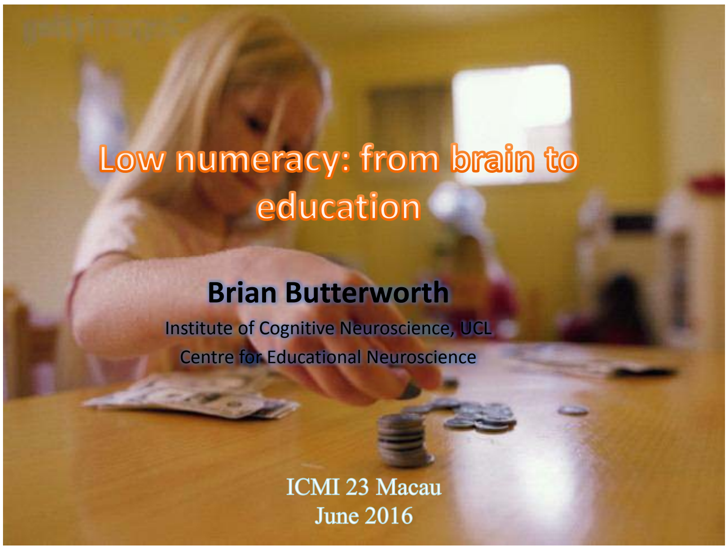


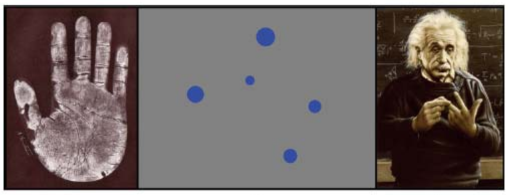
Neglected topics at ICMI23

- The cognitive 'starter kit' for learning arithmetic
- Individual differences
- Digital technologies to help learners

Whole numbers as cardinal numbers 'numerosities'

- Back to basics

Arithmetic is about sets and numerosities



Arithmetic is about sets and their numerosities

- Sets
  - A set has definite number of members ("numerosity" of a set)
  - Adding or taking away a member changes the numerosity
  - Other transformations conserve numerosity
  - Numerical order can be defined in terms of sets and subsets
  - Arithmetical operations can be defined in terms of operations on sets
- We learn about counting and arithmetic using sets
  - And about the meaning of number terms

"God created the integers. All else is the work of man." Kronecker

- Not a testable hypothesis

The ‘number module’ hypothesis

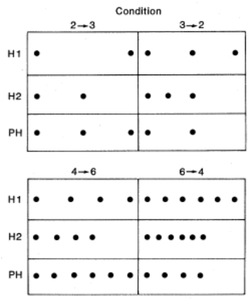
- This is testable
- It states that humans inherit a number module – a **core capacity** for processing numerosities
- There are ancestral versions in other species
  - Chimps, mammals large and small , reptiles, fish and insects
  - This system is adaptive in foraging, mating, fight or flight, navigation
- Like any inherited organ or capacity the process of inheritance can go wrong

Arithmetical development starter kit

- Domain-general cognitive capacities – including sufficient working memory capacity, reasoning abilities, etc.
- **Number module**
- Individual differences in the number module are reflected in differences in arithmetical development

The number module in human infants

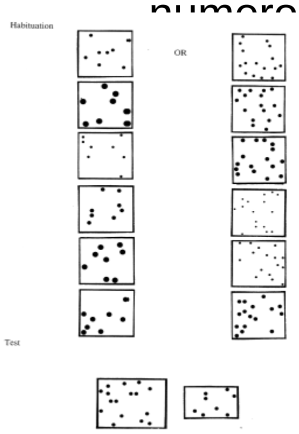
Primate B: human infants



Participants: 72 infants 22 wks av
Stimuli: see picture
Method: Habituation with H1 or H2 until 50% decrement in looking time averaged over three successive trials. PH (post habituation) using same criterion.
Result: infants look longer in PH for 3 vs 2, but not 6 vs 4
Implication: “subitizing underlies infants’ performance in the small number conditions”

Starkey & Cooper, 1980, *Science*

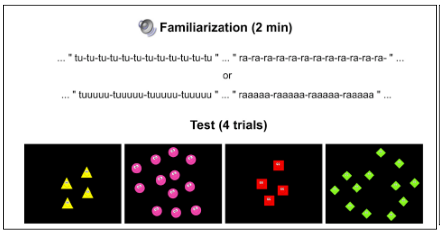
Approximations of larger numerosities

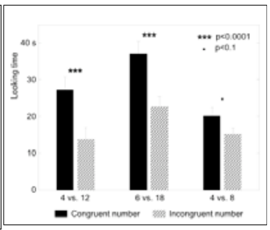


Participants:16 6mth olds
Stimuli: non-numerosity dimensions - dot size & arrangement, luminance, density - randomly varied during habituation
Method: Measure looking time during habituation, and then during test.
Results: Infants look longer at 8 vs 16, but not 8 vs 12.
Implication: Infants cannot be using non numerical dimensions, but can make discriminations if the ratio is large enough (2:1, but not 3:2) True representations of number used, but not object-tracking system; “infants depend on a mechanism for representing approximate but not exact numerosity”

Xu & Spelke, 2000

Newborns represent abstract number

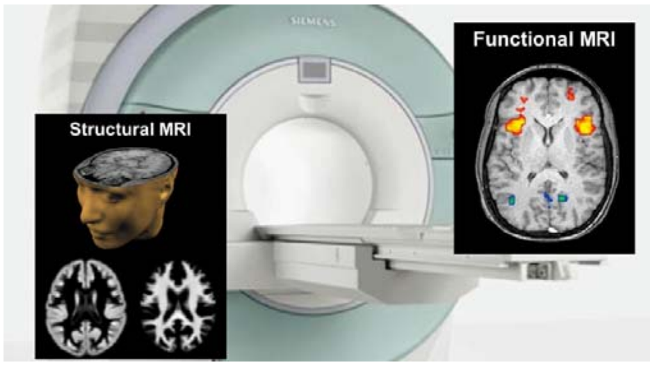




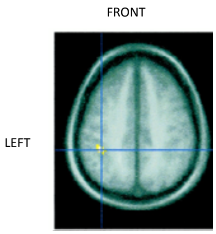
Izard et al, 2009, *PNAS*

Specialised brain network for arithmetic and numerosity processing

Magnetic Resonance Imaging (MRI)

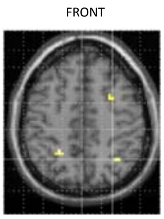


What's the difference between these fMRI images of the brain?



