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Optimal Capital Structure in Real Estate Investment: A Real Options Approach

Jyh-Bang Jou *

School of Economics and Finance, Massey University (Albany), Private Bag 102 904, North Shore City 0745, Auckland, New Zealand; Graduate Institute of National Development, National Taiwan University, Taiwan; Tel: 64-9-4140800, Ext. 9429, Fax: 64-9-4418156; E-mail: J.B.Jou@massey.ac.nz

Tan (Charlene) Lee

Department of Accounting and Finance, University of Auckland, Private Bag 92109; Owen G Glenn Building, 12 Grafton Road, Auckland, New Zealand; Tel: 64-9-373-7599,Ext.87190,Fax:64-9-373-7406;E-mail: tan.lee@auckland.ac.nz

This article employs a real options approach to investigate the determinants of an optimal capital structure in real estate investment. An investor has the option to delay the purchase of an income-producing property because the investor incurs sunk transaction costs and receives stochastic rental income. At the date of purchase, the investor also chooses a loan-to-value ratio, which balances the tax shield benefit against the cost of debt financing resulting from a higher borrowing rate and a lower rental income. An increase in the sunk cost or the risk of investment will not affect the financing decision, but will delay investment. An increase in the income tax rate or a decrease in the depreciation allowance will encourage borrowing and delay investment, while an increase in the penalty from borrowing, a decrease in the investor's required rate of return, or worse real estate performance through borrowing, will discourage borrowing and delay investment.

Keywords

Optimal Capital Structure; Real Estate Investment; Real Options; Transaction Costs

* *Corresponding author*

1. Introduction

This article investigates the investment and financing decisions of a real estate investor who considers the acquiring of an income-producing property through debt financing. The existing literature that theoretically investigates this issue includes Cannaday and Yang (1995, 1996), Gau and Wang (1990), and McDonald (1999).[1] All of these articles assume that the investor must purchase the property now or never. Our article significantly differs from them because we allow a property investor to have the option to delay the purchase.

This article, which belongs to the burgeoning literature that applies the real options approach to investment (Dixit and Pindyck, 1994), assumes that an investor chooses an optimal date to maximize the net expected present value of an income-generating property. The investor receives the stochastic income generated from the service of this property, but incurs sunk costs such as statutory costs and third-party charges (Brueggeman and Fisher, 2006). The interaction of these sunk costs and the stochastic cash flow confers on the investor an option value to delay the purchase of property. Consequently, the investor will not purchase the property until s/he is sufficiently satisfied with the current income generated by the service of the property. At the optimal date of purchasing, the investor also chooses a loan-to-value ratio that involves the tradeoff as follows: the investor enjoys tax deductible benefits from interest payments and capital depreciation, but will be charged a higher mortgage rate when the loan-to-value ratio increases, and may receive a lower income because the potential tenants may be willing to pay less as they realize that their landlord is highly indebted, and thus, highly susceptible to bankruptcy.[2]

Aside from allowing the investor to delay the purchase of property, our article also departs from the existing literature in the following respects. First, we assume that property value is endogenously determined, while Cannaday and Yang (1995; 1996), and McDonald (1999) assume that the purchase price and the net selling price of a property are both exogenously determined. Our assumption is more plausible because the evolution of the stochastic income generated by the service of a property determines the dynamic evolution of the property value. Second, we assume that debt financing may adversely affect real estate performance, such that investment and financing decisions interact with each other. As such, factors that characterize the evolution of the property

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[1] Ever since the seminal paper by Modigliani and Miller (1958), the determinants of corporate borrowing have been a heated topic in the corporate finance literature. See, for example, the survey paper by Harris and Raviv (1991), and Myers (2003). This topic has received little attention, however, in the real estate investment literature. See the discussions in Gau and Wang (1990) and Clauretie and Sirmans (2006, Chapter 15).

[2] This tradeoff significantly differs from that addressed in the finance literature, which also allows the tax advantages of borrowing, but considers the costs associated with either financial distress, or the conflict of interest between equity and debt holders. See, for example, Harris and Raviv (1991) and Myers (2003).

value will also affect the optimal level of debt. In contrast, Cannaday and Yang (1995; 1996), and McDonald (1999) abstract from this adverse effect, and thus, the investment and financing decisions are independent.[3]

The remaining sections are organized as follows. We first present the basic assumption of the model, and then derive the conditions for the investment timing and the loan-to-value ratio decided by an investor who indefinitely holds the property. We further consider the polar case where debt financing does not affect real estate performance, in which we derive some testable implications with regards to the determinants of debt financing. We then move to a more general case, in which debt financing adversely affects real estate performance, but find that most of our theoretical predictions become indefinite. Consequently, we employ plausible parameters in order to carry out some numerical comparative-statics testing in the following section. The last section concludes and offers suggestions for future research.

2. The Model

The model presented in this section extends that of McDonald (1999), which in turn, resembles that of Cannaday and Yang (1995, 1996). We depart from these studies by allowing non-negligible transaction costs, uncertainty in demand, as well as endogenously determined property values. Consider an investor who chooses an optimal date to purchase a commercial property, as well as the percentage of debt to finance the purchase. For ease of exposition, we consider the *interest only* mortgage loan. That is, we assume that the investor pays only interest in the holding period, and repays the principal when selling the property. Suppose that we start at time t_0. Then, the expected net present value of this investment is given by:

$$W(P(t_0),T,M) = E_{t_0}\Big[\int_T^{T+\bar{t}} ATCF(s)e^{-\rho(s-t_0)}ds + ATER(T+\bar{t})e^{-\rho(T+\bar{t}-t_0)} - (EI(T)+f)e^{-\rho(T-t_0)}\Big], \tag{1}$$

where T is the date on which the property is purchased; $ATCF(s)$ is the after-tax cash flow from the net operating income at time t; $ATER\,(T+\bar{t})$ is the after-tax equity reversion from selling the property at time $(T+\bar{t})$, where $\bar{t}$ is the holding period of the real estate investment; ρ is the equity investor's required rate of return; $EI\,(T)$ is the initial equity investment; and f is the transaction cost.

[3] Our article also differs from Gau and Wang (1990) and McDonald (1999), as these two studies allow for the cost associated with bankruptcy (Stiglitz, 1972) when the investor fails to pay off debt obligations. Our article, however, abstracts from this bankruptcy cost.

Each of the four terms in Equation (1) is defined as follows. The after-tax cash flow for the investor is written as:

$$ATCF\ (s) = (1-\tau)P(s) + \tau\delta H(T)/n - (1-\tau)MH\ (T)r(M\ ), \qquad (2)$$

where $T < s < T + \bar{t}$. The term τ is the (constant) income tax rate, δ is the proportion of the property that is depreciable capital (that is, not land), M is the loan-to-value ratio, n is the length of the depreciation period (39 years for commercial real estate in the U.S.)[4] $r(M)$ is the borrowing rate(where $r'(M)>0$), $H(T)$ is theinitial housing price at time T, and $P(s)$ is the net operating income generated from the property investment at time s, which follows the geometric Brownian motion as given by:

$$dP\,(s) = \mu(M\,)P(s)ds + \sigma P(s)dZ\,(s)\,, \qquad (3)$$

where $\mu(M)$ is the expected growth rate of $P(s)$, expressed as a non-positive function of M, σ is the instantaneous volatility of the growth rate, and $dZ(s)$ is an increment to a standard Wiener process.The housing price at time s,$H(s)$, is equal to the expected discounted present value of the net operating income, and is thus given by:

$$H(s) = \frac{P(s)}{\rho - \mu(M)}\,. \qquad (4)$$

Note that both the interest payments, $MH\ (T)\ \ r\ (M)$, and straight-line depreciation permitted under the tax code, $\delta H(T)/n$, are tax deductible. Upon investment, the property investor trades the tax shield benefits with two types of costs associated with debt financing when choosing a loan-to-value ratio. The first one, which is already addressed in Cannaday and Yang (1995, 1996), and McDonald (1999), indicates that the borrowing rate increases with the loan-to-value ratio, given that the investor is more likely to default when borrowing more. This positive relation is supported by the empirical study of Maris and Elayan (1990). The second one, which is novel to the literature, indicates that the expected growth rate of the net operating income is non-increasing with the loan-to-value ratio. This non-positive relation indicates that those who intend to rent commercial property may be willing to pay less when they realize that their landlord bears more debt and is thus, more susceptible to bankruptcy. This is plausible because those who rent in a commercial property, such as a shopping mall, would typically rather stay at the same place for a long period of time so that they can attract loyal customers.[5]

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[4] Note that depreciation is only allowed for the period of n even if the holding period $\bar{t}$ is longer than n.

