INTERNATIONAL REAL ESTATE REVIEW
2004 Vol. 7 No. 1: pp. 31 - 55

Modelling Households' Savings and Dwellings Investment – A Portfolio Choice Approach

Gábor Vadas
Model Development Unit, Conjunctural Assessment and Projections Division, Economics Department, National Bank of Hungary, H-1850 Budapest V: Szabadság tér 8/9., Phone: (36)-1-428-2600/1131, Fax: (36)-1-428-2590,
E-mail: vadasg@mnb.hu

A house is generally considered as a 'roof over one's head', however, housing can be regarded as an investment or asset. Our paper focuses on this function of dwellings and develops a stochastic portfolio choice model for the housing market, which is easy to incorporate into medium and large-scale macro models. Theoretical results suggest that house prices move in line with households' income, although house prices have a higher variance than income does. On the other hand the positive correlation between the return on housing investment and consumption not only implies positive relationship between the portfolio share of housing investment and excess return but also renders the housing wealth inappropriate in consumption smoothing. We use UK data to test these theoretical implications of the model. In this case, empirical results strengthen the model framework.

Keywords

households' behaviour, housing investment, saving, portfolio decision, house price

Introduction

Housing market is a frequent topic of discussion among policy-makers and researchers. The most typical issue relates either to the financial sector or to the development of the mortgage loan market with its implications for the financial sector. The main concern is raised by financial sector stability.

The first obvious link between the housing market and financial stability is the level of households' mortgage loans or, in other words, the level of households' indebtedness. The higher the level of indebtedness, the higher the risk of default in the event of a change in mortgage interest rates. This influences the financial position of lenders. The second link is established by changes in house prices. If mortgage repayment is tied to the value of collateral, namely dwellings, changes in house prices alter monthly repayment by changing the risk premium. Increasing house prices reduces, while decreasing house prices increases the risk premium. Thus, changes in house prices either increase or decrease the amount of monthly repayment, thereby influencing the ability to repay, and the possibility of default.

However, the financial sector is not the only one in which interest rates and house prices play a critical role. House prices are also an important factor in households' life and are essential for understanding the housing market. The importance of the housing market may be even more crucial for households than for the financial sector. Dwellings have several functions – it is a roof over your head, a component of households' wealth, a property which can be used as collateral, a potential form of investment or an asset, etc. This paper focuses on the asset function. Its aim is to examine housing theoretically as an investment and develop a model that can handle this kind of investment decision and be a building block of medium or large-scale macro models. The appropriateness of this approach is tested on UK data.

The paper is structured as follows. In Section 2, we overview the related literature of research and show some selected works on modelling household behaviour. In Section 3, we develop a portfolio choice model in a stochastic environment, which can be applied to describe housing investment. Section 4 defines a set of data, while the implication of the model is empirically tested and verified in the Section 5. Section 6 summarises the results and indicates some future work.

Related Literature

The importance of the housing market from the perspective of households is unassailable on both the micro and macro levels. However, its mechanism is different from that of the financial sector. First, changes in house prices and the existence of housing wealth influence households' behaviour. On the one hand, rise in house prices imply increasing wealth, which makes higher consumption possible through the wealth effect. The credit channel has a broadly similar effect. The rise in house prices increases housing wealth and so the available collateral for loan, which, in turn, induces higher consumption expenditure.

On the other hand, we can consider housing investment as any other 'normal' investment. The owner of a house can realize income from tenants and from changes in house prices. Increasing house prices can provide a higher return on real estate than financial investment does, and force households to reallocate their portfolios. Second, focusing on the macroeconomic view, housing investment amounts to a sizeable share of nation-wide investment expenditure.

Several theoretical and empirical studies seek to incorporate these effects into their models. Westaway (1992) provides a comprehensive general equilibrium model, which incorporates the flow of housing services into the utility function. Aoki et al. (2002) go one step further and not only use housing services in the utility function, but also apply the financial accelerator developed by Bernanke et al. (2000). The main point of the financial accelerator is that house prices influence housing wealth that households can use as collateral in borrowing. If house prices increase, housing wealth and available collateral do so as well. Consequently, households can borrow at a lower financial premium.

Households' financial accelerator leads us to the empirical approach of the housing wealth effect. The Bank of England (2000) (hereafter BoE) model uses the modified version of error correction equation, originally suggested by Hendry and Ungern Sternberg (1981) (hereafter HUS). [1] In the BoE model, households' wealth consists of not only net financial but also housing wealth. One can easily see the similarity between the effect of the financial accelerator and that of incorporating housing wealth into the error correction model (ECM) form. When house prices rise, total housing wealth does so too, which implies a positive adjustment to consumption through the error correction mechanism? Case et al. (2001) and Girouard and Blöndal (2001) also found empirically significant positive relationship between housing wealth and household expenditure.

Relying on the above-mentioned, one could easily conclude that these empirical models solve the problem entirely. However, we have to admit that,

---

[1] The general form of the HUS equation is

$$\Delta c_t = \theta_0 + \theta_1 \Delta y_t - \theta_2 (c_{t-1} - \alpha_1 y_{t-1} - \alpha_2 w_{t-1}) + \varepsilon_t ,$$

where small letters denote the natural logarithm of consumption (c), income (y), and wealth (w). The variant of the HUS formula is used in the ECB model (see Fagan at al. (2001)), and a similar ECM formula can be found in the NIGEM model (see NIESR, Jakab-Kovács (2002)). Muellbauer and Lattimore (1995) also recommend this approach in their empirical studies.