Isaacs et al, 2001, *Brain*

Structural



Castelli et al, 2006, *PNAS*

Functional

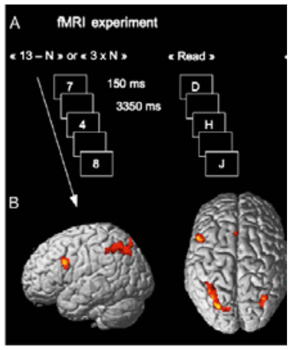
These are always pictures of a **comparison**

Special brain network for arithmetic

Read $1\ \square\ 0$
Retrieve 6×4
Compute 37×14

RETRIEVE+COMPUTE vs READ

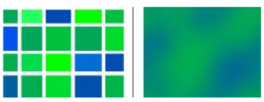
Zago et al, 2001, *Neuroimage*



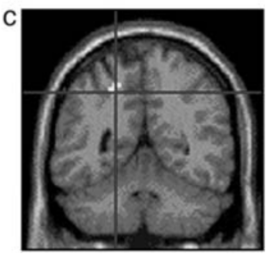
SUBTRACTION+MULTIPLICATION vs READ

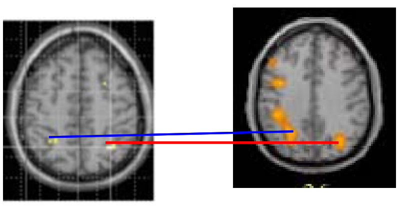
Andres et al, 2011, *Neuroimage*

Numerosity processing part of calculation network



Task in the brain scanner: more green or more blue?





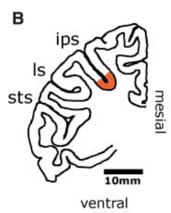
Numerosity processing in the Intraparietal Sulcus

Castelli et al, *PNAS*, 2006

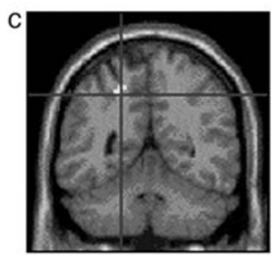
The calculation network

Zago et al, *Neuroimage*, 2001

Homology in monkey cortex



Monkey cortex
Nieder, Diester, & Tudusciuc, *Science*, 2006



Human cortex
Castelli, Glaser, & Butterworth, *PNAS*, 2006

Neurological patients show these areas are necessary

Patient	Lesion	Language	Reasoning	Number skils
CG Cipolotti, Denes & Butterworth (*Brain*, 1991)	**Left parietal lobe damage** Rest of brain OK	Intact	Intact	Can count to 4; can't calculate with numbers >4
IH Cappelletti, Kopelman & Butterworth (*Cognitive Neuropsychology*, 2002)	**Left parietal lobe OK** Rest of brain degenerating	Speech: severe Comprehension of single words at chance	Untestable	Single and multi-digit calculation almost flawless

Why is any of this relevant to maths ed?

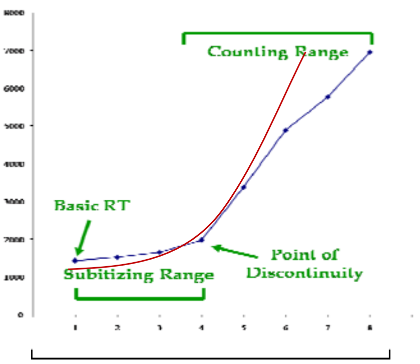
Inherited core capacity for processing numerosity of sets

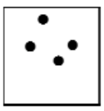
- Numerosity processing underlies the development of arithmetic and there are some simple tests for individual differences in it, which will help to identify very early which children are going to have difficulty

Testing individual differences

- A simple test of numerosity processing capacity

Enumerating sets: the 'size effect'





Data from Butterworth et al, 1999

Individual differences in the number module

Numerosity processing in the development of arithmetic

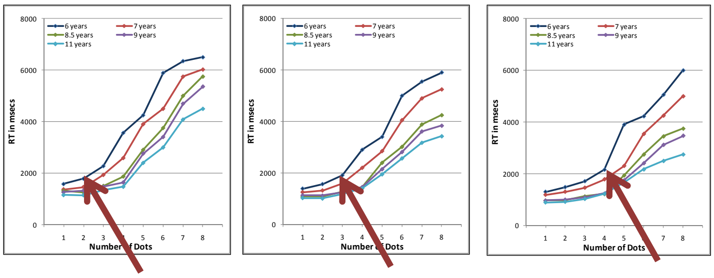
- Melbourne longitudinal study
- 159 children from 5½ to 11, tested 7 times, over 20 cognitive tests per time;
- item-timed calculation, dot enumeration & number comparison (adjusted for simple RT) at each time, Raven's Coloured Progressive Matrices

Reeve et al, 2012, *J Experimental Psychology: General*

Cluster analysis

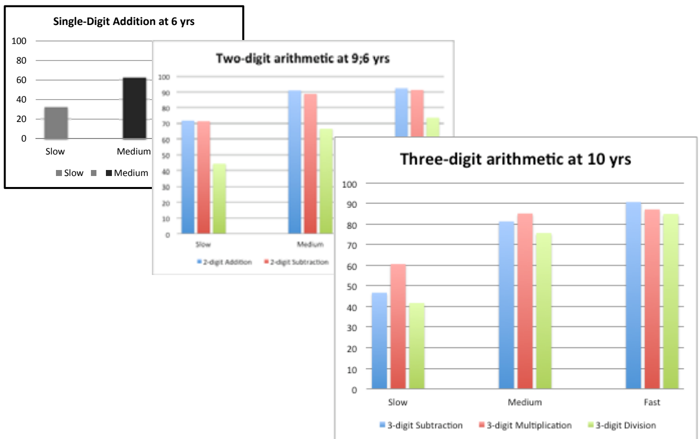
- Children improve with age. How to assess whether they improve relative to peers?
- Criterion or cluster analysis?
- Is a learner always in the same cluster?
  - Cluster based on parameters of the dot enumeration measure, adjusted for basic RT
  - At each age, there were exactly three clusters, which we labelled Slow, Medium and Fast
  - Ordinal correlations show that cluster membership stable

Enumeration times by age & cluster



SLOW MEDIUM FAST

Cluster at K predicts arithmetic to age 10 yrs



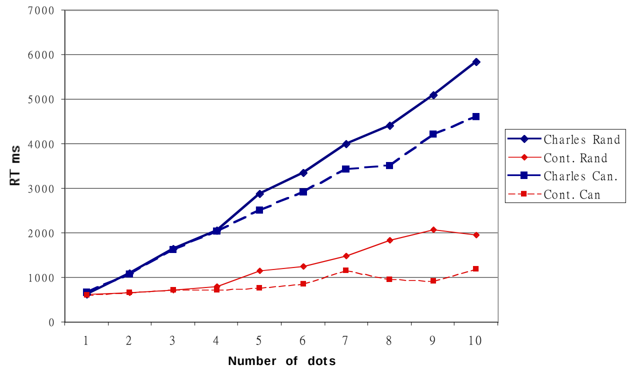
Arithmetical development starter kit

- Domain-general cognitive capacities – inc. sufficient working memory capacity, reasoning, et.
- **Number module** is key

What happens when the number module is defective?