[5] This assumption is also plausible for a competitive commercial property market where landlords who substantially borrow may need to lower the rent to attract potential tenants.

The after-tax equity reversion for the investor at time $T+\bar{t}$ is given by:[6]

$$ATER(T+\bar{t}) = H(T+\bar{t}) - MH(T) - \tau[H(T+\bar{t}) - H(T) + (\delta H(T)\bar{t}/n)], \quad (5)$$

where $H(T+\bar{t})$ is the selling price on date $T+\bar{t}$ at which the investor receives the payment. On this date, however, the investor must also pay off the loan balance, $MH(T)$, and pay taxes on the capital gain of $H(T+\bar{t}) - H(T) + (\delta H(T)\bar{t}/n)$ In addition, the amount of equity investment at time T is simply:

$$EI(T) = (1-M)H(T), \quad (6)$$

Finally, the transaction cost f is also novel to the literature. As Brueggeman and Fisher (2006, Chapter 4) suggest, a mortgage loan borrower, who is also the buyer of a property in our framework, incurs statutory costs and third-party charges. The former includes certain charges for legal requirements that pertain to the title transfer, recording of the deed, and other fees required by state and local law. The latter includes charges for services, such as legal fees, appraisals, surveys, past inspection, and title insurance. All of these changes, however, are unrecoverable after the property is purchased.[7]

Given that the investor incurs sunk costs in purchasing a property and that the property offers a stochastic cash flow in the future, the investor must thus wait for a sufficiently good state of nature to purchase the property, as the real options literature suggests (Dixit and Pindyck, 1994). Specifically, the investor simultaneously chooses a date T and a loan-to-value ratio M, so as to maximize the expected net present value of the investment. This problem is defined as:

$$W^*(P(t_0), t_0) = \underset{T,M}{Max}\ E_{t_0} W(P(t_0), t_0, T, M). \quad (7)$$

As indicated by Dixit and Pindyck (1994, p.139), when the net operating income is stochastic, we are unable to find a non-stochastic timing of investment. Instead, the investment rule takes the form where the investor will not purchase the property until the net operating income $P(t_0)$ reaches a critical level, denoted by P^*. At that instant, the investor will choose a loan-to-value ratio, denoted by M^*. Consequently, the initial purchase price of the property, $P^*/(\rho-\mu(M^*))$ as given by Equation (4), is endogenously determined. Our model thus significantly departs from that in the literature as we endogenize the value of the property.

$V_2(P(t), t)$ is denoted as the gross value of investment, i.e.,

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[6] Note that Equation (5) applies to the case in which $\bar{t} \le n$. When $n > \bar{t}$, we need to impose $n = \bar{t}$.

[7] Broadly speaking, the property buyer also incurs the transaction sunk cost such as opportunity cost in the form of time spent on negotiating with both the property seller and the mortgage loan provider.

$$V_2(P(t),t) = E_t \int_t^{t+\bar{t}} ATCF(s)e^{-\rho(s-t)}ds + ATER(T+\bar{t})e^{-\rho(T+\bar{t}-t)}, \qquad (8)$$

where $t \geq T$, and $V_1(P(t))$ is denoted as the investor's option value from waiting in the region where $P(t_0) < P^*$. The investor's option value is time-independent, i.e., $\partial V_1(\cdot)/\partial t = 0$, because the investor has some leeway in choosing the timing of investment rather than being forced to purchase the property during a finite period of time. By applying Ito's lemma, $V_1(P(t))$ satisfies the ordinary differential equation given by:

$$\frac{\sigma^2}{2}P(t)^2\frac{d^2V_1(P(t))}{dP(t)^2} + \mu(M)P(t)\frac{dV_1(P(t))}{dP(t)} = \rho V_1(P(t)), \qquad (9)$$

By contrast, if $P(t_0) \geq P^*$ and $t \geq t_0$, then the investment is made, and thus, $V_2(P(t), t)$ satisfies the partial differential equation given by:

$$\frac{\sigma^2}{2}P(t)^2\frac{\partial^2 V_2(P(t),t)}{\partial P(t)^2} + \mu(M)P(t)\frac{\partial V_2(P(t),t)}{\partial P(t)} + \frac{\partial V_2(P(t),t)}{\partial t} + \qquad (10)$$
$$(1-\tau)P(t) + \frac{\tau\delta}{n}\frac{P^*}{(\rho-\mu(M))} - (1-\tau)M\frac{P^*}{(\rho-\mu(M))}r(M) = \rho V_2(P(t),t)$$

The boundary condition is given by:

$$V_2(P(T+\bar{t}), T+\bar{t}) = ATER(T+\bar{t}). \qquad (11)$$

Equation (10) has an intuitive interpretation. If we treat $V_2(P(t), t)$ as an asset value, then the expected capital gain of the investment (the sum of the first three terms on the left-hand side) plus the dividend (the sum of the last three terms on the left-hand side) must be equal to the return required by the investor (the term on the right-hand side). Equation (11) simply says that when the investor sells the property, the value of the property must be equal to the after-tax equity reversion for the investor.

Appendix A shows that when an investor holds a property for an infinite time horizon, then the investment and financing decisions for the investor respectively satisfy the two equations given by:

$$D(P^*, M^*) = -(1-\frac{1}{\beta_1})\frac{P^*}{(\rho-\mu(M^*))}A_0 + f = 0, \qquad (12)$$

and

$$H(P^*, M^*) = \frac{\mu'(M^*)A_0}{(\rho-\mu(M^*))} + [1 - \frac{(1-\tau)}{\rho}(r(M^*) + M^*r'(M^*))] = 0, \qquad (13)$$

where

$$A_0 = M^* - \tau + (1 - e^{-\rho n})\frac{\tau\delta}{n\rho} - \frac{(1-\tau)}{\rho} M^* r(M^*), \tag{14}$$

and

$$\beta_1 = \frac{1}{2} - \frac{\mu(M^*)}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\mu(M^*)}{\sigma^2})^2 + \frac{2\rho}{\sigma^2}} > 1. \tag{15}$$

Equation (12) is derived based on the condition that an investor balances the immediate benefit from purchasing a property against the benefit from waiting for a more favorable state of nature. Equation (13) is derived based on the condition that an investor trades off the benefit from the tax advantages of debt financing against the adverse effect of debt financing that results from a higher borrowing rate and a possible lower expected growth rate of the net operation income. We can simultaneously use Equations (12) and (13) to derive the solution for the choice of the loan-to-value ratio, M^*, and that for the critical level of the net operating income that triggers investment, P^*.

