HPM 2020
Macao, China
Plenary Speaker Information (with theme)
[Revised: 18 January 2020]

Michael N. Fried
Position/Title: Associate Professor in the Program for Science and Technology Education
Organization: Ben Gurion University of the Negev, Beer-Sheva, ISRAEL
Theme 1: Theoretical and/or conceptual frameworks for integrating history in mathematics education

Title:
History of Mathematics as a Way of Relating to Mathematics of the Past: The Case of Edmond Halley and Apollonius

Abstract:
The history of mathematics is a discipline which aims to give an account of the mathematics of the past by trying to understand the texts and artifacts passed on to us. But within mathematical texts we often, if not typically, find mathematicians engaging with texts of their *own* past, implicitly or explicitly. Understanding, how mathematicians refer to and use past mathematics, in short, their relationship with the mathematics of the past, is crucial in giving an account of their mathematical activity *tout court*. But reflecting on how others related to the past in the past we must consider our own relationship to mathematics of the past which is part, at least, of our own mathematical activity. It is in this way that studying the history of mathematics becomes, for students studying mathematics, a form of self-reflection. This self-reflection on how we stand towards our predecessors, I want to argue, is a central justification for a role of history of mathematics in mathematics education. In the lecture, I will try to bring out this idea of history of mathematics as a way relating to the past by looking at a historical instance, namely, Edmond Halley's reconstruction of lost works by Apollonius. Reconstruction of lost texts is only one avenue for relating to mathematics of the past, but it one is revealing and suggestive of other ways of relating to mathematics of the past, not the least because an account of any text, lost or not, is always to some degree a reconstruction of it.

Marc Moyon
Position/Title: Maître de Conférences (Lecturer)
Organization: University of Limoges, FRANCE
Theme 2: History and epistemology in students' and teachers' mathematics education: Classroom experiments and teaching materials

Title:
"I would like to introduce history in my mathematics lessons but I do not know how to do it!"

Abstract:
The objective of my talk is to present several elements—between an inventory of the current situation and proposals for work with teachers in initial and in-service training—on the introduction of a historical perspective into mathematics teaching. Defended for decades within the IREM (Institut de recherche sur l'enseignement des mathématiques) in the French educational landscape, the history of mathematics is now officially recognized in the French curriculum, especially as a means of differentiation. Is it nevertheless sufficient to allow each teacher to initiate a reflection necessary for the success of such a project, to integrate an approach allowing such an introduction? In my talk, I will first rely on a survey conducted with mathematics teachers (for pupils aged 10 to 18) about this introduction. I will then focus on French mathematics schoolbooks in order to question their effectiveness as tools for the introduction of a historical perspective. Finally, I will discuss various plans that we have put in place to best meet the needs of teachers, describing our objectives and modalities of action in various institutional frameworks.

Mary Flagg
Position/Title: Associate Professor
Organization: University of St. Thomas, Houston, Texas, USA
Theme 3: Original sources in the classroom and their educational effects

Title:
Using Original Sources in the Classroom to Enrich the Mathematical Learning Experience

Abstract:
History without context may not be enough to help students understand that new mathematics may arise from struggling with a difficult problem. Students may assume that all formulas were created in the polished form found in their textbook. Without context, students never experience the struggle to embrace a new idea. I want to help my students discover that they have the potential to do new mathematics. I introduce this possibility by giving my students the opportunity to discover mathematics from the pen of its creator. I have been privileged to participate in the TRIUMPHS research grant (funded by the National Science Foundation in the United States): TRansforming Instruction in Undergraduate Mathematics via Primary Historical Sources. TRIUMPHS Primary Source Projects are curriculum modules for standard undergraduate mathematics courses that introduce important concepts through guided reading of excerpts from the original sources on the subject. I have written two Primary Source Projects: a linear algebra project introducing elimination from the *Jiuzhang Suanshu* (*Nine Chapters on the Mathematical Art*) and a project on the Euclidean algorithm for finding the greatest common divisor, using both Euclid's *Elements* and the algorithm from *The Nine Chapters on the Mathematical Art*. Original sources are challenging, presenting daunting problems of language, archaic notation, and unfamiliar terminology. However, guided reading of an original source gives the students the opportunity to see the early development of an idea. I will share how using TRIUMPHS Primary Source Projects, both mine and others, is changing my understanding of undergraduate education, inspiring more active pedagogy in my classroom, and benefitting my students.

