq m \leq +\infty$ ,  $n \geq 1$  and  $\zeta_k \neq \zeta_l$  if  $k \neq l$ . Moreover let  $g$  be an arbitrary positive integer. For every  $\zeta_k$ , give a positive integer  $s(\zeta_k)$  with  $1 \leq s(\zeta_k) \leq g$  and  $(k(1), k(2), \dots, k(s(\zeta_k))) \in (\mathbb{Z}_+)^{s(\zeta_k)}$  with  $\sum_{j=1}^{s(\zeta_k)} k(j) \leq g$ . Then there exists a bounded holomorphic map  $F$  from the unit ball of  $\mathbb{C}^n$  to  $\mathbb{C}^g$  such that the radial cluster set of  $F$  at  $\zeta_k$

$$C_r(F, \zeta_k) = \bigcap_{T < 1} \overline{\{F(t\zeta_k) : T < t < 1\}}$$

is a direct product of  $s(\zeta_k)$  closed balls of a positive radius whose dimensions are  $k(1), k(2), \dots, k(s(\zeta_k))$  respectively for every  $k$ .

The aim of this talk is to see some extension of this theorem.

# Conformal monogenic signals

Uwe Kahler  
ukaehler@ua.pt  
University of Aveiro, Portugal

**Abstract:**In the last ten years higher-dimensional analogues of classic analytic signals have received a wide-ranging interest. The possibility of using phase-space information for identifying textures in images and movies has given rise to a multitude of such signals. But up to now most of these approaches allowed only the identification of intrinsically 1D-signals. Recently, G. Sommer and L. Wietzke suggested a new definition, called conformal monogenic signal, by mapping the original image on the sphere. After a short review of hypercomplex analytic signals we are going to take a look into such signals from the point of view of Clifford analysis.

## Isoperimetric Inequalities For Conformal Capacity Of Condensers

Vinokurova Tatyana  
olga @imcs. dvgu.ru  
40-32, Sipyagina street, Vladivostok, Russia, 690065

**Abstract:** A condenser in the  $\overline{R}^n$  is a pair  $C = (E_0, E_1)$  of closed nonempty disjoint sets. The sets are called the plates of the condensers  $C$ . The conformal capacity  $cap_n C$  is defined as the infimum of the Dirichlet integral

$$I(v) := \int_{R^n} |\nabla v|^n dx$$

taken over all admissible functions  $v$ , i.e. real-valued functions satisfying the Lipschitz condition and taking the value  $k$  on the  $E_k$ ,  $k = 0, 1$ .

In this report we discuss some new isoperimetric inequalities for conformal capacity of condensers in  $\overline{R}^n$ . As corollaries the estimates for logarithmic capacity of plane sets are proved. For proofs we use symmetrization methods [1]. For example, the application of polarization with respect to the hypersphere and spherical symmetrization gives following result.

Lets the ball  $\overline{B}(\infty, R) := \{x \in R^n : |x| > 1\}$  is in the plate  $E_0$ ,  $R > 1$ , some continuum contained the points  $\bar{a} = (a, 0, \dots, 0)$ ,  $\bar{b} = (b, 0, \dots, 0)$ ,  $a < b < 1$  belongs to  $E_1 - \{x \in R^n : |x| < 1\}$  onto unit sphere  $S(0, 1)$ . We define  $\lambda(a, b, \rho, R)$  as conformal capacity of the condenser  $C^* := ([\bar{a}, \bar{b}] \cup L(\bar{1}, \rho), \overline{B}(\infty, R))$ , where  $L(\bar{1}, \rho)$  is the spherical cap on the  $S(0, 1)$  of the measure  $\rho$  with center  $\bar{1} = (1, 0, \dots, 0)$ .

Then the inequality

$$cap_n C \geq \lambda(a, b, \rho, R)$$

is true.

[1] Dubinin V.N. Symmetrization in the geometric theory of functions of a complex variable  
 Russian Mathematical Surveys, 1994, 49:1, 1-79.