To compare our model with those in the existing literature, we first investigate the polar case where debt financing does not affect real estate performance at all, i.e., $\mu'(M) = 0$. From Equation (13), this condition implies that:

$$\rho - (1-\tau)(r(M) + Mr'(M)) = 0. \tag{16}$$

Equation (16), which is exactly the same as that in McDonald (1999), suggests that an investor will choose a higher loan-to-value ratio, if the investor either requires a higher rate of return, faces a lower income tax rate, or is penalized less when borrowing more.

Let us switch to the case where debt financing adversely affects real estate performance, i.e., $\mu'(M^*) < 0$. Given this premise and the requirement that $\partial H(P^*, M^*)/\partial M^* < 0$, it follows that $M^* < M_a$, where M_a is defined as the M that satisfies Equation (16). In other words, when debt financing adversely affects real estate performance, then the loan-to-value ratio chosen by the investor will be lower than its counterpart when debt does not affect real estate performance at all.

We assume that an investor simultaneously makes the investment and the financing decision. In order to make comparisons with the results of the literature, we will first separately investigate each decision, assuming that the other decision is exogenously given. Differentiating $H(P^*, M^*) = 0$ in Equation (13) with respect to its various underlying parameters yields the following results.

Proposition 1 *Given the timing in the purchase of a property, the investor will take on more debt* (M^* *increases*) *if:* (*i*) *the investor is allowed to depreciate capital less rapidly* (*n increases*)*;* (*ii*) *the investor is penalized less through debt financing* ($r'(M)$ *decreases*)*;* (*iii*) *the investor expects borrowing to exhibit* *a less adverse impact on real estate performance (the absolute value of $\mu'(M)$ is smaller); and (iv) the investor has less depreciable capital (δ decreases).*[8]

Proof: See Appendix B.

The result of Proposition 1(ii) is the same as that in the literature such as McDonald (1999), and the reason for Proposition 1(iii) is obvious. The result for Propositions 1(i) and 1(iv) may seem to counter intuition at first sight because tax deductions from depreciation allowance will be lower as the investor is either allowed to depreciate capital less rapidly (n increases) or has less depreciable capital (δ decreases). However, it is the interaction effect between $\mu'(M)$ and δ or n that matters for the financing decision. As suggested by Equation (13), an increase in n or a decrease in δ will mitigate the negative impact on real estate performance which results from an increase in the loan-to-value ratio, thus encouraging the investor to borrow more.

Differentiating $D(P^*, M^*) = 0$ in Equation (12) with respect to its various underlying parameters yields the following results.

Proposition 2 *Given an investor's loan-to-value ratio, the investor will delay the purchase of a property (P^* increases) if: (i) the investor incurs a larger transaction cost (f increases); (ii) the investor is allowed to depreciate capital less rapidly (n increases); (iii) the investor is penalized more through debt financing ($r'(M^*)$ increases); (iv) the investor expects to receive less return through debt financing (the absolute value of $\mu'(M^*)$ is larger); (v) the investor has less depreciable capital (δ decreases); and (vi) the investor faces a higher risk in purchasing the property (σ increases); and (vii) the investor faces a higher income tax rate (τ increases).*[9]

Proof: See Appendix C.

Propositions 2(i) and (vi) are the standard results of the real options literature (see, for example, Dixit and Pindyck, 1994), which indicate that greater uncertainty and/or irreversibility will delay investment. The other scenarios stated in Proposition 2 follow because an investor will receive less return from investing immediately.

Propositions 1 and 2 help us investigate how the various forces affect the investment and financing decisions for the case where these two decisions are interacting with each other. We, however, can only reach definite comparative -statics results for the two exogenous forces, namely, the sunk costs and the risk of investment, as stated below in Proposition 3.

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[8] Furthermore, we find that an investor's incentive to borrow is ambiguously affected if the investor either faces a higher income tax rate (τ is higher) or requires a higher rate of return (ρ is higher). See Equations (B5) and (B6), respectively.

[9] Furthermore,we find that an investor's incentive to purchase property is ambiguously affected if the investor requires a higher rate of return, as suggested by Equation (C7).

Proposition 3 *An investor who incurs a larger sunk cost of investment or faces a higher risk of investment will not alter the loan-to-value ratio, but will delay investment and receive a higher net investment value.*

Proof: See Appendix D.

We use Figure 1 to explain the results of Proposition 3. Suppose that we start from an initial point E_0, which is the intersection of line I_0I_0 and line F_0F_0. In the figure, line I_0I_0 characterizes the optimal condition for the choice of investment timing as shown by Equation (12). Note that we assume that M^* exhibits a negative effect on P^* in this figure (our result will be qualitatively the same even if M^* exhibits a non-negative effect on P^*).[10] Furthermore, line F_0F_0, which characterizes the optimal condition for the financing decision as shown by Equation (13), is vertical because P^* will not affect M^* at all. Proposition 2 indicates that an investor who incurs a higher transaction cost or faces a higher risk of investment will delay the purchase of a property. This is shown in Figure 1, where the optimal timing decision characterized by line I_0I_0 will shift upward to line I_1I_1, while the optimal debt financing decision characterized by line F_0F_0 will remain unchanged. Thus, the investor will wait for a better state to invest, but will not alter the loan-to-value ratio. The net value of investment will increase, given that the investor purchases the property at a better state of nature.

The results of Proposition 3 imply that neither irreversibility nor uncertainty will affect an investor's choice of the loan-to-value ratio. This comes from our assumption that an investor has the option to delay the purchase of a property, but not the option to default the loan. As a result, the investor will choose the same loan-to-value ratio regardless of the state of nature at which the investor purchases the property. If we allow the investor to have the default option (see e.g., Kau et al., 1993), then these two exogenous forces will also affect the debt financing decision of the investor because different states of nature will entail different likelihoods of default.[11]

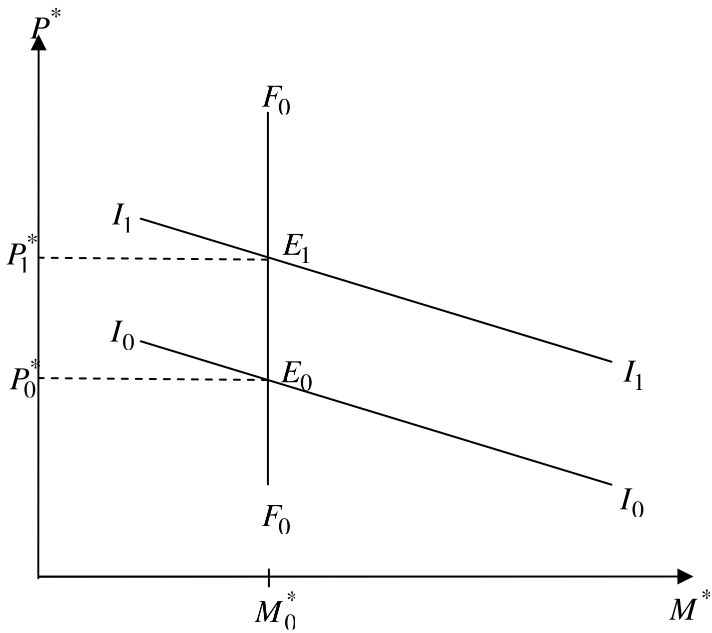
We will use plausible parameters to employ a numerical analysis to make our theoretical predictions stated in Propositions 1-3 more vivid. We consider both cases, that is, where the holding period is infinite and finite. Appendix E shows the procedures to find the solutions for the latter case.

[10] See Equation (C8) which indicates that M^* exhibits an ambiguous effect on P^*.

[11] If we allow the option to default, then an investor will both purchase a property at an earlier date and borrow more because the investor will receive the (put) option value to default, which also increases the benefit from borrowing.