Man Keung Siu
Position/Title: Professor
Organization: The University of Hong Kong, Hong Kong SAR, CHINA
Theme 4: Mathematics and its relation to science, technology, and the arts: Historical issues and interdisciplinary teaching and learning

Title:
Mathematical World (or Worlds?) in the Context of HPM

Abstract:
Without engaging in a detailed analysis of categorization of separate areas nor going into a deeper philosophical discussion we usually hold the conception that there is one single mathematical world, which is different from, though closely related to, the real world we live in. Experience in the mathematical world can appear so foreign to daily life that many people feel turned off by this world. Even though many people are aware of the utility and power of mathematics they do not see any relevance of the subject to them. In both the historical and pedagogical aspects it may be worthwhile to look at the issue in a pluralistic way from different angles. Among others there is, for instance, a world of school mathematics, a world of tertiary mathematics, a world of daily life mathematics, a world of recreational mathematics, a world of proof and proving, a world of mathematics in science and technology, and a world of mathematics in humanities. In fact, the history of mathematics affords some means to mirror these different worlds in the context of HPM to offer a fuller view of the subject, rather than like what is depicted in the ancient Indian fable of the proverbial elephant felt by the six blind men. Is mathematics a useful science? A vibrant science? An amusing science? A rigorous science? A heuristic science? An experimental science? A humane science? Or even, is mathematics a subject in science or in arts? This lecture will attempt to examine this issue through examples gleaned from the history of mathematics.

Johanna Pejlare
Position/Title: Senior Lecturer
Organization: Chalmers University of Technology and the University of Gothenburg, SWEDEN
Theme 6: Topics in the history of mathematics education

Title: Algebra in Swedish Mathematics Textbooks During the Era of Great Power

Abstract:
During the end of the 16th century, Swedish mathematics was almost 100 years behind in development, compared to France, Italy, and England. During the 17th century Swedish mathematicians, while travelling in Europe, began to learn from foreign mathematicians, and brought this knowledge to Sweden. In the early 18th century, Swedish mathematics had become as advanced as in most other countries in Europe. Even though there has been an increased interest in the study of the history of Swedish mathematics education, the history of Swedish mathematics and mathematics education from this period is still relatively unexplored. The present paper focuses on the 17th century in Sweden, a period often referred to as the *Swedish Empire*, or the *Era of Great Power*, due to the exercised territorial control over much of the Baltic region during this period of time. The aim of this talk is to provide an understanding of mathematics education during this crucial period in Swedish history. Of particular interest is the utilization of algebra in textbooks by a variety of authors during this period, as well as the investigation of the possible influence upon them from mathematicians from the European continent and from England. We will consider examples from various levels of education: school, university, as well as education of mining engineers and prospective officers.

The beginning of the Swedish Empire is usually symbolized as the reign of King Gustavus Adolphus, who ascended the throne in 1611, half a century after the King Gustav Vasa laid the foundations of the Swedish state. Only a few years earlier did it become possible to teach mathematics at the cathedral schools, as long as the teaching did not have a negative influence on other subjects. From this period, we find the first mathematical text in Swedish, a handwritten manuscript on arithmetic by Hans Larsson Rizanesander (1574–1646). In this manuscript, Rizanesander includes the rule of three, which is a tool utilized to verbalize the process of solving problems based on proportions, and thus can be considered as a rhetorical algebra.