## Differential Subordinations and Superordinations for Integral and Differential Operators

Teodor Bulboacă

bulboaca@math.ubbcluj.ro

Faculty of Mathematics and Computer Science, Babes-Bolyai University, 400084  
 Cluj-Napoca, Romania

**Abstract:** For the following classes of integral operators on  $\mathcal{K} \subset H(U)$ ,

$$A_{\beta,\gamma}[f](z) = \left[ \frac{\beta + \gamma}{z^\gamma} \int_0^z f^\beta(t) t^{\gamma-1} dt \right]^{1/\beta},$$

$$I_{h;\beta}[f](z) = \left[ \beta \int_0^z f^\beta(t) h^{-1}(t) h'(t) dt \right]^{1/\beta},$$

$$A_{\alpha,\beta,\gamma}^{\phi,\varphi}[f](z) = \left[ \frac{\beta + \gamma}{z^\gamma \phi(z)} \int_0^z f^\alpha(t) \varphi(t) t^{\delta-1} dt \right]^{1/\beta},$$

we determined sufficient conditions for the appropriate functions and parameters, such that the next *sandwich-type theorems* hold (the symbol “ $\prec$ ” stands for the subordination):

$$z \left[ \frac{g_1(z)}{z} \right]^\beta \prec z \left[ \frac{f(z)}{z} \right]^\beta \prec z \left[ \frac{g_2(z)}{z} \right]^\beta \Rightarrow$$

$$z \left[ \frac{A_{\beta,\gamma}[g_1](z)}{z} \right]^\beta \prec z \left[ \frac{A_{\beta,\gamma}[f](z)}{z} \right]^\beta \prec z \left[ \frac{A_{\beta,\gamma}[g_2](z)}{z} \right]^\beta,$$

$$\left[ \frac{zh'(z)}{h(z)} \right]^{1/\beta} g_1(z) \prec \left[ \frac{zh'(z)}{h(z)} \right]^{1/\beta} f(z) \prec \left[ \frac{zh'(z)}{h(z)} \right]^{1/\beta} g_2(z) \Rightarrow$$

$$I_{h;\beta}[g_1](z) \prec I_{h;\beta}[f](z) \prec I_{h;\beta}[g_2](z),$$

and respectively

$$z\varphi(z) \left[ \frac{g_1(z)}{z} \right]^\alpha \prec z\varphi(z) \left[ \frac{f(z)}{z} \right]^\alpha \prec z\varphi(z) \left[ \frac{g_2(z)}{z} \right]^\alpha \Rightarrow$$

$$z\phi(z) \left[ \frac{A_{\alpha,\beta,\gamma}^{\phi,\varphi}[g_1](z)}{z} \right]^\beta \prec z\phi(z) \left[ \frac{A_{\alpha,\beta,\gamma}^{\phi,\varphi}[f](z)}{z} \right]^\beta \prec z\phi(z) \left[ \frac{A_{\alpha,\beta,\gamma}^{\phi,\varphi}[g_2](z)}{z} \right]^\beta.$$

All the above results are *sharp*, i.e. the right-hand side will be the *largest* function and the left-hand side will be the *smallest* function so that the right-hand side, respectively the left-hand side of all the above implications hold, for all  $f$  functions satisfying the differential subordination, respectively the differential superordination of the assumption.

Similar *sharp* results results for generalized *Briot-Bouquet* differential operators are given..

## On boundary regularity of solutions to degenerate Beltrami equations

Toshiyuki Sugawa

sugawa@math.is.tohoku.ac.jp

Graduate School of Information Sciences, Tohoku University

Aramaki Aza Aoba, Aoba-ku, Sendai, 980-8579, JAPAN

**Abstract:** The theory of degenerate Beltrami equations is currently one of the most active fields in Mathematics. I would like to talk about the boundary regularity of the solutions to degenerate plane Beltrami equations with emphasis on the modulus techniques. In the classical case of Beltrami equations, a homeomorphic solution of the open unit disk (or an open half-plane) onto itself always has a homeomorphic extension to the boundary by the famous Mori theorem. In the case of degenerate Beltrami equations, such an extension might not exist. We will give sufficient conditions for a solution to have a homeomorphic (or even smooth in some sense) extension to the boundary.

## Density estimates on composite polynomials

Chiu Yin Tsang

h0347529@graduate.hku.hk

Department of Mathematics, the University of Hong Kong,

Pokfulam, Hong Kong

**Abstract:** J.F. Ritt introduced the concepts of prime and composite polynomials and proved three fundamental theorems on factorizations (in the sense of compositions) of polynomials in 1922. In this talk, we shall give a density estimate on the set of composite polynomials. This kind of density estimation was first used by Smale in

his work on the efficiency of Newton's method. In fact, Smale found a density estimate on a set of bad polynomials which fails to arrive at an approximate zero when applying the Newton's method. This is a joint work with Wai Shun Cheung and Tuen Wai Ng.