Naturally, the ECM form is not a completely different approach from the general equilibrium model. When there is liquidity constraint, ECM can be derived from utility maximising problem (see Jakab-Vadas (2001) appendix) or minimising a quadratic loss function (see Hendry and Ungern Sternberg (1981)).

although the influence of housing wealth on consumption has been modelled, we do not know much about the portfolio decisions of households. Nor are we familiar with the factors, which determine housing wealth in the HUS approach, namely the changes in house prices and housing investment. Although the determinants of house price evolution have a long description in empirical literature, these studies generally focus on house prices equation only, hardly taking into consideration its implication to household behaviour or more specifically housing investment (see Chen and Patel (1998), Hsueh (2000)). The BoE model formally contains housing investment whose growth rate has not been modelled, however. Instead, housing investment simply equals the business investment rate thus there is no link between house prices and housing investment.

In the following, we investigate the factors, which influence house prices and households' investment expenditure. Furthermore, we develop a model that can handle this incompleteness and is easy to be incorporated into medium or large-scale macro models.

The Model

Our model is based on a stochastic portfolio choice model of Cochrane (2001). For reasons of tractability, we consider only two types of investment:

– financial investment having R^{I}_{t+1} return at t+1 on every invested currency unit in period t, and
– real estate investment having R^{II}_{t+1} payoff at t+1 on every invested unit bought at price P in period t.

The payoff (R^{II}_{t+1}) consists of two components – the price and other income (dividend per unit, V) in period t+1, thus $R^{\mathrm{II}}_{t+1} = P_{t+1} + V_{t+1}$. The optimisation problem of households is

$$\max_{w_1, w_2} \sum_{j=0}^{\infty} E_t \left[\beta^j \; U(C_{t+j}) \right] \tag{1}$$

$$\text{s.t.} \quad C_t + W^{\mathrm{I}}_t + P_t W^{\mathrm{II}}_t = R^{\mathrm{I}}_t W^{\mathrm{I}}_{t-1} + R^{\mathrm{II}}_t W^{\mathrm{II}}_{t-1} + Y_t$$

where C, Y, W^{I}, and W^{II} denote consumption, income, and the amount invested in financial and real assets, respectively. Solving Formula (1), the first-order condition for an optimal consumption and portfolio choice can be obtained:

$$1 = E\left[\beta \frac{U'(C_{t+1})}{U'(C_t)} R^{\mathrm{I}}_{t+1}\right] \tag{2}$$

and

$$P_t = E\sum_{j=0}^{\infty} \beta^j \frac{U'(C_{t+j})}{U'(C_t)} V_{t+j} \tag{3}$$

which is the well-known asset pricing formula. Let the first asset be the risk-free asset, $R^{\mathrm{f}}_{t+1} = R^{\mathrm{I}}_{t+1}$ and let the second be housing investment, $R^{\mathrm{h}}_{t+1} = R^{\mathrm{II}}_{t+1}$. Recalling that $P_{t+1} + V_{t+1} = R^{\mathrm{h}}_{t+1}$ and focusing on time t and time t+1 period, we get

$$1 = R^{\mathrm{f}}_{t+1} E\left[\beta \frac{U'(C_{t+1})}{U'(C_t)}\right] \tag{4}$$

and

$$P_t = E\left[\beta \frac{U'(C_{t+1})}{U'(C_t)} \left(P_{t+1} + V_{t+1}\right)\right] = E\left[\beta \frac{U'(C_{t+1})}{U'(C_t)} R^{\mathrm{h}}_{t+1}\right] \tag{5}$$

from Eqs. (2) and (3).

In the first step in applying the portfolio choice model for housing investment, we should rethink the role of dwellings.

Firstly, even if housing investment has several special properties, e.g. the requirement of a considerable amount of initial money, large transaction costs, uncertainty about quality, the uniqueness of every unit, relative illiquidity, long implementation time etc., it can be regarded as an investment form. One can state that most households are likely to buy a flat in order to live in it rather than sell it afterwards. Moreover, the above mentioned special properties of dwellings rule out that housing can be an asset. Naturally, this argument is valid to a certain extent. It cannot be ruled out that, if the payoff of housing investment is significantly higher than that of other assets, agents are willing to invest in it.

Secondly, in the microeconomic sense, a house is not simply a ‘roof over one’s head’. Arrondel and Lefebvre (2001) define the dual attitude of households’ decision on housing: a source of housing services and an asset, i.e. housing is taken into consideration in investment decisions. Xiao Di

(2001) examines the roles of dwellings in the USA, where one of these treats housing investment as a form of investment competitive vis-à-vis financial investment. In sum, despite the special properties of dwellings, actors are willing to buy or sell assets if such an activity is profitable irrespective of the type of the asset in question. As a consequence, payoff affects the demand for dwellings.

Finally and most importantly one should realize the fundamental difference between housing investments on micro and macro level. While each house purchase is investment in the micro-economic sense, this is not true macro-economically. It should be noted that, when a household buys a second-hand flat from another household, though this is dwelling investment on the household level, it does not appear so on the aggregate household level. This transaction is a simple asset-change between two households. Briefly, only the first sale of dwellings is considered as housing investment in the macro-economic sense. In other words, dwelling investment of the household sector equals new housing supply.