During the turbulent times of the reformation in the 16th century, the Uppsala University was closed, but in 1595 the university reopened, and in the early 17th century activity there flourished. Also, new universities were established (and conquered) as the Empire expanded. Nevertheless, the 17th century was also a time of oppositions at the university. In the early 17th century, Petrus Ramus' (1515–1572) controversial ideas questioning the Aristotelian theories that were then dominant in the academic world had a great impact on the formation of the university. Martinus Erici Gestrinius (1594–1648) was the first Swedish mathematician who had

an influence on mathematical development, when, in 1637, he contributed a commented edition on Euclid’s *Elements*. Of particular interest is that Gestrinius associates some of the geometrical propositions with quadratic equations and in this way for the first time presented cossic algebra to the Swedish audience.

The Swedish Empire ends with the death of King Karl XII in 1718 when territories are lost. The year 1718 also constitutes the “year of algebra”: two Swedish textbooks on algebra were published in this year, one by Emanuel Swedenborg (1653–1735) and one by Anders Gabriel Duhre (c. 1680–1739). Common to these two textbooks is that they both present algebra using Descartes’ notation. Moreover, both address not only students at the university level, but also a wider audience in need of applying mathematics for the benefits of society.

Luis Saraiva
Position/Title: Associate Professor
Organization: University of Lisbon, PORTUGAL
Theme 7: History of Mathematics in China and Eastern Asia

Title: Matteo Ricci and the Introduction of Euclid's *Elements* in China

Abstract:
The Society of Jesus had the purpose to convert China to the Christian faith. To this aim it was thought that the centralized structure of the Chinese society would make this task simpler. While most Jesuit missionaries worked in the provinces to convert ordinary people, a few of them resided in the capital where they developed a network of acquaintances among high officials that could be used to protect the missions from the occasional hostility of the local administration.

To this aim, the Society's members were forbidden to explicitly discuss religion or faith, but they would do their best to be recognized as excellent scholars, and above all as excellent astronomers. Astronomy was known to be a main factor in Chinese society: the calendar was an imperial monopoly, and an accurate calendar was a sign of legitimacy for the reigning dynasty.

The first Jesuits to work in China as missionaries believed that from the recognition of their excellence in the sciences and of the superiority of Western astronomy and mathematics over the ones then practiced in China, Chinese literati could be convinced that this was due to a superior religion, and would embrace the Christian faith. Matteo Ricci (1552–1610) is regarded as the founder of the Jesuit mission in China. He was respected by Chinese scholars and officials, due to not only his mastery of the Chinese language and literati culture, but also to his scientific knowledge and ability to manipulate scientific instruments that later were offered to highly placed Chinese politicians. He was the first Westerner to be invited to the Forbidden City.

Ricci thought that the translation of Euclid's *Elements* to Chinese, and its transmission to Chinese literati would serve well the Society's aims in China. For him, the understanding of Euclid's *Elements* was a step on the way to the successful transmission of the Jesuits' religious teachings to the Chinese.

In my talk I will summarize the contents of Euclid's *Elements,* its importance in the Western World, and I will refer the two first quality Latin translations of the *Elements* in the 16th century, respectively by Federico Commandino (1509–1575) and Christophorus Clavius (1538–1612). We will summarize the work developed by Matteo Ricci in China, with his Chinese students Qu Taisu (1549–1612), Wang Kentang (1549–1613), Li Zhizao (1565–1630), and Xu Guangqi (1562–1633), mainly concerning their acquaintance with Western Science and the translation and circulation of Euclid's *Elements.* The first six chapters were published for the first time in 1607

with the title *Jihe yuanben* (which can be rendered as '*Elements of Quantity*'), with a second edition in 1611, after Matteo Ricci's death, edited by Xu Guangqi with the Jesuits Sabatino de Ursis (1575–1620) and Diego de Pantoja (1571–1618).

I will also analyze why there was only a limited appropriation of Euclid's *Elements* among Chinese scholars versed in mathematics.