## Finding all meromorphic solutions of certain third order non-linear autonomous ODE

Tuen Wai NG

ntw@maths.hku.hk

Department of Mathematics, The University of Hong Kong  
Pokfulam Road, Hong Kong

**Abstract:** In this talk, we shall present a new method based on Nevanlinna theory or Wiman-Valiron theory to find ALL the meromorphic solutions of certain third non-linear autonomous differential equations. In particular, we shall apply this method to the Falkner-Skan differential equation

$$f''' + ff'' + \beta(1 - f'^2) = 0.$$

This third order ODE describes the boundary-layer flow over a wedge of included angle  $\beta\pi$  and it has been studied intensively in fluid mechanics. The solution  $f$  if it exists, is called the shape function, or the dimensionless stream function and its first derivative, after suitable normalization, represents the velocity.

## Contour -solid results for finely meromorphic functions

Tuğba Akyel Güner

takyel@gyte.edu.tr

Gebze Institute of Technology

**Abstract:** In [1] and [2], certain contour-solid theorems for analytic functions were extended onto meromorphic functions and strengthened with taking into account zeros and multivalence of functions. In [3], these results have been extended to finely holomorphic functions which are defined on finely open subsets of complex plane. In this study, we obtain new contour-solid theorems for finely meromorphic functions and different types of majorants.

### References:

1. P.M. Tamrazov and T. Aliyev, A contour-solid problem for meromorphic functions, taking account of nonunivalence, *Ukrain Math. Zh.* 39(1987), p. 683-690.

2. T. Aliyev and P. Tamrazov, Contour-solid theorems for meromorphic functions taking multivalence into account. In: Progress in Analysis. Proceedings of 3-d International ISAAC Congress, Vol. I, 469-476. World Scientific, Singapore, 2003.

3. P.M. Tamrazov, Finely holomorphic and finely subharmonic functions in contour-solid problems. Annales Academia Scientiarum Fennica. Mathematica, Vol. 26, 2001, 325-360.

In [3], these results have been extended to finely holomorphic functions which are defined on finely open subsets of complex plane. In this study, we obtain new contour-solid theorems for finely meromorphic functions and different types of majorants.

## An Integral Representations for some hypergeometric Exton functions

Turaev Mamasali

mtouraev@yahoo.com

Department of Mathematics, Dongguk University

**Abstract:** Several Euler type of Integral Representations with the special functions in the kernel, are derived for some hypergeometric Exton functions in this talk.(joint work with J. Choi and A. Hasanov ).

## The Hawaii conjecture and its generalizations

Mikhail Tyaglov

tyaglov@gmail.com

tyaglov@math.tu-berlin.de

Technische Universität Berlin, Institut für Mathematik

**Abstract:** The object of the talk is to present a proof of the following statement which was conjectured by T.Craven, G.Csordas and W.Smith [1] and is known now as the Hawaii conjecture.

**Hawaii Conjecture.** If a real polynomial  $p$  has exactly  $2m$  nonreal zeros, counting multiplicities, then its logarithmic derivative has at most  $2m$  critical points, counting multiplicities.

We show that this conjecture is true not only for real polynomials but also for all real entire functions of genus  $1^*$  with finitely many nonreal zeros. A real entire function  $f$  is of genus  $1^*$  if  $f(z) = e^{-az^2}g(z)$ , where  $a \geq 0$  and  $g$  is a real polynomial or a real entire function of genus 0 or 1.

In the talk, we will also discuss some open problems concerning generalizations of the Hawaii conjecture and the Newton inequalities.



1. T. Craven, G. Csordas, W. Smith, The zeros of derivatives of entire functions and the Polya-Wiman conjecture, *Ann. of Math.*,125, No. 2, 405-431 (1987).