Core deficit persists into adulthood

Charles 30 y.o. vs controls

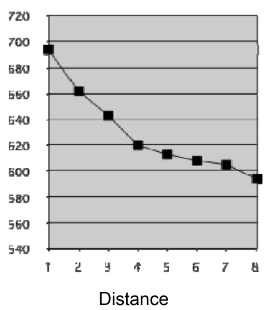


What is dyscalculia?

- It's not just being very bad at maths?
- In the same way that dyslexia is not just being very bad at reading.
- It can be quite specific – that is, the dyscalculic can be average or very good at every other school subject

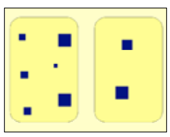
More tests of numerosity processing

Comparing numerosities: the 'distance effect'



Data from Butterworth et al, 1999

Non-Symbolic



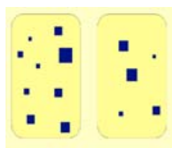
Symbolic

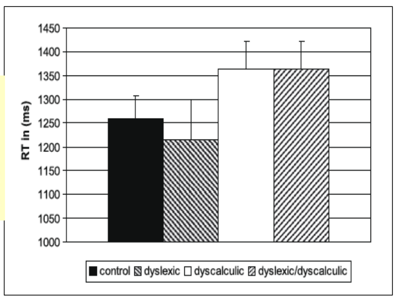
3	8

3	8

Taller number?

Dot comparison





Landerl et al, 2009, *J Exp Child Psychology*

Case JB

- 9years 7 months old, Right Handed male. Normal in all school subjects except maths, which he finds impossible. Not dyslexic. Counts up to 20 slowly. Can read and write numbers up to 3 digits.
- Failed Britsh Abilities Scale arithmetic questions
- Knows that 4 is the next number after 3 (has a sense of ordinality)
- Believes that 3+1 is 5
- Dot enumeration: 1-3 accurate. Guesses larger numbers
- Cannot say which of two numbers is bigger

What it's like for the dyscalculic learner (9yr olds)

Moderator: *How does it make people feel in a maths lesson when they lose track?*
Child 1: *Horrible.*
Moderator: *Horrible? Why's that?*
Child 1: *I don't know.*
Child 3 (whispers): *He does know.*
Moderator: *Just a guess.*
Child 1: *You feel stupid.*

Focus group study (lowest ability group)
Bevan & Butterworth, 2007

What it's like for the dyscalculic learner

Child 5: *It makes me feel left out, sometimes.*
Child 2: *Yeah.*
Child 5: *When I like - when I don't know something, I wish that I was like a clever person and I blame it on myself –*
Child 4: *I would cry and I wish I was at home with my mum and it would be - I won't have to do any maths -*

What it's like for their teacher

- KP: … they kind of have a block up, as soon as we get to starting to do it. Then they seem to just kind of phase out.
- ML1: In a class of thirty I've got six. You've got a lot of problems. And when I'm on my own, I don't – I really feel very guilty that I'm not giving them the attention they need.
- JL: …lots of times they're trying to cover it up … they'd rather be told off for being naughty than being told off that they're thick.

Prevalence of dyscalculia

- This is important for policy: how much additional support will society need to provide?

Numerosity processing in a prevalence study of arithmetical disorders and dyscalculia

- Havana study: 11562 children in Havana Centro; 1966 tested individually with dot enumeration and timed arithmetic.

- Reigosa Crespo, Valdés Sosa, Butterworth, et al, 2012, *Developmental Psychology*

Prevalence of dyscalculia:Testing for core deficit

- **Calculation disorder** based on timed arithmetic – **9.4%**
  - **No gender difference**
- **Dyscalculic** (calculation disorder PLUS poor numerosity processing as measured by timed dot enumeration) – **3.4%**
  - **Male:Female numerosity processing 2.4:1**

Inherited?

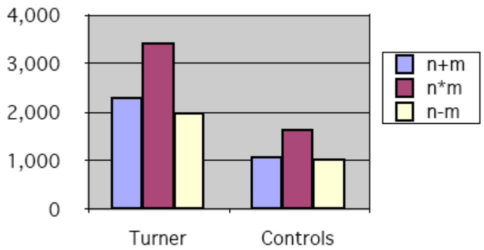
Heritability of numerical abilities

Numerosity processing disabilities more common in boys
- 2.4:1 (Reigosa Crespo et al 2012 *Developmental Psychology*)

Twin studies
- If one twin has very low numeracy, then 58% of monozygotic co-twins and 39% of dizygotic co-twins also very low numeracy (Alarcon et al, 1997, *J Learning Disabilities*)
- Also in Ranpura et al. (2013 *Trends in Neuroscience & Education*, under review)
- One-third of genetic variance in 7 year olds specific to mathematics (Kovas et al, 2007, *Monograph of the Society for Research in Child Development*)

X chromosome disorders
- Damage to the X chromosome can lead to parietal lobe abnormalities with numeracy particularly affected. Numerosity processing always affected.
  - Turner's Syndrome. (e.g. Bruandet et al., 2004; Butterworth et al, 1999; Molko et al, 2004)
  - Fragile X (Semenza, 2005);
  - Klinefelter (and other extra X conditions). (Brioschi et al, 2005)

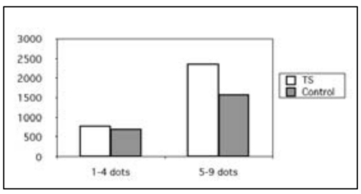
Calculation

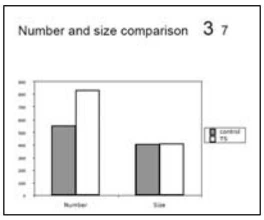


Butterworth et al 1999 *Brain & Language*

Calculation abilities

7 TSs with normal+ IQ and normal language abilities





Butterworth et al 1999 *Brain & Language*

Heritability of numerosity processing ability AND calculation

104 MZ, 56 DZ Mean Age 11.8 yrs

40 behavioural tests; Structural scans for all

Exclusions: gestational age < 32 weeks; Cognitive test < 3SD; Motion blurring on MRI

Zygosity assessed using molecular genetic methods

Ranpura et al 2013 *Trends in Neuroscience & Education, under review*

Factors for the whole sample

Factor 1 (24% of total variance) Number processing: WOND-NO, Addition (IE), Subtraction(IE), Multiplication (IE), Dot enumeration

Factor 2 (19%) Intelligence: IQ measures, Vocabulary, and working memory (span)

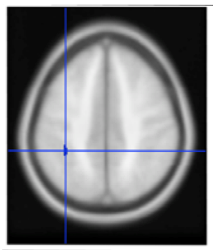
Factor 3 (12%) Speed: Processing speed, Performance IQ

Factor 4 (9%) Fingers: finger sequencing, tapping, hand-position imitation preferred hand, non-preferred hand

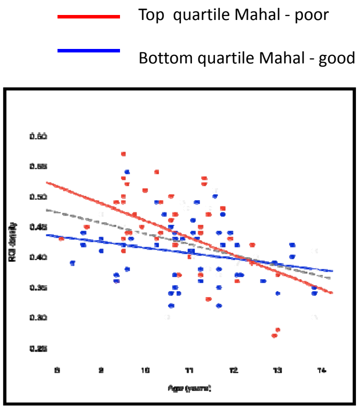
Mahalanobis distance to identify outliers from sample mean on basis of numerical dimension of Factor 1.