Yiwen ZHU, Panel Coordinator
Position/Title: Associate Professor
Organization: Sun Yat-sen University, The city of Guangzhou, Guangdong Province, CHINA
Themes 5 and 7: Cultures and mathematics fruitfully interwoven and History of Mathematics in China and Eastern Asia

Panel members:
Assistant Professor Shuyuan PAN, CHINA
Professor Shirong GUO, CHINA
Professor Alexei Volkov, TAIWAN, CHINA

Title:
History of Mathematics Education in China: Its Features, Influences, and Modern Values

Abstract:
This panel aims at exploring the history of mathematics education in East Asia from four aspects:

(1) the main characteristics of mathematics in ancient and medieval China, such as *The Nine Chapters on Mathematical Procedures* (*Jiuzhang suanshu* 九章算術), and their educational value and significance, which serves as a basis for the panel by introducing the feature of mathematics in ancient China and studying its value in modern mathematics education;

(2) the transmission and teaching of mathematics during the Han (202 BCE–220 CE) and the Tang (618–907) dynasties in China, which introduces the feature of mathematics education and its connection to Confucianism in the ancient and medieval China;

(3) the mathematics introduced from Europe into China and its reception in the 16th–18th centuries, which re-examines that as a continuous process of mathematics education in a cross-cultural context;

(4) the mathematics examination in 19th century Vietnam, which studies how mathematics education system in ancient China influenced other Asian counties.

The first speaker is Shirong GUO. As we know, Wu Wentsun (1919–2017), a Chinese mathematician and historian of mathematics, studied and compared the main characteristics of both Western and Chinese mathematics, and came to the conclusion that Western mathematics represented by Euclid's *Elements* and Chinese (or Eastern) mathematics represented by the *Nine Chapters on Mathematical Procedures* are of different styles and characteristics. He pointed out that Western mathematics emphasized deductive reasoning more and therefore can be regarded as axiomatic mathematics; meanwhile, Eastern mathematics paid more attention to algorithms and therefore mechanical mathematics. He also summarized the characteristics of algorithmization, constructivity, and mechanization of ancient Chinese mathematics. In addition to the characteristics that Wu mentioned, what are the other important features of traditional Chinese mathematics? This is a question worthy of further study.

In this part, I will discuss and analyze the main characteristics of Chinese mathematics from several aspects, such as the macro-concept of Chinese mathematics, the Chinese mathematical reasoning system and thought different from the West, the mathematical models in Chinese mathematics, some characteristics of traditional Chinese algorithms, the practical strategies of ancient Chinese mathematicians, and so on, as well as their educational value and significance. It should be emphasized that the fact that Eastern and Western mathematics have their different characteristics does not mean that Eastern and Western mathematics are totally opposite. Wu Wentsun's emphasis on algorithm in Eastern mathematics does not mean that there is no algorithm in Western mathematics, nor does his emphasis on deductive reasoning in Western mathematics mean that Chinese mathematics is weak in proof and deduction. It is about which aspects they pay more attention to. Therefore, when we study the main characteristics of Chinese traditional mathematics, we should also pay attention to some important aspects of both sides.

The second speaker is Yiwen ZHU. Previous research, in particular Li Yan's 李儼 (1892–1963) study, has investigated the mathematics education system from the Han dynasty to the Qing dynasty, i.e., the 2nd century BCE to the 19th century. In recent years, I have studied the mathematical practice in Confucian canonical texts in the early Tang dynasty, i.e., seventh century, which was different from the one in the mathematical texts, such as *The Nine Chapters on Mathematical Procedures* and *Mathematical Procedures of the Five Canonical Texts* (*Wujing suanshu* 五經算術). These results invite me to rethink the transmission and teaching (e.g., including pedagogy) of mathematics during the Han and Tang dynasties. In this part, I will explore this issue in two steps. First, I will analyze the relationship between Zheng Xuan 鄭玄 (127–200) and mathematics. As a master of Confucianism in the Han dynasty, Zheng introduced and used *The Nine Chapters on Mathematical Procedures* in his Confucian study. Second, I will argue that Zheng Xuan's mathematical commentaries produced two consequences. On one hand, these commentaries created room for developing mathematics in Confucian study for later generations, such as Kong Yingda 孔穎達 (574–648) and Jia Gongyan 賈公彥 (fl. 650–655); on the other hand, Zheng Xuan's idea to bring mathematics and Confucianism together influenced scholars' mathematical commentarial practice, such as Liu Hui 劉徽 (fl. 263) and Li Chunfeng 李淳風 (602–670). In the early Tang dynasty, Kong's and Jia's subcommentaries as well as Liu's commentary and Li's subcommentary were all used as part of textbooks in different schools of the Imperial University. Therefore, although mathematics was always an independent discipline, the transmission and teaching of mathematics had a close relation to Confucian study during the Han and Tang dynasties.