Figure 1 The Effect of an Increase in Either the Sunk Cost or the Risk of Investment.

This graph shows that either change will move the equilibrium point from E_0, the intersection of I_0I_0 (the line that represents the optimal condition of the investment decision) and F_0F_0 (the line that represents the optimal condition of the financing decision), to E_1. As a result, choices of the loan-to-value ratio will remain unchanged at M_0^*; while the critical level of the net operating income that triggers investment will increase from P_0^* to P_1^*.



3. Numerical Analysis

We assume that $r(M) = r_0 + \lambda_1 M$, and $\mu(M) = \mu_0 + \lambda_2 M$, such that $r'(M) = \lambda_1 (>0)$ and $\mu'(M) = -\lambda_2 (<0)$. Our chosen benchmark case is as follows: sunk cost f = 1; income tax rate τ=20%; required rate of return on equity ρ=12% per year; the number of years allowed for depreciation for tax purposes n=39 years; proportion of depreciable capital δ=0.5; minimum borrowing rate r_0=7% per year; as an investor increases the loan-to-value ratio by 1%, then the mortgage rate that the investor faces will be increased by 0.05%, i.e., λ_1= 0.05; the net operating income is expected to grow at most 2%, i.e., μ_0=2% per year; an investor expects the growth rate of the net operating income to decline by 0.01% if the investor increases loan-to-value increases by 1%, i.e., λ_2 = 0.01;

the instantaneous volatility of that growth rate is equal to 15% per year, i.e., $\sigma = 15\%$ per year; and the holding period is infinite, i.e., $\bar{t} = \infty$.[12]

Table 1 Determinants of the Investment Timing and Loan-to-Value Ratio.

Benchmark case: $f = 1$, $\tau = 20\%$, $\rho = 12\%$ per year, $n = 39$ years, $\delta = 0.5$, $r_0 = 7\%$ per year, $\lambda_1 = 0.05$, $\mu_0 = 2\%$ per year, $\lambda_2 = 0.01$, $\sigma = 15\%$ per year, $\bar{t} = \infty$, $M^* = 79.52\%$, $P^* = 4.5324$, and $W^* = 0.4483$.

	Variation in f				
	0.5	0.75	1	1.25	1.5
M^*	0.7952	0.7952	0.7952	0.7952	0.7952
P^*	2.2662	3.3993	4.5324	5.6655	6.7986
W^*	0.2241	0.3362	0.4483	0.5604	0.6724
	Variation in τ				
	10%	15%	20%	25%	30%
M^*	0.6257	0.7058	0.7952	0.8956	1.0
P^*	2.5503	3.4425	4.5324	5.3023	4.9560
W^*	0.4636	0.4563	0.4483	0.4395	0.4307
	Variation in ρ				
	11.5%	11.75%	12%	12.25%	12.5%
M^*	0.6497	0.7204	0.7952	0.8744	0.9586
P^*	6.2215	5.4605	4.5324	3.6560	2.9249
W^*	0.4765	0.4622	0.4483	0.4349	0.4218
	Variation in n				
	31	35	39	43	47
M^*	0.7945	0.7949	0.7952	0.7955	0.7957
P^*	3.9527	4.2511	4.5324	4.7966	5.0444
W^*	0.44835	0.44831	0.44829	0.44826	0.44824
	Variation in δ				
	0.4	0.45	0.5	0.55	0.6
M^*	0.7958	0.7955	0.7952	0.7949	0.7946
P^*	5.1664	4.8286	4.5324	4.2704	4.0371
W^*	0.44823	0.44826	0.44829	0.44831	0.44834
	Variation in λ_1				
	0.045	0.0475	0.05	0.0525	0.055
M^*	0.8800	0.8354	0.7952	0.7587	0.7254
P^*	2.6944	3.4243	4.5324	6.4160	10.3262
W^*	0.4409	0.4448	0.4483	0.4515	0.4545

(Continued…)

[12] According to Goetzmann and Ibbotson (1990), during the period of 1969 to 1989, the annual standard deviation for REITs on commercial property was equal to 15.4%. We use this as a proxy for the volatility of the growth rate of the net operating income.

(*Table 1 continued*)

	Variation in λ_2					
	0	0.005	0.01	0.015	0.02	
M^*	0.8000	0.7975	0.7952	0.7931	0.7910	
P^*	4.4236	4.4764	4.5324	4.5916	4.6239	
W^*	0.5263	0.4852	0.4483	0.4152	0.3853	
	Variation in σ					
	10%	12.5%	15%	17.5%	20%	
M^*	0.7952	0.7952	0.7952	0.7952	0.7952	
P^*	4.0940	4.3071	4.5324	4.7697	5.0189	
W^*	0.3082	0.3763	0.4483	0.5241	0.6037	
	Variation in $\bar{t}$					
	10	15	20	25	30	∞
M^*	0.7968	0.7963	0.7915	0.7954	0.7961	0.7952
P^*	5.2597	5.2804	5.2835	5.3169	5.3322	4.5324
W^*	0.0615	0.1496	0.3106	0.4519	0.5361	0.4483

Note: M^*: the optimal loan-to-value ratio; P^*: the critical level of the net operating income that triggers investment; W^*: the net value of investment; f: the sunk cost of investment; τ: the income tax rate; ρ: an investor's required rate of return; n: the number of years allowed for depreciation for tax purposes; δ: the proportion of depreciable capital; r_0: the minimum borrowing rate; λ_1: the size of the effect of debt financing on the borrowing rate; μ_0: the maximum expected growth rate of the net operating income; λ_2: the size of the effect of debt financing on that expected growth rate; σ: the instantaneous volatility of that expected growth rate; and $\bar{t}$: the holding period.

Given this set of benchmark parameter values, we find that the investor will not purchase a property until the net operating income reaches 4.5324 (P^* = 4.5324). At that instant, the investor will use 79.52% debt to finance this purchase (M^* =79.52%), and will receive a net value equal to 0.4483 (W^* = 0.4483.[13] We also find that the P^* and M^* defined in Equation (12) are negatively correlated, as shown by line I_0I_0 in Figures 1, 2, and 3.

Table 1 shows the results for f changes in the region (0.5, 1.5), τ in the region (10%, 30%), ρ in the region (11.5%, 12.5%), n in the region (31, 47), δ in the region (0.4, 0.6), λ_1 in the region (0.045, 0.055), λ_2 in the region (0, 0.02), σ in the region (10%, 20%), and $\bar{t}$ in the region of (10, ∞), holding all the other parameters at their benchmark values.

[13] The ratio of the sunk cost, f, to the housing price, $P^*/(\rho\text{-}\mu(M^*))$, is equal to 2.38%, which is a little lower than the average level (see, e.g., 5-6% estimated by Stokey, 2009, p.108). Either a lower tax rate or lower degree of uncertainty will drive this ratio close to the average level (See Table 1).

Figure 2 The Effect of an Increase in Either the Tax Rate or the Length of Depreciation for Tax Purposes, or A Decrease in Depreciable Capital.

This graph shows that each change will move the equilibrium point from E_0 to E_1, such that choices of the loan-to-value ratio will increase from M_0^* to M_1^*, and the critical level of the net operating income that triggers investment will increase from P_0^* to P_1^*.