Highly significant predictor of dyscalculia as defined by significant discrepancy between FSIQ and WOND-NO (Isaacs at al, 2001) .

Grey matter and age



Significant difference in grey matter density here



Heritability of cognitive measures

Based on a comparison of MZ and DZ twin pairs in the usual way

	h^2 Genetic factor	c^2 Shared environment	e^2 Unique environment
Timed addition	0.54	0.28	0.17
Timed subtraction	0.44	0.38	0.18
Timed multiplication	0.55	0.31	0.15
Dot enumeration	0.47	0.15	0.38

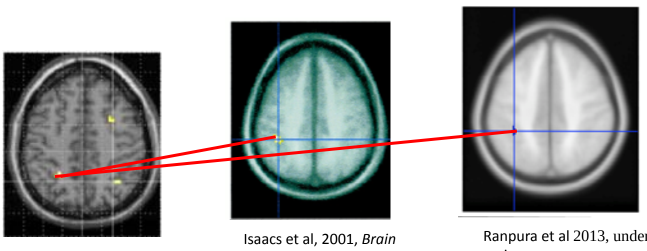
Heritability of numerosity processing ability AND calculation

Cross Twin Cross Trait genetic correlations for Dot Enumeration:

Is the relationship between dot enumerations and calculation closer for MZ (identical twins) than DZ (fraternal twins)

	$h_1h_2r_G$
Addition Efficiency	0.54
Subtraction Efficiency	0.28
Multiplication Efficiency	0.36
Finger Sequencing	0.25

Abnormal structure in numerosity network in low numeracy



Brain areas for numerosity Processing
Castelli et al, 2006, *PNAS*

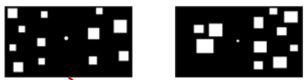
Isaacs et al, 2001, *Brain*

Ranpura et al 2013, under review

	h_2	c_2	e_2
ROI	0.28	0.34	0.38

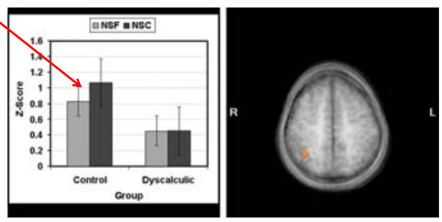
	$h_1h_2rG_2$
ROI & Number factor	0.34

Here are differences in activation



NSC – close   NSF - far

12 year olds: dyscalculics and matched controls



Price et al, 2007, *Current Biology*

Interim conclusions

- To identify dyscalculics is very simple. Just assess core abilities such as dot enumeration (rather than just standardized arithmetic tests)
- Deficiencies in core abilities may have a heritable component
- This won't be true for all atypical learners, and it doesn't mean that appropriate teaching won't help
- Interventions should target strengthening these core abilities (not just more of the same

Numerosity processing as a target for intervention

From neuroscience to education

From diagnosis to educational remedy?

No clear logical pathway

→ use established pedagogical principles

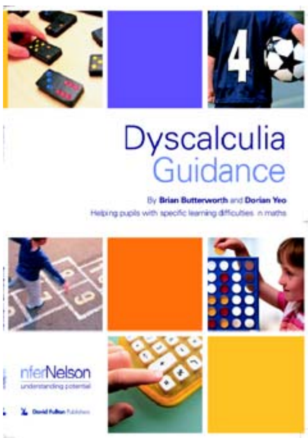
→ use ideas from best practitioners

→ use technology to capture and test ideas

Pedagogic principles

- Constructionism –construct action to achieve goal (Papert)
- Informational feedback (Dayan)
- Concept learning through contrasting instances (Marton)
- Reinforcement of learned associations (Gagné)
- Build new tasks on what has been learned (Ausubel)
- Direct attention to salient properties (Frith)
- Adapt each task to be just challenging enough (Vygotsky)
- Generalise concepts through attention to invariant properties (Marton)

Ideas from best practitioner



Adaptive technologies based on cognitive neuroscience

Number Race
(Räsänen, Wilson, Dehaene, etc)
http://sourceforge.org

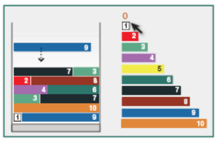
Calcularis (Kucian et al)
http://www.dybuster.com/calcularis

Meister Cody (Kuhn et al)
www.meistercody.com

Adaptive technologies

Number Bonds, Dots2Track, etc (Laurillard et al)
http://number-sense.co.uk

Smartphone app: "Number Bonds by Thinkout"



Example intervention 1

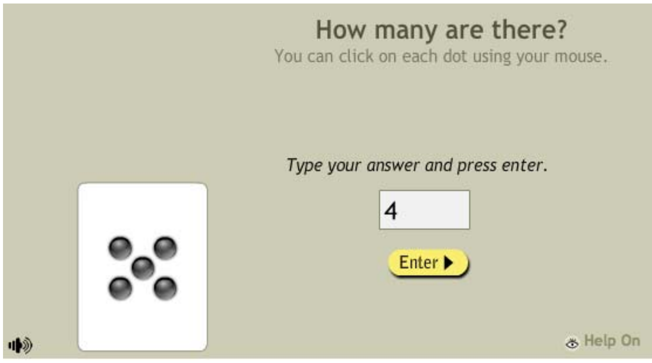
'Dots-to-track'

- Uses regular dot patterns for 1 to 10
- Links patterns to representation on number line and to written digit and to sound of digit

Aims to help the learner

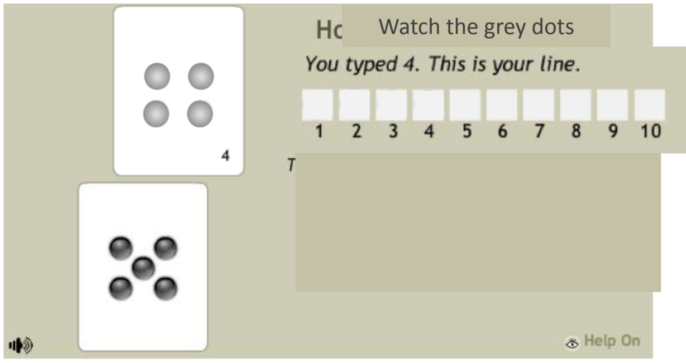
- *recognise* rather *than* count dot patterns
- see regular patterns within random collections
- using learning through practice, not