The third speaker is Shuyuan PAN. From the late 16th century on, “Western learning” (*xixue* 西學), namely, knowledge imported from Europe, gradually raised considerable attention among Chinese literati and provided intellectual resources in scholarly mathematical activities in China. It thus became a key factor for the remarkable transformation that mathematics underwent at that time. Undoubtedly, the mathematical knowledge introduced by the European Jesuits who came to China was closely connected with their educational background, that is to say, the mathematics training and relevant textbooks and references prescribed by the curricula which were implemented in colleges of the Society of Jesus. In the late Ming dynasty, this knowledge was introduced basically through Jesuits and their Chinese collaborators’ translation, which substantially performed as a series of teaching-learning interactions between them. However, during the Kangxi reign (1662–1722), in the early Qing dynasty, even before translation was carried out, mathematical knowledge was first introduced when the emperor received instruction by Jesuits as his imperial tutors. Either the Chinese collaborators on translation, or Kangxi emperor, naturally transformed themselves into tutors, to impart their newly acquired knowledge to others, while in most occasions, the texts translated in the late Ming and early Qing dynasties played a role of guidelines for studying the most novel, essential, and useful knowledge, and these texts were further rewritten or adapted into new mathematical works, to allow beginners to understand more easily. This part aims at yielding an understanding of the mathematical knowledge introduced from Europe into China and its reception in the 16th–18th centuries—a continuous process of mathematics education in a cross-cultural context.

The fourth speaker is Alexei Volkov. The system of mathematics instruction developed in the Mathematics College (*Suanxue* 算學, lit. “School of Computations”) that existed in China during the Sui (581–618), Tang (618–907), and Song (960–1279) dynasties became a model for similar educational institutions in Korea and Japan in the late first and early second millennium CE, while in Vietnam, Chinese-style state mathematics examinations were held from the 11th to the late 18th century. The extant sources contain short descriptions of the textbooks, instruction methods, and examination procedures adopted in these countries as well as information about the number of instructors and students. Unfortunately, there exist only brief descriptions of the examinations that took place in China, yet a model examination paper preserved in a Vietnamese mathematical treatise of 1820 allows us to reconstruct the examination procedure used in Vietnam.

The mathematics examinations were supposed to test the capability of an examinee to solve mathematical problems similar to those studied in the classroom. The process of generalization of solutions of the algorithms provided in mathematical textbooks can be studied in comparison with procedures of analogical reasoning found in Chinese scientific and philosophical texts. Such a generalization was mentioned, although briefly, in Chinese mathematical texts of the early first millennium CE, whose authors explicitly praised the capability of the learner to arrive at mathematically valid methods by analogy with the solutions of the problems provided in the textbooks. Moreover, mathematical textbooks were designed as collections of problems accompanied by algorithms for their solutions. Numerical parameters used by the compilers of the textbooks in sequences of generic problems often form particular structures. Their analysis, with the help of the modern concept of “didactical variable”, may offer interesting insights into the role of these parameters. For instance, the parameters found in a series of problems in a Vietnamese textbook arguably provided the learners with information concerning the possible generalization of the method supposed to be used for their solution.