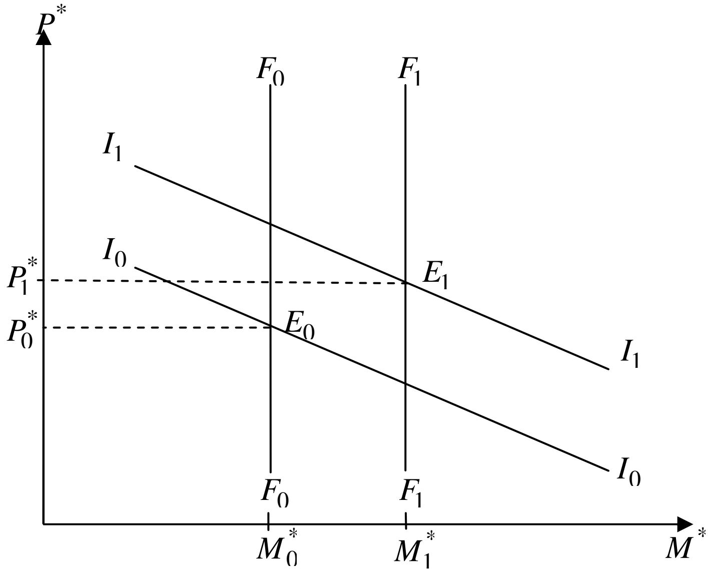
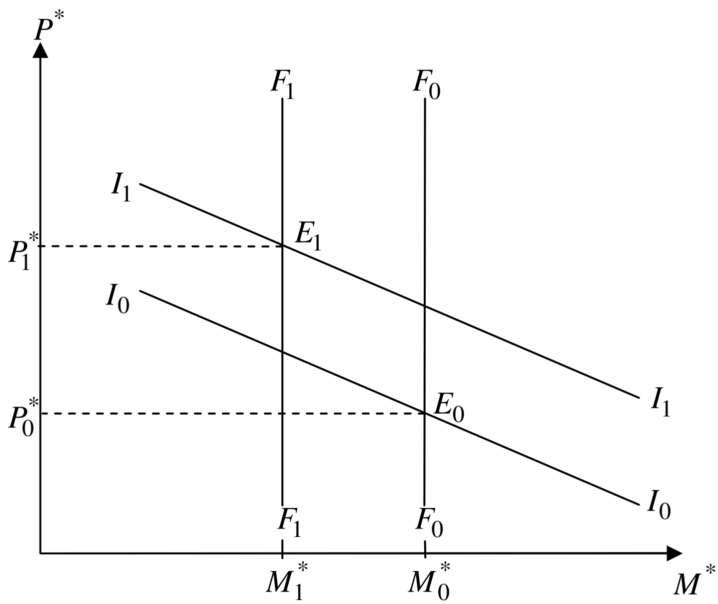


Table 1 indicates the following results. First, (a) an investor will wait for a better state to purchase a property and receive a higher net value (both P^* and W^* increase), but will choose the same level of debt (M^* remains unchanged) if the investor incurs a higher transaction cost (f increases) or faces a higher risk (σ increases). These results conform to those stated in Proposition 3. Second, (b) an investor will wait for a better state to purchase a property and use more debt, but receive a lower net value (both P^* and M^* increase, but W^* decreases), if the investor faces a higher income tax rate (τ increases), is allowed to depreciate capital less rapidly (n increases), or has less depreciable capital (δ decreases). Third, (c) an investor will wait for a better state to purchase the property and receive a higher net value, but use less debt (both P^* and W^* increase, and M^* decreases), if the investor either requires a lower rate of return (ρ decreases) or is penalized more through debt financing (λ_1 increases). Fourth, (d) an investor will wait for a better state to purchase the property, but will use less debt and receive a lower net investment value (P^* increases, and both M^* and W^* decrease), if borrowing exhibits a more adverse impact on real estate performance (λ_2 increases). Finally, (e) an investor will choose almost the same debt-to-loan value ratio for all holding periods. However, this is not the case for the choice of investment timing. When the holding period is shorter than thirty years, the investor will wait for a better state to purchase a property and receive a higher net value (both P^* and W^* increase) if the investor holds the property longer ($\bar{t}$ increases). However, for holding periods longer than thirty years, both P^* and W^* will then decline toward their respective steady-state levels.

Figure 3 The Effect of an Increase in Penalty Through Borrowing, A Decrease in the Investor's Required Rate of Return, or a More Adverse Effect of Debt Financing on Real Estate Performance.

The graph shows that each change will move the equilibrium point from E_0 to E_1, such that choices of the loan-to-value ratio will decrease from M_0^* to M_1^*, and the critical level of the net operating income that triggers investment will increase from P_0^* to P_1^*.



The reason for Result (b) is as follows. Consider an investor who is allowed to depreciate capital less rapidly (n increases) or has less depreciable capital (δ decreases). Each leads to a direct effect that forces the investor to purchase the property later, given the debt level, as suggested by Propositions 2(ii) and (v), respectively. This is shown in Figure 2 where line I_0I_0 shifts upward to I_1I_1. Each change also leads the investor to use more debt as shown by Propositions 1(i) and (iv), respectively, such that the investor will be induced to purchase at an earlier date. This is shown in Figure 2 where line F_0F_0 shifts rightward to line F_1F_1. The equilibrium point thus shifts from E_0 to E_1, which indicates that the investor delays the purchase and borrows more. An increase in the loan-to-value ratio, in turn, decreases the net value through lowering real estate performance. Similar arguments as the above also apply to the case where an investor faces a higher income tax rate (τ increases).

The reason for Results (c) and (d) is as follows. Suppose that an investor is penalized more through debt financing (λ_1 increases). Proposition 2(iii) indicates that an investor will delay purchasing, given debt levels. This is shown by a shift from line I_0I_0 upward to line I_1I_1 in Figure 3. Proposition 1(iii), on the other hand, indicates that the investor will borrow less, given the investment timing. This is shown by a shift of line F_0F_0 leftward to line F_1F_1 in Figure 3. The equilibrium point shifts from E_0 to E_1, thus suggesting that the investor will delay the purchase and also borrow less. Similar arguments as above can apply to the case where the investor requires a lower rate of return (ρ decreases) or debt exhibits a more adverse effect on real estate performance (λ_2 increases). The net investment value will increase when either λ_1 increases or ρ decreases because the investor invests at a better state of nature. By contrast, the adverse effect of an increase in λ_2 will outweigh the positive effect that results from investing at a better state of nature such that the net investment value will decrease as a result.

Finally, the reason for Result (e) is as follows. Consider that an investor increases the holding period in the region capped by thirty years. The value of the option to wait thus becomes more valuable as the holding period increases. As a result, the net investment value also increases. Nonetheless, the above pattern will eventually reverse when the holding period is longer than thirty years. The reason is obvious. Given that an investor enjoys tax deduction benefits from depreciation allowance for at most thirty nine years, the investor is unable to continuously receive a higher net value from the investment over an infinite horizon.

4. Conclusion

This article employs a real options approach to investigate the determinants of an optimal capital structure in real estate investment. We have assumed that an investor incurs transaction costs when purchasing an income-producing property that yields a stochastic net operating income. We find several testable implications as follows. First, an investor who incurs a larger sunk cost or faces a higher risk of investment will not alter the loan-to-value ratio, but will delay investment. Second, an investor who either faces a higher income tax rate or receives lower depreciation allowance for tax purposes will borrow more and delay investment. Finally, an investor who either pays more penalties from borrowing, requires less return for equity investment, or has worse real estate performance through borrowing will borrow less and delay investment.

This article builds a simplified model, and thus, can be extended in the following ways. First, this article implicitly assumes that an investor has a monopolized right to purchase a certain income-producing property (see, for example, Dixit and Pindyck, 1994). A more sophisticated model may allow different investors to compete for a certain property, or allow the seller of the property to play a more active role. Second, this article abstracts from several aspects of real estate financing, such as variable mortgage rates and prepayment penalties. It deserves further investigation of whether these factors also matter for the determinants of optimal capital structures.