Learner constructs the answer, rather than selects it



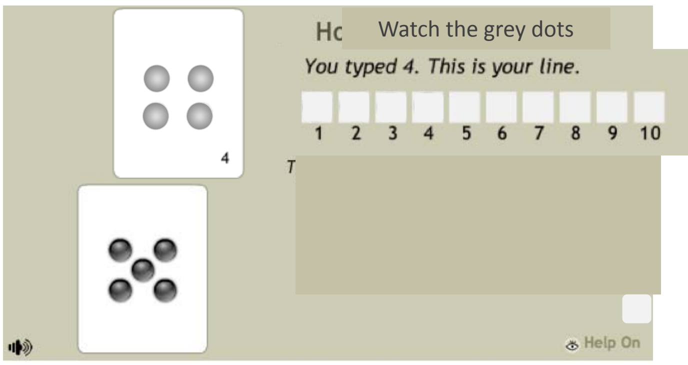
Pedagogic principle: constructionism

Feedback shows the effect of their answer as the corresponding pattern



Pedagogic principle: informational feedback

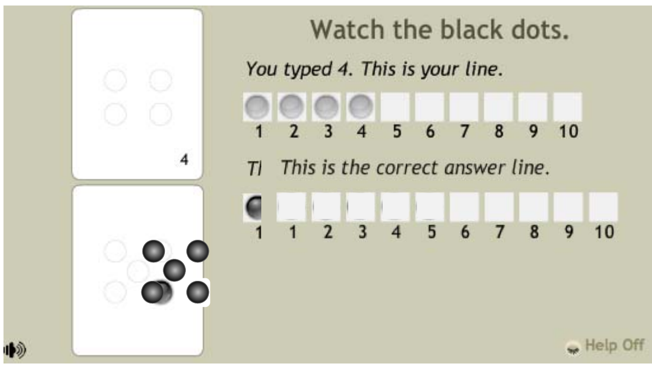
And counts (with audio) their pattern onto the number line



Pedagogic principle: concept learning through contrasting instances

2

Then counts (with audio) the target pattern onto the number line

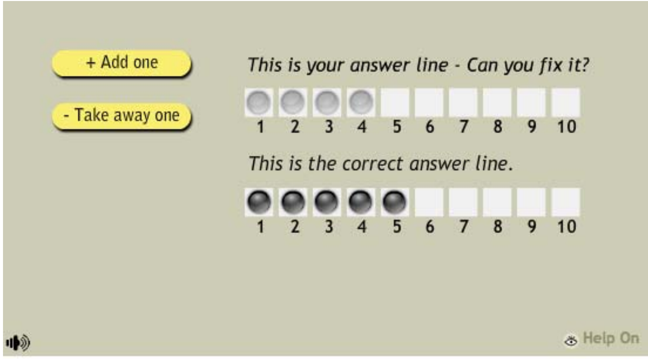


Pedagogic principle: concept learning through contrasting instances

Slide 66

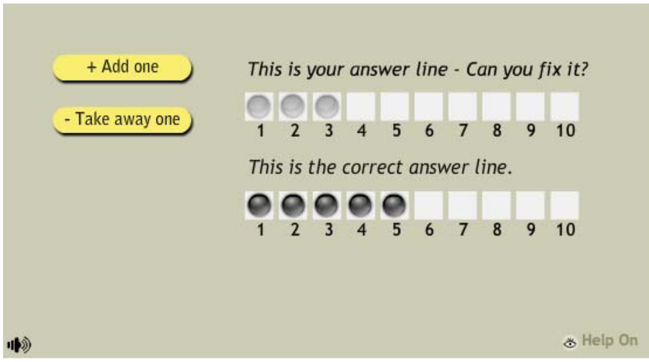
2 Diana Laurillard, 20/05/2012

The learner is then asked to construct
the correct answer on their line



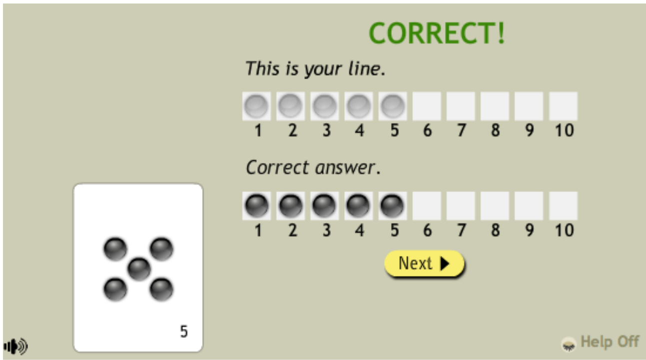
Pedagogic principle: constructionism

Again the feedback shows
the effect of a wrong answer



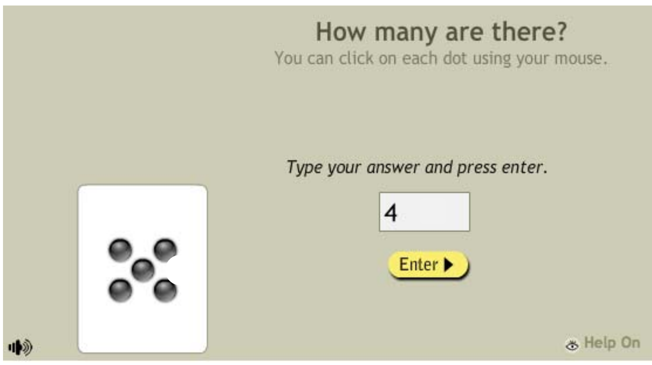
Pedagogic principle: constructionism

The correct answer matches
the pattern to digit and number line



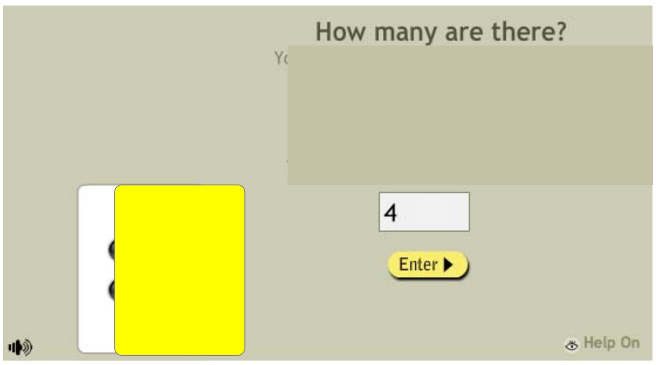
Pedagogic principle: reinforce associated representations