To take the special characteristics of dwellings into consideration, we define the components of payoff, similar to Cho (1996) and Muellbauer and Murphy (1997). Let

$$R^{\mathrm{h}}_{t+1} = \left[P_{t+1} + V_{t+1}\right] - \left[\delta_{t+1} + m_{t+1} + t^{\mathrm{p}}_{t+1} + r^{\mathrm{m}}_{t+1}\right] \tag{6}$$

where R^{h} denotes the payoff on dwelling, V is the rental fee from tenant, δ is depreciation, m is maintenance cost per unit, t^{p} and r^{m} are property tax and mortgage interest paid. The first brackets represent the benefit from owning house, while the second brackets represent the cost of asset holding. To keep the equation tractable, let us return to the standard notation of payoff R^{h} rather than use this long expression.

Although the demand side is determined, the supply of housing is not modelled explicitly, thus we apply a neutral approach that does not distort our results. In the short run, new housing supply is obviously inelastic, thus increasing demand pushes up house prices. In the long run, the simplest approaches are the perfectly elastic or inelastic supply. The perfectly elastic supply can be ruled out, since housing construction uses a completely constrained resource, i.e. land. The inelastic supply also cannot be maintained, given that the number of dwellings is rising in cities. A plausible approach is that the new housing supply equals the growth rate of population in the long run.

To understand how our model works, let us assume that income and preferences are unchanged and the housing market is at its long-run equilibrium, so that house prices are constant and housing starts are equal to growth in population. Assume an unexpected rise in income, which, in turn, increases the demand for new residence. This leads to an increase in house prices, additional new construction occurs and the city grows in size. At a new equilibrium, the city is physically larger, house prices remain constant at a higher level and housing starts are equal to the long-run steady-state rate again. In this case, house prices will increase in line with income.

To put differently, suppose that the housing market was in steady state, which means the return on housing and other forms of investment are equal. When house prices increase, ceteris paribus, the return on housing increases relative to other asset types, which, in turn, encourages builders to step up production (i.e. to invest more). This intuition is in line with Mayer and Somerwille's (1996). They argue that house prices are a stock variable equilibrating the quantity of housing with total demand. Housing starts (which is a new housing supply) are a flow variable, representing changes in the stock of housing, thus housing starts are a function of other flow variables, such as changes in house price.

To formalize this intuition between income and house prices as well as between house prices and dwellings investment, we apply widely used constant elasticity of substitution (CES) utility function:

$$U(C_t) = \frac{1}{1-\gamma} C^{1-\gamma} .$$

Substituting the utility function into Eq. (5) and assuming that rental fee moves in line with house prices,[2] namely $V_t = \varphi P_t$, so that housing prices are the determinant of dwelling investment payoff and rearranging the equation we obtain an equation which describes the link between consumption and house prices

$$1 = E\left[\beta \left(\frac{1}{\Delta c_{t+1}} \right)^{\gamma} (1+\phi)\Delta p_{t+1} \right] \tag{7}$$

where small letters denote the logarithm of variables. Assume $\gamma \rightarrow 1$, i.e. the utility function is logarithmic. Knowing that households adjust consumption to their permanent income, which is approximated by current income, we get

[2] This assumption is supported by our data. For more details, see the data section.

$$1 = E\left[\beta \frac{1}{\Delta y_{t+1}} (1+\phi)\Delta p_{t+1}\right] \tag{8}$$

Equations (7) and (8) provide some important results of the model. First, Δp_t is proportional to Δy_t, implying co-movement between house prices and income in the long run, which can be captured empirically by an error correction model. Furthermore, if $\gamma \to 1$, the degree of homogeneity equals one. These theoretical results support several empirical studies where the relationship between house prices and income is described in this manner (see Cho (1996), Muellbauer and Murphy (1997), Malpezzi (1998), and the BoE model (2000)). Equation (7) underpins Pain and Westaway's (1994, 1996) approach theoretically, since in their paper housing demand is conditioned on consumer expenditure, and not on disposable income. The disequilibrium between housing demand and supply influences house prices. Pain and Westaway argue that conditioning on consumption ensures that the permanent income measure used in determining the level of consumption is consistently reflected in housing demand.

Second, the ratio of house price volatility to income volatility depends on the value of β and φ. Moreover, if $\left[\beta(1+\phi)\right]^{-1} > 1$, which is true for any reasonable parameter value, the variance of house prices is larger than that of income.[3] The latter one implies that if consumption function contains housing wealth, e.g. in the BoE (2000) model, and the house prices are modelled correctly, then the income variance generates larger fluctuation in consumption compared to the case in which consumption function contains only financial wealth.

In the next step, we derive the portfolio share of two assets.[4] For a more convenient notation, let D_{t+1} denote the stochastic discount factor and define the payoff of housing investment as in Eq. (7):

$$D_{t+1} = \beta \frac{U'(C_{t+1})}{U'(C_t)}\ ,\ R^h_{t+1} = (1+\phi)P_{t+1} \big/ P_t\ .$$

Equation (5) has the following form:

---

[3] The inference is even more obvious if Eq. (8) is re-arranged in deterministic case to obtain the following form $\Delta p_{t+1} = \frac{1}{\beta(1+\phi)} \Delta y_{t+1}$.