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Appendix A The Case for $\bar{t} = \infty$

If $\bar{t} = \infty$, then $\partial V_2(P(t),t)/\partial t = 0$, and $E_{t_0} ATER(T+\bar{t})e^{-\rho(T+\bar{t}-t_0)} = 0$. For this case, suppose that $V_1(P(t))$ and $F_2(P(t))$ denote the option value of waiting in the region where $P(t_0) < P^*$ and the property value in the region where $P(t_0) \geq P^*$, respectively. Substituting $V_1(P(t))=P(t)^\beta$ into Equation(8) yields the quadratic equation for solving β:

$$\phi(\beta) = -\frac{\sigma^2}{2}\beta(\beta-1) - \mu(M)\beta + \rho = 0 \cdot \tag{A1}$$

Consequently, the solution for $V_1(P(t))$ in Equation (9) is given by:

$$V_1(P(t)) = A_1 P(t)^{\beta_1} + A_2 P(t)^{\beta_2}, \tag{A2}$$

where β_1 and β_2 are, respectively, the larger and smaller roots of β in Equation (A1), and A_1 and A_2 are constants to be determined.

Similarly, if $P\ (t_0) \geq P^*$ such that investment is made, then we can rewrite Equation (10) as:

$$\frac{\sigma^2}{2}P(t)^2\frac{\partial^2 F_2(P(t))}{\partial P(t)^2}+\mu(M)P(t)\frac{\partial F_2(P(t))}{\partial P(t)}+(1-\tau)P(t)+ \\ \frac{\tau\delta}{n}\frac{P^*}{(\rho-\mu(M))}-(1-\tau)M\frac{P^*}{(\rho-\mu(M))}r(M)=\rho F_2(P(t)). \tag{A3}$$

The solution for $F_2(P)$ in Equation (A3) is given by:

$$F_2(P(t))=B_1P(t)^{\beta_1}+B_2P(t)^{\beta_2}+(1-\tau)\frac{P(t)}{(\rho-\mu(M))} \\ +(1-e^{-\rho n})\frac{\tau\delta}{n\rho}\frac{P^*}{(\rho-\mu(M))}-\frac{(1-\tau)}{\rho}\frac{MP^*r(M)}{(\rho-\mu(M))}, \tag{A4}$$

where B_1 and B_2 are constants to be determined.

The terms A_1, A_2, B_1, B_2, and the critical level of the net operating income that triggers investment, P^*, are simultaneously solved from the boundary conditions as follows:

$$\lim_{P(t)\to 0}V_1(P(t))=0, \tag{A5}$$

$$\lim_{P(t)\to 0}B_1P(t)^{\beta_1}+B_2P(t)^{\beta_2}=0, \tag{A6}$$

$$\lim_{P(t)\to \infty}B_1P(t)^{\beta_1}+B_2P(t)^{\beta_2}=0, \tag{A7}$$

$$V_1(P^*)=F_2(P^*)-(1-M)\frac{P^*}{(\rho-\mu(M))}-f, \tag{A8}$$

and

$$\frac{\partial V_1(P^*)}{\partial P(t)}\Big|_{t=t_0}=\frac{\partial F_2(P^*)}{\partial P(t)}\Big|_{t=t_0}-\frac{(1-M)}{(\rho-\mu(M))}. \tag{A9}$$

Equation (A5) is the limit condition, which states that the investor's option value from delaying the purchase is worthless as the net operating income approaches its minimum permissible value of zero. This condition requires that $A_2=0$. Equations (A6) and (A7) are two other limit conditions, which respectively state that after an investor purchases a property, the investor's option value from abandoning the property is worthless, when the net operating income is extremely bad and extremely good. These two conditions require that $B_1=B_2=0$. Equation (A8) is the value-matching condition, which states that at the optimal timing of purchasing ($t_0=T$ in our case), the investor is indifferent between exercising and not exercising the investment. Equation (A9) is the smooth-pasting condition, which guarantees that the investor will not derive any arbitrage profits by deviating the optimal exercise strategy.

Define $A_0 = M^* - \tau + (1-e^{-\rho n})\tau\delta / np - (1-\tau)M^* r(M^*)/\rho$. Multiplying Equation (A9) by P^*/β_1, and then subtracting Equation (A8) from it yields:

$$D(P^*, M^*) = -(1-\frac{1}{\beta_1})\frac{P^*}{(\rho-\mu(M^*))}A_0 + f = 0, \tag{A10}$$

$$A_1 = \frac{1}{\beta_1(\rho-\mu(M^*))}A_0 P^{*1-\beta_1}, \tag{A11}$$

and $A_2 = 0$. Furthermore, the choice of M is found by setting the derivative of $V_1(P^*)$ in Equation (A2), or equivalently, $F_2(P^*) - (1-M)\frac{P^*}{(\rho-\mu(M))} - f$, with respect to M equals to zero. This yields:

$$H(P^*, M^*) = \frac{\mu'(M^*)A_0}{(\rho-\mu(M))} + [1 - \frac{(1-\tau)}{\rho}(r(M^*) + M^* r(M^*))] = 0. \tag{A12}$$

The second-order conditions require that:

$$\partial D(P^*, M^*)/\partial P^* < 0, \tag{A13}$$

$$\partial H(P^*, M^*)/\partial M^* < 0, \tag{A14}$$

and

$$\begin{aligned}&\partial D(P^*, M^*)/\partial P^* \cdot \partial H(P^*, M^*)/\partial M^* - \\ &\partial D(P^*, M^*)/\partial M^* \cdot \partial H(P^*, M^*)/\partial P^* > 0.\end{aligned} \tag{A15}$$

Appendix B Proof of Proposition 1

Totally differentiating $H(P^*, M^*) = 0$ in Equation (13) with respect to n, $r'(M^*)$, $\mu'(M^*)$, δ, τ, and ρ yields:

$$\frac{\partial M^*}{\partial n} = \frac{\Delta_1}{-\Delta} > 0, \tag{B1}$$

$$\frac{\partial M^*}{\partial r'(M)} = \frac{\Delta_2}{-\Delta} < 0, \tag{B2}$$

$$\frac{\partial M^*}{\partial \mu'(M)} = \frac{\Delta_3}{-\Delta} > 0, \tag{B3}$$

$$\frac{\partial M^*}{\partial \delta} = \frac{\Delta_4}{-\Delta} < 0, \tag{B4}$$

$$\frac{\partial M^*}{\partial \tau} = \frac{\Delta_5}{-\Delta} \gtrless 0, \tag{B5}$$

$$\frac{\partial M^*}{\partial \rho} = \frac{\Delta_6}{-\Delta} \begin{matrix} > \\ < \end{matrix} 0, \tag{B6}$$

where $\Delta = \partial H(P^*, M^*) / \partial M^* < 0$,

$$\Delta_1 = \frac{\partial H(P^*, M^*)}{\partial n} = \frac{-\mu'(M^*)\tau\delta}{(\rho - \mu(M^*))n^2\rho}(1 - e^{-\rho n} - \rho n e^{-\rho n}) > 0, \tag{B7}$$

$$\Delta_2 = \frac{\partial H(P^*, M^*)}{\partial r'(M^*)} = \frac{-(1-\tau)M^*}{\rho} < 0, \tag{B8}$$

$$\Delta_3 = \frac{\partial H(P^*, M^*)}{\partial \mu'(M^*)} = \frac{A_0}{(\rho - \mu(M^*))} > 0, \tag{B9}$$