## Covering and distortion theorems for complex polynomials

Vladimir Dubinin

dubinin@iam.dvo.ru

Institute of Applied Mathematics. Far East Branch Russian Academy of Sciences. 7, Radio St., 690041, Vladivostok, Russia

**Abstract:** We discuss the geometric function theory approach to the proofs of the covering and distortion theorems for complex polynomials. First, some applications of the classical Schwarz lemma are mentioned. We refine Dochev's result about the images of the ellipses under polynomial mappings [1] and Pawłowski's theorem on the zeros of a polynomial and its derivative [2]. Second, we consider an analog of the Koebe-Bieberbach One-quarter theorem for the univalent functions. This result follows from Hayman's symmetrization theorem [3] and from the dissymmetrization of the domains [4]. Also, the finite increment theorems for the polynomials are proved [5]. In particular, the constant in the dual mean value problem [6] is made more precise. Finally, we solve the weakened version of Sheil-Small's problems [7, ch.10.3]. Our proof is based on symmetrization methods [4].

### references:

- [1] Dochev K. Some extremal properties of polynomials, *Soviet Math. Dokl.* 4 (1964), 1704-1706.
- [2] Pawłowski P. On the zeros of a polynomial and its derivatives, *Trans. Amer. Math. Soc.* 350:11 (1998), 4461-4472.
- [3] Hayman W.K. *Multivalent functions*, Second ed., Cambridge Univ. Press, Cambridge, 1994.
- [4] Dubinin V.N. Symmetrization in the geometric theory of functions of a complex variable, *Russian Math. Surveys.* 49:1 (1994), 1-79.
- [5] Dubinin V.N. On the finite increment theorem for complex polynomials, *Math. Notes.* (2010), to appear.
- [6] Dubinin V. and Sugawa T. Dual mean value problem for complex polynomials, *Proc. Japan Acad. Ser. A.* 85:9 (2009), 135-137.
- [7] Sheil-Small T. *Complex polynomials*, Cambridge Univ. Press, Cambridge, 2002.

# On some potential like operators in the theory of elliptic equations

Vladimir Vasilyev  
vladimir.b.vasilyev@gmail.com  
Bryansk State University ,Bezhitskaya 14, Bryansk 241036, Russia

**Abstract:** For solvability studying pseudo differential equation

$$(Au)(x) = f(x), \quad x \in K, \quad (1)$$

where  $A$  is an elliptic pseudo differential operator with the symbol  $A(\xi)$ ,  $\xi \in \mathbf{R}^2$ ,  $K$  is a quadrant in a plane  $\mathbf{R}^2$ , the author has introduced the wave factorization concept [?] for a symbol. It has permitted to describe solvability situations for equation (1) in the Sobolev-Slobodetskii spaces  $H^s(K)$ . Key point in this question took the index  $\varkappa$  of wave factorization. This index determined the type of boundary value problem, which was well defined in this situation. So, particularly, if  $\varkappa - s = n + \delta$ ,  $n \in \mathbf{Z}$ ,  $n < 0$ ,  $|\delta| < 1/2$ , the equation (1) was over-determined, and for a well-posedness for a such problem one needs additional unknown potentials or solvability conditions for the right hand side  $f$ . The case  $n = -1$  was considered earlier [?]. Here one considers more general situations.

**References:** V.B. Vasil'ev, *Wave factorization of elliptic symbols: theory and applications. Introduction to the theory of boundary value problems in non-smooth domains*, Kluwer Academic Publishers, Dordrecht-Boston-London, 2000.

# Solving boundary and eigenvalue problems for operators with variable coefficients using pseudoanalytic functions

Vladislav Kravchenko  
vkravche@yahoo.com  
Center for Research and Advanced Studies of the National Polytechnic Institute

**Abstract:** The aim of this talk is to present new solution techniques for boundary and eigenvalue problems for second order elliptic operators with variable coefficients. The results are based on recent developments in pseudoanalytic function theory [1] and involve construction of complete systems of solutions of the equation as well as the associated reproducing kernels. Numerical examples are considered.

[1] V. V. Kravchenko, *Applied pseudoanalytic function theory*, Birkhauser (2009).