[4] This is where the main difference between the deterministic and stochastic case lies. In the deterministic case, the arbitrage condition between assets determinates only the entire amount of the money invested, but it cannot determine the portfolio shares.

$$1 = E\left[D_{t+1} R_{t+1}^{\mathrm{h}}\right] \tag{9}$$

As $E\left[xy\right] = E\left[x\right]E\left[y\right] + \operatorname{cov}(x, y)$, Eq. (9) can be rewritten as

$$1 = E\left[D_{t+1}\right] E\left[R_{t+1}^{\mathrm{h}}\right] + \operatorname{cov}\left[D_{t+1}, R_{t+1}^{\mathrm{h}}\right] \tag{10}$$

It follows from Eq. (4) that $E\left[D_{t+1}\right] = 1 / R^{\mathrm{f}}$, thus

$$E\left[R_{t+1}^{\mathrm{h}}\right] = R_{t+1}^{\mathrm{f}}\left[1 - \operatorname{cov}\left(D_{t+1}, R_{t+1}^{\mathrm{h}}\right)\right] \tag{11}$$

Substitute the form of stochastic discount factor back

$$E\left[R_{t+1}^{\mathrm{h}}\right] = R_{t+1}^{\mathrm{f}}\left[1 - \beta \frac{\operatorname{cov}\left(U'(C_{t+1}), R_{t+1}^{\mathrm{h}}\right)}{U'(C_t)}\right] \tag{12}$$

and the second period constraint yields

$$E\left[R_{t+1}^{\mathrm{h}}\right] = R_{t+1}^{\mathrm{f}}\left[1 - \beta \frac{\operatorname{cov}\left(U'(Y_{t+1} + R_{t+1}^{\mathrm{f}} W_t^{\mathrm{f}} + R_{t+1}^{\mathrm{h}} W_t^{\mathrm{h}}), R_{t+1}^{\mathrm{h}}\right)}{U'(C_t)}\right] \tag{13}$$

It is easy to see how Eq. (13) determines the portfolio shares of different assets. Given a risk-free interest rate and the payoff of the second asset (in our case, housing investment), households have to set the amount of assets in such a way to equalize the above equation through the covariance term. It should be noted that the value of covariance is not independent of the amount of assets. When the R_{t+1}^{h} covaries positively with consumption and thus negatively with $U'(C_{t+1})$, households require higher expected return relative to risk-free rates.[5] When the R_{t+1}^{h} covaries negatively with consumption and thus positively with $U'(C_{t+1})$, households are satisfied with lower expected return relative to risk-free rates (e.g. insurance). The other aspect of Eq. (13) explains how house prices determine the invested amount. When house prices covary positively with consumption the increasing return

[5] Note that the sign of relationship would not be altered if we incorporate the housing stock in utility function. In that case the nominator would be

$$\operatorname{cov}\left(U'(Y_{t+1} + R_{t+1}^{\mathrm{f}} W_t^{\mathrm{f}} + R_{t+1}^{\mathrm{h}} W_t^{\mathrm{h}}, W_{t+1}^{\mathrm{h}}), R_{t+1}^{\mathrm{h}}\right).$$

Since the correlation is determined by $R^{\mathrm{h}}W^{\mathrm{h}}$ term thus the additional, stand alone W^{h} term does not have influence on correlation and thus the above-described mechanism.

on housing investment induces higher housing investment.[6]

Note that the sign of correlation between the rate of return and consumption has another implication. If the payoff of real asset correlated positively with consumption and thus with income then the real asset cannot smooth consumption over time. As a result, contrary to the financial wealth, housing wealth cannot be the source of consumption smoothing.

In order to obtain more detailed view, we rearrange Eq. (12) as

$$E\left[R_{t+1}^{\mathrm{h}}\right]-R_{t+1}^{\mathrm{f}}=-\beta\frac{\operatorname{cov}\left(U'(Y_{t+1}+R_{t+1}^{\mathrm{f}}W_{t}^{\mathrm{f}}+R_{t+1}^{\mathrm{h}}W_{t}^{\mathrm{h}}),R_{t+1}^{\mathrm{h}}\right)}{\left(R_{t+1}^{\mathrm{f}}\right)^{-1}U'(C_{t})} \tag{14}$$

Note that the variance of the consumption period t+1 depends on the variance of the risky-asset payoff at t+1 if the only source of uncertainty is the return of this asset. Assume that the risky-asset payoff is normally distributed, thus consumption growth also follows normal distribution. Using that $R_{t+1}^{\mathrm{f}}=\beta E_{\mathrm{t}}\left[\left(C_{t+1}/C_{t}\right)^{\gamma}\right]$ and $E\left[\mathrm{e}^{z}\right]=\mathrm{e}^{E[z]+\frac{1}{2}\sigma^{2}(z)}$ if z is normally distributed yields

$$E\left[R_{t+1}^{\mathrm{f}}\right]=\exp\left(\ln\beta+\gamma E\left[\Delta c_{t+1}\right]+\frac{\gamma^{2}}{2}\sigma^{2}\Delta c_{t+1}\right) \tag{15}$$

where σ^2 is the variance of the risky asset. Substituting Eq. (15) into Eq. (14), we obtain

$$E\left[R_{t+1}^{\mathrm{h}}\right]-R_{t+1}^{\mathrm{f}}=-\beta\frac{\operatorname{cov}\left(U'(Y_{t+1}+R_{t+1}^{\mathrm{f}}W_{t}^{\mathrm{f}}+R_{t+1}^{\mathrm{h}}W_{t}^{\mathrm{h}}),R_{t+1}^{\mathrm{h}}\right)}{\exp\left(-\ln\beta\text{-}\gamma E\left[\Delta c_{t+1}\right]-\gamma^{2}\sigma^{2}\Delta c_{t+1}/2\right)\;U'(C_{t})} \tag{16}$$