$$\Delta_4 = \frac{\partial H(P^*, M^*)}{\partial \delta} = \frac{\mu'(M^*)\tau}{(\rho - \mu(M^*))n\rho}(1 - e^{-\rho n}) < 0, \tag{B10}$$

$$\Delta_5 = \frac{\partial H(P^*, M^*)}{\partial \tau} = \frac{\mu'(M^*)}{(\rho - \mu(M^*))}[-1 + \frac{M^* r(M^*)}{\rho} + (1 - e^{-\rho n})\frac{\delta}{n\rho}] + \frac{1}{\rho}(r(M^*) + M^* r'(M^*)) \begin{matrix} > \\ < \end{matrix} 0, \tag{B11}$$

and

$$\Delta_6 = \frac{\partial H(P^*, M^*)}{\partial \rho} = \frac{-\mu'(M^*)}{(\rho - \mu(M^*))^2} A_0 + \frac{\mu'(M)}{(\rho - \mu(M^*))}[\frac{(1-\tau)}{\rho^2} M^* r(M^*) + \frac{\tau\delta}{n\rho^2}(1 + n\rho)e^{-\rho n} - 1] + \frac{(1-\tau)}{\rho}(r(M^*) + M^* r'(M^*)) \begin{matrix} > \\ < \end{matrix} 0. \tag{B12}$$

Q. E. D.

Appendix C Proof of Proposition 2

Equation (12) implies that:

$$P^* = \frac{f\rho}{A_0}(1 - \frac{1}{\beta_2}), \tag{C1}$$

where we have used the relationship $\beta_1\beta_2(\rho - \mu(M^*)) = (\beta_1 - 1)(\beta_2 - 1)\rho$. Differentiating P^* with respect to f, n, δ, σ, τ, ρ and M^* yields:

$$\frac{\partial P^*}{\partial f} = \frac{P^*}{f} > 0, \tag{C2}$$

$$\frac{\partial P^*}{\partial n} = \frac{f\rho}{A_0^2}(1-\frac{1}{\beta_2})\frac{\tau\delta}{n^2\rho}(1-e^{-\rho n}-\rho n e^{-\rho n})] > 0, \tag{C3}$$

$$\frac{\partial P^*}{\partial \delta} = \frac{-f\rho}{A_0^2}(1-\frac{1}{\beta_2})(1-e^{-\rho n})\frac{\tau}{n\rho}] < 0, \tag{C4}$$

$$\frac{\partial P^*}{\partial \delta} = \frac{f\rho}{A_0\beta_2^2}\frac{\partial \beta_2}{\partial \sigma} > 0, \tag{C5}$$

$$\frac{\partial P^*}{\partial \tau} = \frac{f\rho}{A_0^2}(1-\frac{1}{\beta_2})[-1+\frac{M^* r(M^*)}{\rho}+(1-e^{-\rho n})\frac{\delta}{n\rho}] > 0, \tag{C6}$$

$$\frac{\partial P^*}{\partial \rho} = \frac{f}{\rho A_0 \beta_2^2}\frac{\partial \beta_2}{\partial \rho} - \frac{f}{\rho A_0}(1-\frac{1}{\beta_2})[\frac{(1-\tau)M^* r(M^*)}{\rho^2} + \frac{\tau\delta}{n\rho^2}(1+n\rho)e^{-\rho n}-1)] + \frac{f}{A_0}(1-\frac{1}{\beta_2}) \gtreqless 0, \tag{C7}$$

and

$$\frac{\partial P}{\partial M^*} = \frac{f\rho}{A_0\beta_2^2}\frac{\partial \beta_2}{\partial M^*} - \frac{f\rho}{A_0^2}(1-\frac{1}{\beta_2})[1-(1-\tau)(r(M^*)+M^* r'(M^*))] \gtreqless 0. \tag{C8}$$

Proposition 2(iii) follows because as $r'(M^*)$ increases, then $r(M^*)$ in Equation (C1) also increases. Proposition 2(iv) follows because as the absolute value of $\mu'(M^*)$ increases, then $\mu(M^*)$ in Equation (C1) will decrease.

Q. E. D.

Appendix D Proof of Proposition 3

Equation (13) indicates that $H(P^*, M^*)$ is independent of f and σ,thus suggesting that the optimal level of M^* is neither related to f nor σ.Equations (C2) and (C5) then suggest that $\partial P^* / \partial f > 0$ and $\partial P^* / \partial \sigma > 0$. Substituting A_1 in Equation (A11) and P^* in Equation (C1) into the left-hand side of Equation (A8) yields the net value of investment equal to $f/(\beta_1 - 1)$. Differencing this value with respect to f and σ yields

$$\frac{\partial(f/(\beta_1-1)}{\partial f} = \frac{1}{(\beta_1-1)} > 0, \tag{D1}$$

and

$$\frac{\partial(f/(\beta_1-1))}{\partial\sigma}=\frac{-f}{(\beta_1-1)^2}\frac{\partial\beta_1}{\partial\sigma}>0\cdot \tag{D2}$$

Q. E. D.

Appendix E The Case for Finite $\bar{t}$

We follow Brennan and Schwartz (1978), and Hull and White (1990) to find P^* and M^*. Let $y(t)=\ln P(t)$ such that $y^*=\ln P^*$, $U(y(t))=V_1(P(t))$ for $y(t)<y^*$ and $Z(y(t),t)=V_2(P(t),t)$ for $y(t_0)\geq y^*$ and $t\geq t_0$. As a result, Equation (9) can be rewritten as:

$$\frac{1}{2}\sigma^2\frac{\partial U^2(y(t))}{\partial y(t)^2}+(\mu(M)-\frac{\sigma^2}{2})\frac{\partial U(y(t))}{\partial y(t)}-\rho U(y(t))=0, if\ \ y(t)<y^*\cdot \tag{E1}$$

Furthermore, Equation (10) can be rewritten as:

$$\begin{aligned}&\frac{1}{2}\sigma^2\frac{\partial^2 Z(y(t),t)}{\partial y(t)^2}+(\mu(M)-\frac{\sigma^2}{2})\frac{\partial Z(y(t),t)}{\partial y(t)}+\frac{\partial Z(y(t))}{\partial t}+(1-\tau)e^{y(t)}\\&+\frac{\tau\delta}{n}\frac{e^{y^*}}{(\rho-\mu(M))}-(1-\tau)M\frac{e^{y^*}}{(\rho-\mu(M))}r(M)=\rho Z(y(t),t),\\&if\ \ y(t_0)\geq y^*, and\ t\geq t_0.\end{aligned} \tag{E2}$$

Equation (11) can also be rewritten as:

$$Z(y(T+\bar{t}),T+\bar{t})=\frac{(1-\tau)e^{y(T+\bar{t})}}{(\rho-\mu(M))}-(M-\tau+\frac{\tau\delta\bar{t}}{n})\frac{e^{y^*}}{(\rho-\mu(M))}\cdot \tag{E3}$$

Finally, Equation (6) can be rewritten as:

$$EI(T)=(1-M)\frac{e^{y^*}}{(\rho-\mu(M))}\cdot \tag{E4}$$

The choice of M is derived by setting the derivative of $W(\cdot)$ in Equation (1) with respect to M equals to zero. Let $G(P(t),t)=\partial V_2(P(t),t)/\partial M$. As a result,

$$\frac{\partial W(\cdot)}{\partial M}=G(P(T),T)+E_{t_0}e^{-\rho(T-t_0)}[1-\frac{(1-M)\mu'(M)}{(\rho-\mu(M))}]\frac{e^{y^*}}{(\rho-\mu(M))}\cdot \tag{E5}$$