# System identification by using adaptive orthogonal system

Wen Mi

ya87405@umac.mo

Department of Mathematics, FST, University of Macau

**Abstract:** We presents two adaptive algorithms for frequency domain identification. These algorithms are based on the rational orthogonal system (Takenaka-Malmquist system):

$$\mathcal{B}_k = \mathcal{B}_{\{\zeta_1, \zeta_2, \dots, \zeta_k\}}(z) = \frac{\sqrt{1 - |\zeta_k|^2}}{1 - \bar{\zeta}_k z} \prod_{l=1}^{k-1} \frac{z - \zeta_l}{1 - \bar{\zeta}_l z}, \quad k = 1, 2, \dots,$$

where  $\zeta_k \in \mathbb{D}$ ,  $\mathbb{D}$  is the unit disc. An adaptive decomposition algorithm based on this system was previously proposed for decomposing functions  $f \in H_2(\mathbb{D})$  in which a greedy sequence  $\{\zeta_l\}_{l=1}^{\infty}$  is obtained maximizing  $|\langle f, \mathcal{B}_{\{\zeta_1, \zeta_2, \dots, \zeta_k\}} \rangle|^2$  at each step. In the presented paper by showing that the conjugate sequence  $\{\bar{\zeta}_l\}_{l=1}^{\infty}$  is also a greedy sequence, we adopts the algorithm through necessary changes for system identification.

# Algebraic properties of solutions of algebraically defined differential equations

Yum-Tong Siu

siu@math.harvard.edu

Harvard University, USA

**Abstract:** Will discuss some recent developments stemming from algebraic properties of solutions of algebraically defined differential equations. The discussion will start with the historic 1778 paper of Euler, the 1873 paper of Schwarz, and the 1885 paper of Picard concerning what is now known as the equivariant variation of Hodge structure. It will end with the recent developments in the curvature properties of direct images of relative pluricanonical bundles and the techniques for the proof of the abundance conjecture.

# p-adic hyperbolicity and Z-integral points on varieties

Julie Tzu-Yueh Wang

jwang@math.sinica.edu.tw

Institute of Mathematics, Academia Sinica, Taipei, Taiwan

**Abstract:** Similar to the correspondence between classical Nevanlinna Theory and Diophantine approximation, we discuss a correspondence between p-adic analytic curves and Z-integral points on varieties. This is a joint work with Ta Thi Haoi An and Aaron Levin.

## Meromorphic maps and transcendence

Mingxi Wang  
wangmx@math.ethz.ch  
ETH, Zuerich

**Abstract:** We prove a generalization of the Schneider-Lang criterion to meromorphic functions on affine algebraic curves. We shall explain that it is closely related to the First Main theorem in Nevanlinna theory. This is joint work with Gisbert Wuestholz.

## A generalized necessary and sufficient condition for a Bedrosian identity and a generalized sampling theorem

Yi Wang  
ywang2@aum.edu  
Department of Mathematics, Auburn University Montgomery, P.O. Box 244023,  
Montgomery, AL, 36124-4023 USA

**Abstract:** We generalize the result of Cerejeiras, Chen and Kahler in this paper. Cerejeiras, Chen and Kahler provides a necessary and sufficient condition for the Bedrosian identity to hold for a class of mono-components based on a generalized sinc function emerging from the Blaschke product of order one. We extend the generalization to any inner function, including the Blaschke product of any order. Moreover, the sampling properties and orthogonality of the generalized sinc function and its Hilbert transform are identified and hence a generalized version of sampling theorem is presented.

## Algebraic degeneracy of non-Archimedean analytic maps to varieties omitting divisors with sufficiently many components

William Cherry  
wcherry@unt.edu  
University of North Texas

**Abstract:** I will discuss joint work with Julie Tzu-Yueh Wang and Ta Thi Hoai An on the algebraic degeneracy of non-Archimedean analytic maps from the affine line into algebraic varieties missing divisors with sufficiently many components. This is a non-Archimedean analog of work of Noguchi and Winkelmann in complex value distribution theory. In my talk I will give some motivation for studying non-Archimedean analogs of value distribution theory and mention some open problems..

## Decomposition of $k$ -monogenic functions in superspace

Yuan Hongfen  
yhf0609@163.com  
College of Mathematics and Information Science ,Hebei Normal University, China

**Abstract:** We give the decomposition of  $k$ -monogenic functions in superspace, which not only induces Amansi theorem, but also contacts with Fischer decomposition. Besides, Painleve theorem and Uniqueness theorem for monogenic functions in superspace are shown. Further, we deduce the two theorems for  $k$ -monogenic functions correspondingly from the decomposition.