Equation (16) provides a richer explanation for portfolio choice. The left-hand side of Eq. (16) defines the excess return, which is equal to the risk

[6] Suppose that the return on housing investment increases. Provided that the house prices covary positively with consumption and thus negatively with ,$U'(C)$ the equality of Eq. (13) can be held if the value of right squared bracket, i.e. cov(.), increases, which is the case when housing investment increases.
Numerous studies assume the same characteristic housing investment function in an exogenous way, and simply integrate it into the models, which they use. Porteba (1984) assumes that $I = f(P)$, where I and P denote housing investment and house prices, respectively. Since his model is deterministic, the arbitrage condition cannot determinate portfolio shares (See Footnote 4), thus an exogenous investment function is necessary in order that the amount of housing investment can be derived.

adjustment. Note that the structure of the optimal portfolio depends not only on the size of payoff, but also on the riskiness (variance) of payoff. First, the mechanism of return is the same as the one mentioned above. When the payoff of the asset covaries positively with consumption, it must promise a higher expected return than risk-free rates do, in which case the excess return is positive. In that case, the asset has to offer a higher expected return, i.e. a higher excess return to induce investors to hold more of this asset. Second, the riskiness and the amount invested into risky asset depend inversely, i.e. higher risks ceteris paribus reduce the share of risky asset.[7] In the section on empirical results, we examine whether the above assumption and properties can be underpinned by empirical evidence.

Data

For the purposes of empirical examination, we test the model for the UK housing market. Two data sets are used: the annual house prices and disposable income figures available from 1948. House prices are published by National Statistics of UK in the following structure:

– 1946 to 1952: a house price index for modern, existing dwellings was calculated by the Co-operative Building Society from 1946 (=100) to 1970. The movements in the index from 1946 to 1953 have been applied to the average 1953 price, in order to impute average prices for 1946-1952.
– 1953 to 1955 derived from the average of two series of UK projected house prices.
– From 1956 to 1965 prices are based on the BS4 survey of mortgage completions for new dwellings. No adjustment has been made to allow for the absence of existing dwellings. Whilst in recent years average prices of new dwellings have often been more than 10% higher than the average for all dwellings, this was not the situation from 1966 to 1974, the first years when BS4 data both for new and all dwellings were available.
– From 1966 to 1992, average prices are based on the 5% survey of building societies. From 1969, the mix-adjusted index is also based on the survey of building societies.
– From 1993 to date, average prices and the mix-adjusted index are based on the 5% survey of mortgage lenders.

Second, we use quarterly data sets (disposable income, house prices, net

[7] On the basis of Eq. (16), increasing variance reduces the value of the denominator. So, for the equality to be held, the nominator has to decrease. Since the return is unchanged, the covariance might be lower if the amount invested in increasing riskiness asset is lower.

financial saving, and dwellings investment) from 1984 Q1 because housing investment data are available from that time.[8] The yearly data set is used parallel to quarterly data to estimate house price equations.

Rental fees are available from 1992 at Housing Finance Review published by the Centre for Housing Policy. We found that the ratio of rental fee to house price is quite stable, thus rental fee before 1992 is computed as the constant share of house prices.

The above data can be understood and measured more easily than housing investment. As we argued earlier housing investment on the micro level is different from housing investment on the macroeconomic one thus only the first sale of dwellings is considered as housing investment in the macro-economic sense. In other words, dwelling investment of the household sector equals new housing supply, for which data are available in national statistics.

Empirical Results

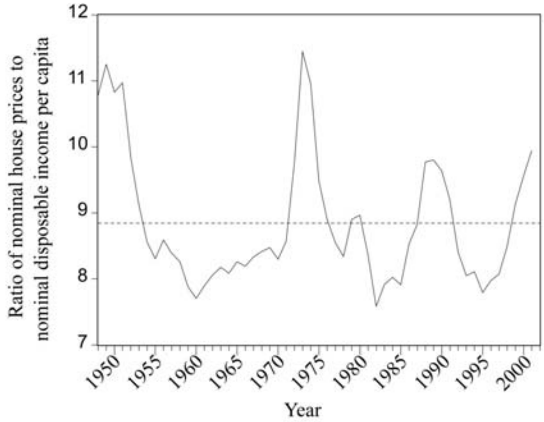
To evaluate its theoretical implications, we examine empirically the two main suggestions of the model. First, whether the house prices move together with income (see Eq. (8)) and whether the error correction approach of house prices and income dynamics is appropriate. Second, whether the excess return and riskiness can determinate the portfolio choice between assets and, if so, in what way. For the sake of tractability we concentrate only on two aggregated assets: net financial savings and housing investment and their development.

House price

Equation (8) states that house prices and income move together in the long term and house prices have higher volatility. Due to the longer implementation time of house building and the higher volatility of house prices compared to income, house prices start to rise more strongly than income in the short term. When new buildings are completed, causing housing supply to increase, house prices decrease and the ratio of house prices to income returns to its steady-state value. Figure 1 displays the ratio of house prices to income per capita, which seems to fluctuate around a stable value in the UK.