Let $g(y(t),t) = \partial Z(y(t),t)/\partial M$ .Differentiating Equation (E2) term by term with respect to M yields:

$$\begin{aligned}&\frac{1}{2}\sigma^2\frac{\partial^2 g(y(t),t)}{\partial y(t)^2}+(\mu(M)-\frac{\sigma^2}{2})\frac{\partial g(y(t),t)}{\partial y(t)}+\mu'(M)\frac{\partial Z(y(t),t)}{\partial y(t)}+\\&\frac{\partial g(y(t),t)}{\partial t}-(1-\tau)\frac{e^{y^*}}{(\rho-\mu(M^*))}[r(M)+Mr'(M)]+\\&[\frac{\tau\delta}{n}-(1-t)Mr(M)]e^{y^*}\frac{\mu'(M)}{(\rho-\mu(M))^2}-\rho g(y(t),t)=0.\end{aligned} \tag{E6}$$

Evaluating Equation (E5) at $T = t_0$ and $M = M^*$ yields:

$$g(y^*,t_0)+[1-\frac{(1-M^*)\mu'(M^*)}{(\rho-\mu(M^*))}]\frac{e^{y^*}}{(\rho-\mu(M^*))}=0\cdot \tag{E7}$$

The boundary condition for $g(y(t),t)$ is derived by differentiating Equation (E3) with respect to M, which yields:

$$\begin{aligned}g(y(T+\bar{t}),T+\bar{t}) &= \frac{(1-\tau)e^{y(T+\bar{t})}\mu'(M)}{(\rho-\mu(M))^2}-\\&(M-\tau+\frac{\tau\delta\bar{t}}{n})\frac{e^{y^*}\mu'(M)}{(\rho-\mu(M))^2}-\frac{e^{y^*}}{(\rho-\mu(M))}.\end{aligned} \tag{E8}$$

We implement the explicit finite difference method (Hull and White, 1990) to solve for M^* and P^*. We begin by choosing a small time interval, Δt, and a small change in $y(t)$, Δy. A grid is then constructed to consider the values of $Z(t)$ when $y(t)$ is equal to

$$y_0, y_0+\Delta y_1,\ldots, y_{\max},$$

and time is equal to

$$t_0, t_0+\Delta t_1,\ldots, t_0+\bar{t},$$

The parameters y_0 and $y_{\max}$ are the smallest and the largest values of $y(t)$, and t_0 is the current time. Let us denote $y_0+i\Delta y_1$ by y_i, $t_0+j\Delta t_1$ by t_j, and the value of the derivative security at the (i, j) point on the grid by $Z_{i,j}$. The partial derivatives of $Z(t)$ with respect to $y(t)$ at node (i, j) are approximately as follows:

$$\frac{\partial Z(t)}{\partial y(t)}=\frac{Z_{i+1,j}-Z_{i-1,j}}{2\Delta y}, \tag{E9}$$

and

$$\frac{\partial^2 Z(t)}{\partial y(t)^2} = \frac{Z_{i+1,j} + Z_{i-1,j} - 2Z_{i,j}}{(\Delta y)^2}, \tag{E10}$$

The time derivative for $Z\,(t)$ is approximately:

$$\frac{\partial Z(t)}{\partial t} = \frac{Z_{i,j} - Z_{i,j-1}}{\Delta t}. \tag{E11}$$

Substituting Equations (E9) to (E11) into Equation (E2) yields:

$$\begin{aligned} Z_{i,j-1} &= aZ_{i-1,j} + bZ_{i,j} + cZ_{i+1,j} + \frac{\Delta t}{(1+\rho\Delta t)}(1-\tau)e^{y_0 + i\Delta y} \\ &+ \frac{\Delta t}{(1+\rho\Delta t)}\left(\frac{\tau\delta}{n} - (1-\tau)Mr(M)\right)\frac{e^{y^*}}{(\rho - \mu(M))}, \end{aligned} \tag{E12}$$

where

$$a = \frac{1}{(1+\rho\Delta t)}\left[\frac{\sigma^2 \Delta t}{2(\Delta y)^2} - \left(\mu - \frac{\sigma^2}{2}\right)\frac{\Delta t}{2\Delta y}\right], \tag{E13}$$

$$b = \frac{1}{(1+\rho\Delta t)}\left[1 - \frac{\sigma^2 \Delta t}{(\Delta y)^2}\right], \tag{E14}$$

and

$$c = \frac{1}{(1+\rho\Delta t)}\left[\frac{\sigma^2 \Delta t}{2(\Delta y)^2} + \left(\mu - \frac{\sigma^2}{2}\right)\frac{\Delta t}{2\Delta y}\right]. \tag{E15}$$

Similarly, Equation (E6) can be written as:

$$\begin{aligned} g_{i,j-1} &= ag_{i-1,j} + bg_{i,j} + cg_{i+1,j} + (Z_{i+1,j} - Z_{i-1,j})\frac{\mu'(M)}{\partial\Delta y}\frac{\Delta t}{(1+\partial\Delta t)} \\ &- \frac{\Delta t}{(1+\rho\Delta t)}(1-\tau)\frac{e^{y^*}}{(\rho-\mu(M))}[r(M) + Mr'(M)] \\ &+ \frac{\Delta t}{(1+\rho\Delta t)}\left[\frac{\tau\delta}{n} - (1-\tau)Mr(M)\right]\frac{e^{y^*}\mu'(M)}{(\rho-\mu(M))^2}. \end{aligned} \tag{E16}$$

We also need to impose the optimal condition for the timing of investment. The solution to $U(y(t))$ in Equation (E1) is given by:

$$U(y(t)) = A_1 e^{\beta_1 y(t)} + A_2 e^{\beta_2 y(t)}, \tag{E17}$$

where A_1 and A_2 are constants to be determined, and β_1 and β_2 are defined in Appendix A. The optimal timing is determined by the following boundary conditions:

$$\lim_{y(t)\to 0} U(y(t)) = 0, \tag{E18}$$

$$U(y^*) = Z(y^*, T) - (1-M)\frac{e^{y^*}}{(\rho - \mu(M))} - f, \tag{E19}$$

and

$$\left.\frac{\partial U(y^*)}{\partial y(t)}\right|_{y(t)=y^*} = \left.\frac{\partial Z(y^*, t)}{\partial y(t)}\right|_{y(t)=y^*} - \frac{(1-M)e^{y^*}}{(\rho - \mu(M))}. \tag{E20}$$

Solving Equations (E18)-(E20) simultaneously yields:

$$A_2 = 0. \tag{E21}$$

$$A_1 = [Z(y^*, 0) - (1-M)\frac{e^{y^*}}{(\rho - \mu(M))} - f]e^{-\beta_1 y^*}, \tag{E22}$$

and

$$A_1\beta_1 e^{\beta_1 y^*} = \left.\frac{\partial Z(y^*, t)}{\partial y(t)}\right|_{y(t)=y^*} - \frac{(1-M)}{(\rho - \mu(M))}e^{y^*}. \tag{E23}$$

The law of motion for $Z(y(t),t)$ shown in Equation (E12) and that for $g(y(t),t)$ shown in Equation (E16) are subject to two optimal conditions shown in Equations (E7) and (E23), respectively, and two boundary conditions shown in Equations (E3) and (E8), respectively. Solving these four conditions simultaneously yields the solutions for A_1, M^*, $g_{i^*,0}$, and $Z_{i^*,0}$, where $Z_{i^*,0}$ is the gross value of investment. We can further use the relation $P^* = e^{y_i^*}$ to find the critical level of the net operating income that triggers investment, as well as the net value of investment, $Z_{i^*,0} - (1-M^*)\frac{P^*}{(\rho - \mu(M^*))} - f$.

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