The next task selected should
use what has already been learned



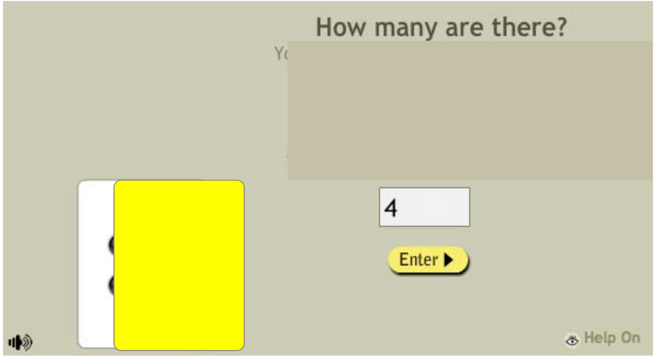
Pedagogic principle: reinforce and build on what has been learned

The next stage encourages *recognition* of the pattern,
rather than *counting*, by timing the display



Pedagogy: focus attention on salience of numerosity rather than sequence

If the learner fails the task it adapts by displaying for
1 sec longer until they can do it, then begins to speed up



Pedagogy: adapt the level of the task to being just challenging enough

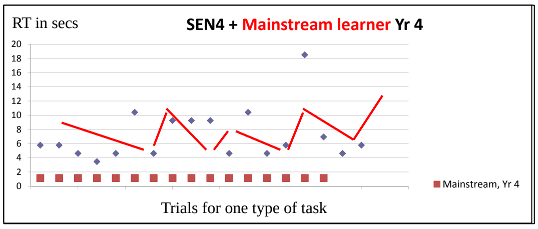
The next stage is to generalise to random collections

Pedagogy: generalise concept of numerosity from patterns to collections

Successive tasks encourage the learner
to see known patterns embedded

Pedagogy: build the concept of the numerosity of a set and its subsets

Adaptation (to 4 learners)



SEN group, Yr 4
- As recognition RTs improve higher numbers are introduced, so RTs slow down then improve, creating saw-tooth pattern of RTs
- Learners *improve their recognition*, but need more time to be as fast as mainstream learners

Mainstream learner, Yr 4
- All patterns are *recognised* within 2 secs

Progress to recognition of pattern

One SEN pupil, Year 4
Time on task: 17.6 minutes over 5 Dots-to-Track enumeration tasks

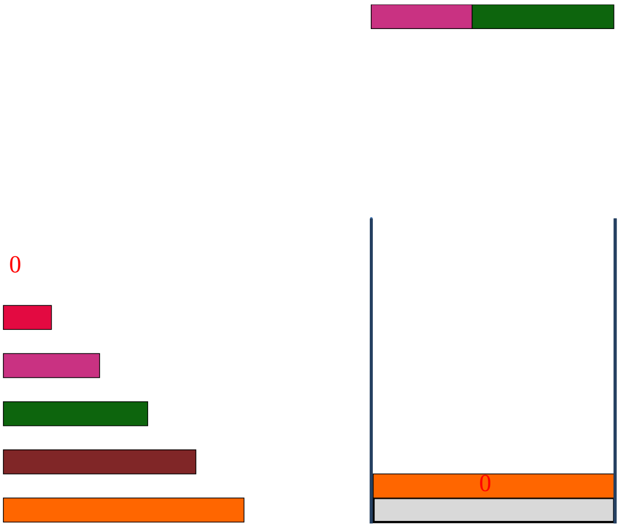
Tasks 1-3 untimed — Task 4 displayed 1s — Task 5 displayed 3s

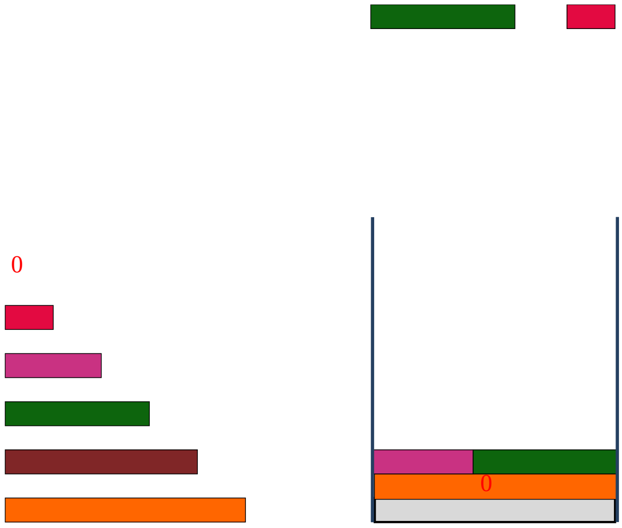
	Task 1	Task 2	Task 3	Task 1s	Task 3s
Errors	1	0	2	5	2
Mean RT	4.9	4.3	3.8	4.4	3.8

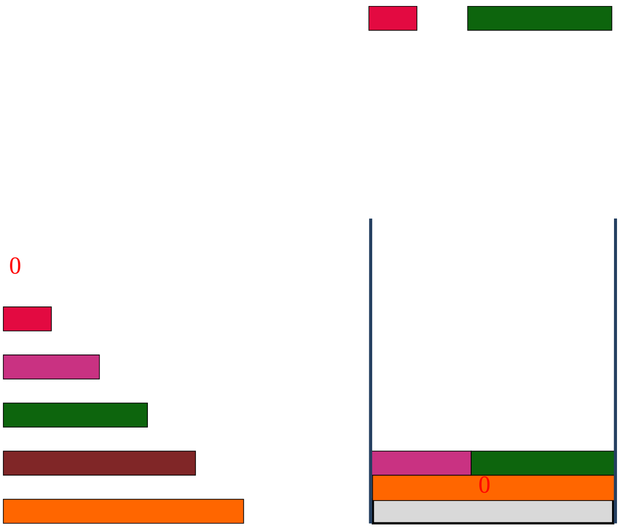
Few errors on untimed tasks, improving RTs
Task timed at 1s to promote recognition of pattern → increases errors and RTs
Next task changes display time to 3s → errors reduce and RTs improve
→ Further trials are needed, reducing time of display until recognition
→ Program must introduce timed display more gradually