[8] The time series, where necessary, is seasonally adjusted.

Figure 1: Ratio of house price to income in the UK from 1948



The charts do not reject our hypothesis about the stable house prices to income ratio. Of course, there can be bubbles and deviations in the short term. The above-mentioned fact implies the applicability of error correction models, where we are able to formalise long-term restrictions allowing short-term dynamics and use econometric tests to verify this question empirically. The estimated framework is the following:

$$\Delta p_t = \gamma_1 + \gamma_2(p_{t-1} + \alpha_1 - \alpha_2 y_{t-1}) + \sum_{i=3}^{4} \gamma_i \Delta p_{t-(i-2)} + \gamma_5 \Delta y_t \qquad (17)$$

where p is the log of house price per square metre. Since the UK time series is long enough, we apply richer short-run dynamic and estimate the long and short-run parameters in the same step by non-linear least squares (NLS) (in this case $\alpha_1 = 0$, because Eq. (17) has an other constant, γ_1).

For the sake of obtaining robust results, the estimation of UK parameters has been performed on both yearly data from 1948 and quarterly data from 1984 Q1. The upper section of Table 1 displays the estimated parameters of the unrestricted house price equation. A glance at the statistics in Eq. (17) seems to be appropriate. All of the parameters are different from zero at standard significance levels.[9] The first important thing is whether the stability of the house price ratio to income or, econometrically, the [1 –1] cointegration vector is an acceptable assumption.[10] This assumption implies the long-run

[9] Except γ_1 and γ_5 in yearly UK model. γ_1 is the constant is the regression, so it is not important and γ_5 will be acceptable after restriction. See more details later.

[10] To test correctly the cointegration assumption, we examined the residuum of $ph_t = \alpha_1+\alpha_2 y_t$ equation when α_2 is estimated and α_2 are restricted to 1. According to the ADF and PP unit root

homogeneity between house prices and income, i.e. the α_2 parameter should be equal to 1. According to the Wald test, the restriction that α_2 equals 1 cannot be rejected for either the yearly or the quarterly data.

Table 1: Estimation results of house price equation

$$\Delta ph_t = \gamma_1 + \gamma_2 (ph_{t-1} + \alpha_1 - \alpha_2 y_{t-1}) + \sum_{i=3}^{n} \gamma_i \Delta p_{t-(i-2)} + \gamma_{n+1} \Delta y_t$$

Parameters	UK (n=4)	
	Quarterly	Yearly
α_1	-	-
	-	-
$\alpha_2 y_{t-1}$	0.76	0.97
	(0.421)	(0.022)
γ_1	–0.15	–0.40
	(0.183)	(0.128)
$\gamma_2 \mathrm{ECM}_{t-1}$	–0.03	–0.25
	(0.016)	(0.086)
$\gamma_3 \Delta ph_{t-1}$	0.48	0.48
	(0.115)	(0.126)
$\gamma_4 \Delta ph_{t-2}$	0.34	–0.22
	(0.120)	(0.139)
$\gamma_5 \Delta y_{t-1}$	0.10	0.95
	(0.184)	(0.224)
Adjusted R^2	0.52	0.50
LM-SC(2) p value:	0.38	0.36
ARCH(1) p value	0.42	0.12
White HET p value	0.63	0.96
Wald test p value of $\alpha_2 = 1$	0.57	0.23
Wald test p value of $\gamma_3 + \gamma_4 = 1$	0.67	0.28
α_1	-	-
	-	-
$\alpha_2 y_{t-1}$	1	1
	-	-
γ_1	–0.23	–0.32
	(0.110)	(0.108)
$\gamma_2 \mathrm{ECM}_{t-1}$	–0.03	–0.17
	(0.015)	(0.059)
$\gamma_3 \Delta ph_{t-1}$	0.49	0.46
	(0.111)	(0.124)
$\gamma_4 \Delta ph_{t-2}$	0.34	–0.28
	(0.118)	(0.128)
$\gamma_5 \Delta y_{t-1} = (1 - \gamma_3 - \gamma_4) \Delta y_{t-1}$	0.172	0.826
	-	-
Adjusted R^2	0.54	0.51
LM-SC(2) p value:	0.39	0.27
ARCH(1) p value	0.42	0.06
White HET p value	0.59	0.99

Standard errors are in brackets.

tests, the residuum is stationary in both cases.

In addition to the long-run homogeneity assumption, we test an additional restriction. To get a correct steady ratio, we should examine whether the sum of short-run parameters equals one. If we ignore this, the short-run dynamics bias the long-run equilibrium (see Appendix A.1). In our case, $\gamma_3+\gamma_4+\gamma_5=1$ is the testable restriction, which means that the sum of the estimated parameters of lagged values of house price and income growth should be equal to one. The Wald test accepts this restriction (the p value of Wald test is 0.67 and 0.28 in UK quarterly and yearly models respectively). Thus we have restricted the parameter and re-estimated Eq. (17).

The new estimated parameters are shown in the bottom section of Table 1. According to the test statistics, the restricted version of Eq. (17) is an acceptable representation of house price movements. Consequently, these empirical results underpin the co-movement of house prices and income derived from our model.