## On quasiconformal Poisson extensions

Guowu Yao  
gwyao@math.tsinghua.edu.cn  
Department of Mathematical Sciences, Tsinghua University,  
Beijing 100084, P.R. China

**Abstract:** Let  $f$  be a Poisson extension of the self-homeomorphism of the unit circle (or the real axis) to the unit disk (or the half-plane). This talk presents some equivalent conditions for  $f$  to be quasiconformal.

# Multipliers and Cyclic Vectors in the Logarithmic $\alpha$ -Bloch Spaces

Ye Shanli

shanliye@fjnu.edu.cn

School of Mathematics and Computer Science, Fujian Normal University, Fuzhou  
350007, P.R. China

**Abstract:** In this paper we discuss the pointwise multiplier, the composition operator in the logarithmic  $\alpha$ -Bloch Spaces  $\mathcal{LB}^\alpha$  and the little logarithmic  $\alpha$ -Bloch Spaces  $\mathcal{LB}_0^\alpha$ , and cyclic vectors in the little logarithmic  $\alpha$ -Bloch Spaces  $\mathcal{LB}_0^\alpha$ . We obtain as follows. (i) A characterization of the pointwise multiplier  $M_\mu$  on  $\mathcal{LB}^\alpha$  and  $\mathcal{LB}_0^\alpha$  is obtained. (ii) Sufficient and necessary conditions of the composition operator  $C_\phi$  being bounded or compact on the  $\mathcal{LB}^\alpha$  and  $\mathcal{LB}_0^\alpha$ . (iii) If  $\alpha \geq 1$ ,  $|f(z)| \geq |g(z)|$  in the open unit disk and  $g$  is cyclic in  $\mathcal{LB}_0^\alpha$  then  $f$  is cyclic in  $\mathcal{LB}_0^\alpha$ . (iv) If  $0 < \alpha < 1$ , then  $f$  is cyclic in  $\mathcal{LB}_0^\alpha$  if and only if  $f$  has no zeros in the closed unit disc.

**Keywords:** Composition operator; Pointwise multipliers, Cyclic vectors, Weighted Bloch space.

## On values of modular forms at algebraic points

Yu Jing

yu@math.ntu.edu.tw

National Taiwan University

**Abstract:** We study the distributions of values of arithmetic modular forms taken at algebraic points. Distribution here is in the sense of algebraic dependence versus independence. Reporting what one can prove in the positive characteristic case which correspond to open problems in the classical case.

## Some results of meromorphic function in an angular domain

Qingcai Zhang

qingcaizhang@yahoo.com.cn

Department of Mathematics, School of Information,  
Renmin University of China Beijing, 100872, China

**Abstract:** We study the problems of meromorphic functions sharing values in an angular domain and obtain some uniqueness results which are the improvement of the well-known five-value theorem and four-value theorem of Nevanlinna. And obtain

some results about Borel directions with shared values.

## Sector Properties of Entire Functions and Laguerre Multiplier Sequences

Matthew N. Chasse  
chasse@math.hawaii.edu  
2153A Atherton Rd. Honolulu, HI 96822, USA

**Abstract:** Let  $S$  be a convex sector in the complex plane, with vertex at the origin. Let  $P$  be the set of polynomials, all of whose zeros lie in  $S$ . We use some composition theorems for sectors to study the class of linear operators which map  $P$  into  $P$ , and provide an extension of a result of Obreshkoff. A new result is presented concerning the problem of characterizing multiplier sequences which can be interpolated by meromorphic functions.

## On certain Riemann boundary value problems in Clifford analysis

Zhongxiang Zhang, Klaus Gürlebeck  
zhangzx9@126.com, guerlebe@fossi.uni-weimar.de  
School of Mathematics and Statistics, Wuhan University  
Ins. Mathematik/physik, Bauhaus-University Weimar,  
Coudraystr. 13, D-99423 Weimar, Germany

**Abstract:** In this paper, we firstly give the Plemelj formula for functions with parameter by following the classical method. Then by using the higher order Cauchy integral representation formulas, some properties for harmonic functions and bi-harmonic functions are presented, for example, the mean value theorem, the Painleve theorem etc.. Finally we consider the Riemann boundary value problems for harmonic functions and bi-harmonic functions in Clifford analysis, the solutions are given in an explicit way.

**Key Words:** Riemann boundary value problem, bi-harmonic function, Plemelj formula.