Number Bonds to 10

- Level 1 Stage 1
- Even numbers, Length, Colour

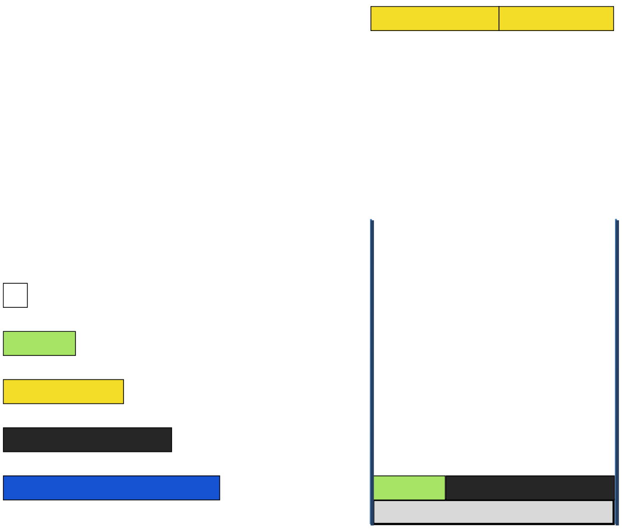






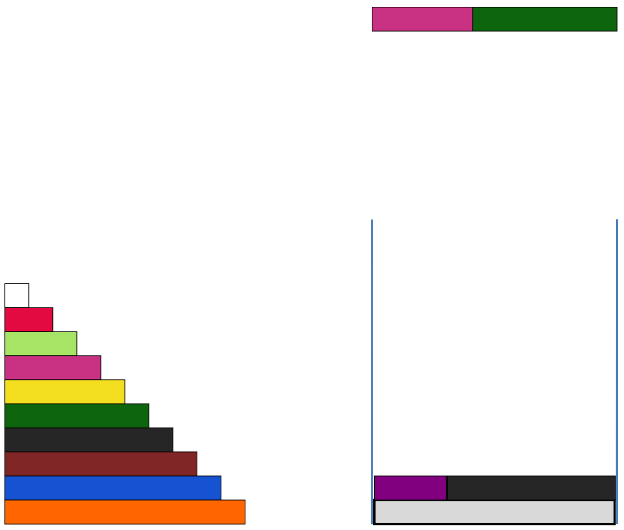
Level 1 Stage 2

- Odd, Length, Colour



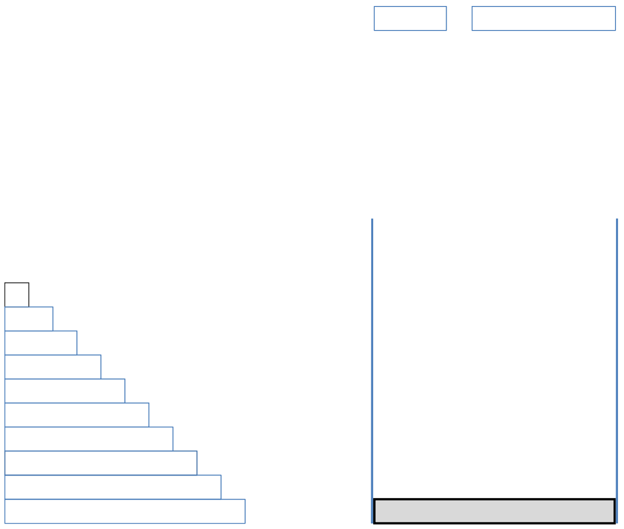
Level 1 Stage 3

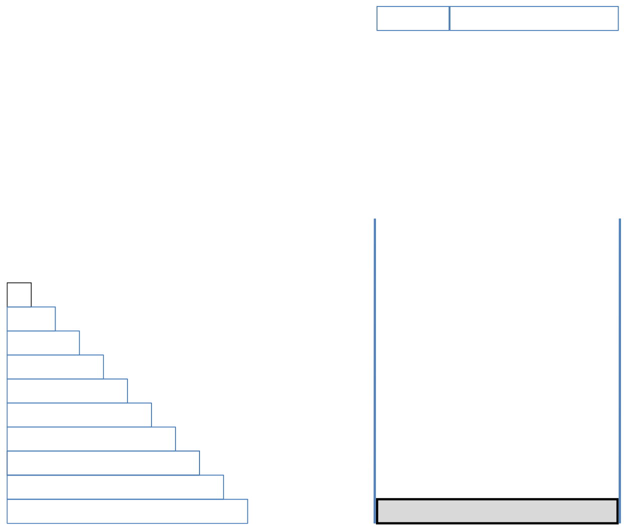
- All, Length, Colour



Level 2 Stage 3

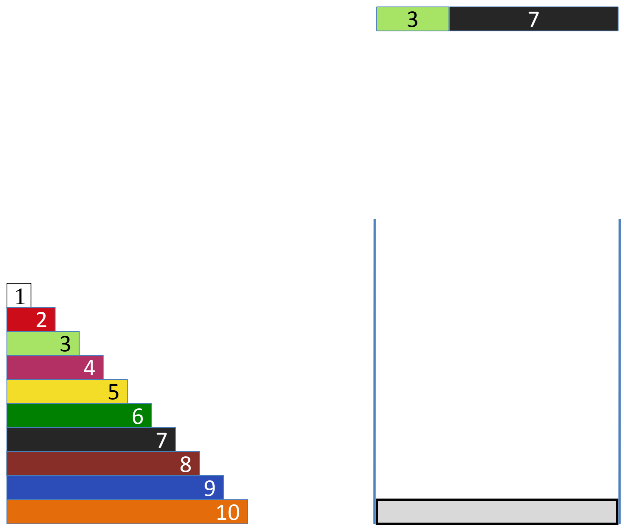
- All, Length

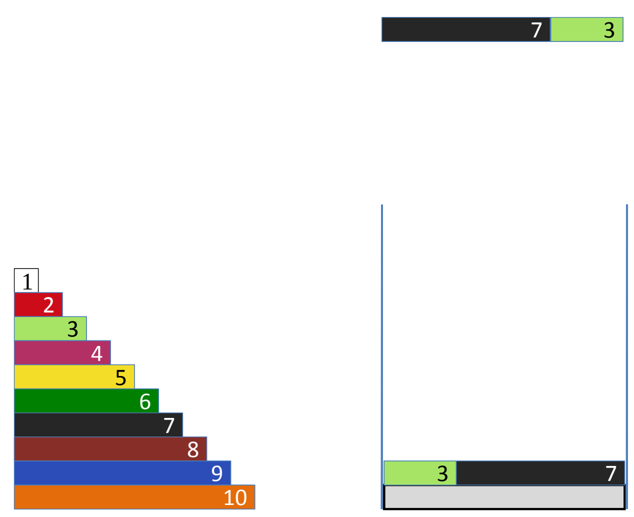


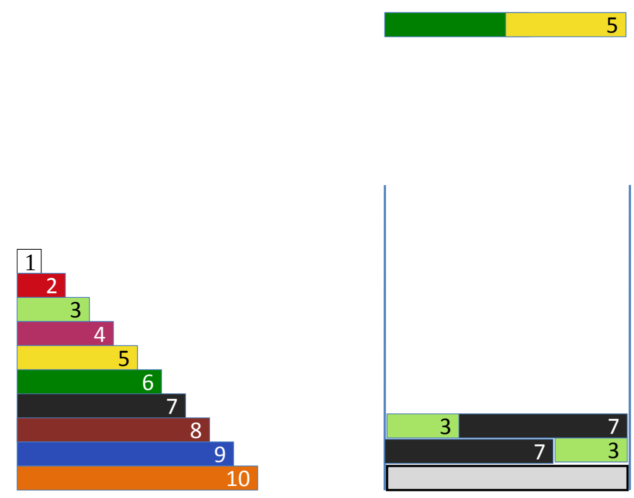


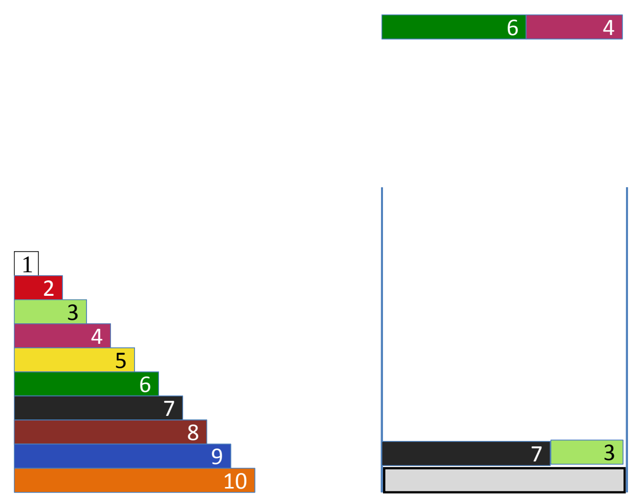
Level 3, Stage 3

- All, Length, Colour, Digits
- (Stages 1 and 2 at each Level use just Even and Odd numbers, respectively)



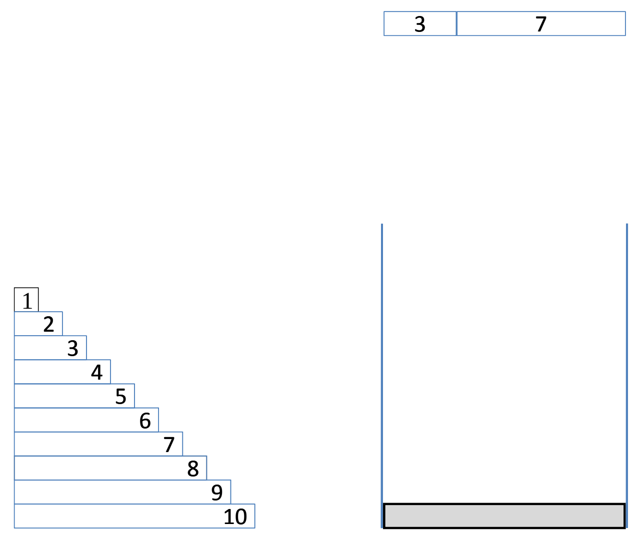


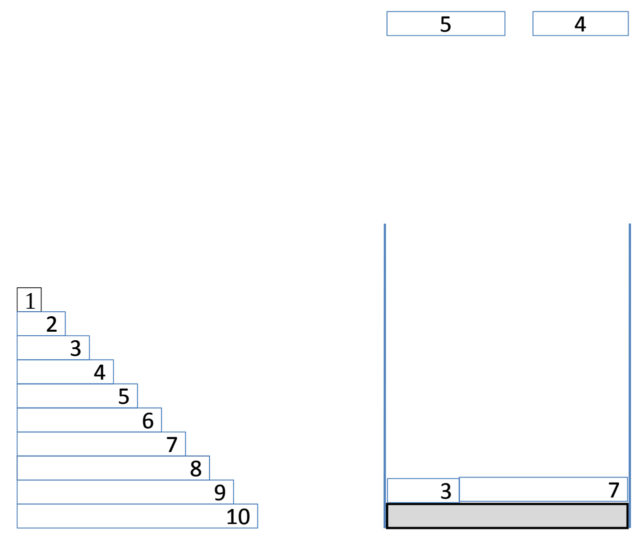




Level 4, Stage 3

- Length, Digits





Level 5, Stage 3

- Digits

3 5

1
2
3
4
5
6
7
8
9

3 6

1
2
3
4
5
6
7
8
9

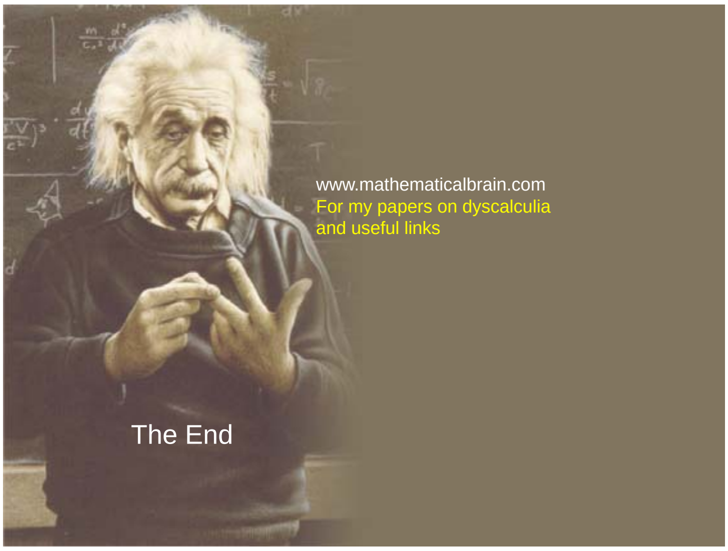
3 7

1
2
3
4
5
6
7
8
9

Summary

- Most of us inherit a number module, which has a specific neural representation
- Dyscalculics have a defective number module and an abnormal neural network in the numerosity-processing region
- Intervention should be designed for individual learners
  - To strengthen numerosity processing
  - Adaptive to their progress
  - With informative feedback and opportunities to re-construct answers
- Digital technologies are useful for this
  - They can be adaptive to individual progress
  - They can collect data on progress for teachers, parents and learners
  - They enable learners to practice in private

Butterworth, Varma & Laurillard 2011 *Science*



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  - Hassan Baajour

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- EU Marie Curie Erasmus Programme
- Emerson House

Useful references

- Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: From brain to education. *Science, 332*, 1049-1053. doi: 10.1126/science.1201536
- Butterworth, B., & Yeo, D. (2004). *Dyscalculia Guidance*. London: nferNelson.
- Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental Dyscalculia and Basic Numerical Capacities: A Study of 8-9 Year Old Students. *Cognition, 93*, 99-125.
- Reeve, R., Reynolds, F., Humberstone, J., & Butterworth, B. (2012). Stability and Change in Markers of Core Numerical Competencies. *Journal of Experimental Psychology: General, 141*(4), 649-666.