Another testable implication of the model is related to the variance of house prices and income. Empirical tests underpin Eq. (8), which implies that the growth rate of house prices has higher variance than income has. Based on Table 2, this hypothesis can be held in the sample period.

Table 2: Test for equality of variances between growth rate of house prices and income

Method	df	Value	Probability
F-test	(71, 71)	5.75	0.00
Siegel-Tukey		5.19	0.00
Bartlett	1	48.17	0.00
Levene	(1, 142)	32.33	0.00
Brown-Forsythe	(1, 142)	31.54	0.00

where $\mathrm{var}(P_t/P_{t-1}) = 0.029$ and $\mathrm{var}(Y_t/Y_{t-1}) = 0.012$

Excess Return

One can see in the stochastic case how excess return and the riskiness of assets determine the portfolio choice. Investors invest more in assets if its payoff (or, equivalently, excess return) increases, provided positive covariance between return and consumption, and invest less if the volatility of expected return rises. Thus, our special interest is in the excess return (ER) on holding real estate above financial assets.

Equation (6) describes the components of return on real estate; however, we apply an empirical approximation of the excess return:[11]

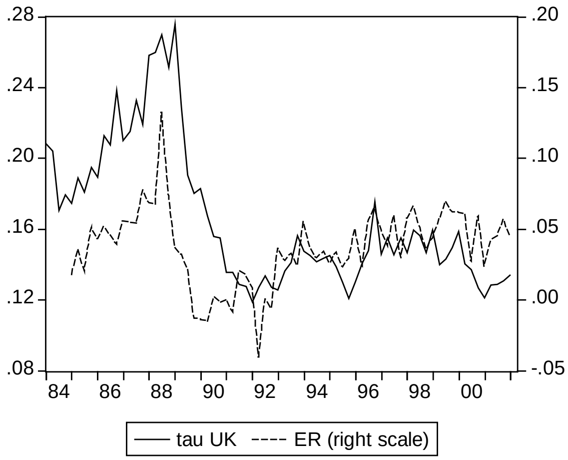
$$\mathrm{ER}_t = \frac{P_t + V_t}{P_{t-1}} - r_{t,3} \tag{18}$$

where P, V, and r_3 denote the house price per square meter, rental fee, and yield on three-month risk-free bond respectively. The return on real asset is defined between t+1 and t, that is why we apply the three-month bond. Let τ denote the ratio between dwelling and financial investment:

$$\tau = \frac{\mathrm{HI}_t}{\mathrm{HI}_t + \mathrm{NFS}_t} \tag{19}$$

where HI is housing investment and NFS is net financial savings. Figure 2 displays historic values of the excess return and τ.

Figure 2: Ratio of housing investment to gross savings and excess return



τ is the rate of housing investment in relation to gross savings (financial savings plus dwelling investment), ER is the excess return.

As mentioned earlier, there is a positive relationship between the portfolio share of housing investment and the return on it, if consumption and return or, more specifically, house prices covary positively. Since both consumption and house prices are not stationary in level (as they increase continuously), we compute the covariance of their growth rates. To be able

[11] Aoki et al. (2002) also define the expected gross return on housing in a similar way

to evaluate the size of the relationship, we present the correlation rather than the covariance:

$$\mathrm{cor}(\Delta p_y^{\mathrm{UK}}, \Delta c_y^{\mathrm{UK}}) = 0.55\,,\ \mathrm{cor}(\Delta p_q^{\mathrm{UK}}, \Delta c_q^{\mathrm{UK}}) = 0.49\,,$$

where the subscripts denote yearly and quarterly data. The covariance is positive and significant on yearly and quarterly UK data, suggesting that a higher return on dwelling investment implies a higher portfolio share of housing.

Since return on housing covaries positively with consumption, Eq. (13) implies that households require a higher return on this asset, thus the excess return has to be positive. According to the test statistics, the sample mean of excess return is positive and differs significantly from zero.[12]

In addition to the covariance analysis, Figure 2 provides some results in respect of households' portfolio decisions between financial and dwelling investments or, in other words, the ratio of housing investment to gross savings. (Note that gross saving is the difference between disposable income and consumption expenditure *id est*. households' funds available for investment). Figure 2 shows the co-movement of excess return and the dwelling investment rate. Due to the long construction time, we expect leading property of excess return compared to the investment rate.

Our expectation is supported by the cross-correlation coefficient[13] (see Figure 3). The high cross-correlation coefficients at $i = -1$ and -2 (0.46 and 0.50 respectively) mean that changes in the excess return lead the changes in the investment ratio. This result is corroborated by the Granger causality test (see Table 3) with a slight difference. Based on the cross-correlation, coefficients from $i = -1$ to -6 lead seem to be appropriate, while the Granger test (see Table 3) shows the only one period lead property compared to the lag values of excess return.

In fact, not only the return on investment, but also the variance of yield, is an important factor in investment decisions. If investors are risk averse, they will prefer an asset with lower volatility in their choice between two assets with the same return. In our case this means that, if there are stronger-than-usual fluctuations in the excess return around its expected value, this will discourage investors from investing in dwellings.[14] We proxy the riskiness of

---

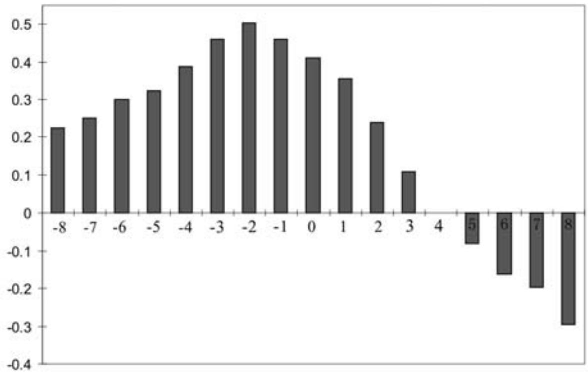
[12] H_0: *Mean*(ER) = 0, t-statistic = 11.35, probability of H_0 = 0.00.

[13] The diagram at Figure 3 displays the cross-correlation coefficients as a function of i, where i denotes the leading or lagging periods of excess return. Negative i means that the excess return leads the housing investment rate, while positive i implies that the excess return lags compared to the housing investment rate.

[14] Owing to the fact that the real interest rate is smoother than changes in house prices, the

real estate investment by the square of excess return minus the sample average of excess return.

Figure 3: Cross-correlation between housing investment rate and excess return



The cross-correlation between the rate of housing investment to gross savings and the excess return, exactly the $f(i)$= Corr(τ, ER(i)) function.

Table 3: Granger causality test between the τ and ER in UK

i = 1		i = 2		i = 3	
lead	lag	lead	lag	lead	lag
0.07	0.37	0.09	0.06	0.07	0.00
i = 4		i = 5		i = 6	
lead	lag	lead	lag	lead	lag
0.01	0.00	0.02	0.01	0.04	0.00

p value of Granger causality test where the lagged and leaded variable is the excess return.

volatility of excess return is driven by house price fluctuation. In the UK sample, the variance of growth rate of real house prices is higher than that of yield on three-month risk-free bonds, and tests have rejected the hypothesis that the variance of these two series is equal (See Table 4).

Table 4: Test for equality of variances between rate of return on housing and yield on three-month risk-free bond

Method	df	Value	Probability
F-test	(68, 71)	36.80	0.00
Siegel-Tukey		3.97	0.00
Bartlett	1	154.26	0.00
Levene	(1, 139)	66.93	0.00
Brown-Forsythe	(1, 139)	63.59	0.00

where $\mathrm{var}(P_t+V_t/P_{t-1}) = 0.026$ and $\mathrm{var}(r_3) = 0.017$

We expect that the higher risk, i.e. the higher value of adjusted excess return squared, will reduce the attractiveness of real estate, causing the share of dwelling investment to decline within gross savings. Table 5 comprises the results of the Granger causality test, suggesting that three-quarter or more lead the squared excess return help to explain the ratio of dwelling investment to gross savings.

Table 5: Granger causality test between the τ and $(\mathrm{ER}-\overline{\mathrm{ER}})^2$ in UK

$i = 1$		$i = 2$		$i = 3$	
lead	lag	lead	lag	lead	lag
0.76	0.18	0.79	0.27	0.02	0.44
$i = 4$		$i = 5$		$i = 6$	
lead	lag	lead	lag	lead	lag
0.01	0.10	0.02	0.09	0.04	0.15

p value of Granger causality test where the lagged and leaded variable is the squared excess return.

Based on the cross-correlation coefficient and the Granger causality test and in order to get an econometrically correct residuum,[15] we use the following equation to explain the ratio of housing investment to gross savings:

$$\tau_t^{\mathrm{UK}} = \eta_1 + \eta_2 \tau_{t-1}^{\mathrm{UK}} + \eta_3 \tau_{t-2}^{\mathrm{UK}} + \eta_4 \mathrm{ER}_{t-1} + \eta_5 (\mathrm{ER}_{t-4} - \overline{\mathrm{ER}})^2 \tag{20}$$

Although Eq. (20) is easy to interpret, the problem we are facing is that the ratio of housing investment to gross savings must be between 0 and 1. The linear model cannot assure the fulfilment of constraint, which causes serious problems in integrating this block into a larger model. The values of the $(1+\mathrm{e}^{-x})^{-1}$ logistic function are between 0 and 1, thus the function form

[15] We tested different specifications. Although the parameters were significant, not all the residuum tests were acceptable in these cases, so we chose the ones that were the best econometrically.

seems to be appropriate:[16]

$$\tau_t^{\mathrm{UK}} = \frac{1}{1+\exp\left[-(\eta_1+\eta_2\tau_{t-1}^{\mathrm{UK}}+\eta_3\tau_{t-2}^{\mathrm{UK}}+\eta_4\mathrm{ER}_{t-1}+\eta_5(\mathrm{ER}_{t-4}-\overline{\mathrm{ER}})^2)\right]} \tag{21}$$

Both Eqs. (20) and (21) provide similar results (See Table 6 and Figure 4). As expected, the estimated sign of η_4 is positive, implying that increasing house prices yield a higher housing investment ratio and, consequently, higher housing investment. The estimated η_5 is negative, implying that higher risk in real estate investment discourages investors from making housing investment, which, in turn, reduces the ratio of dwellings to gross savings.

Figure 4 displays the fitted value of the ratio of housing investment to gross savings. So as to measure the additional explanatory power of excess return over the auto-regressive structure, we compare the increment in adjusted R^2. Restricting the η_4 and η_5 to 0 and estimating Eq. (20) we find that the adjusted R^2 is 0.86. When excess return and its variance are used as explanatory variables in the estimation, the adjusted R^2 increases to 0.89.

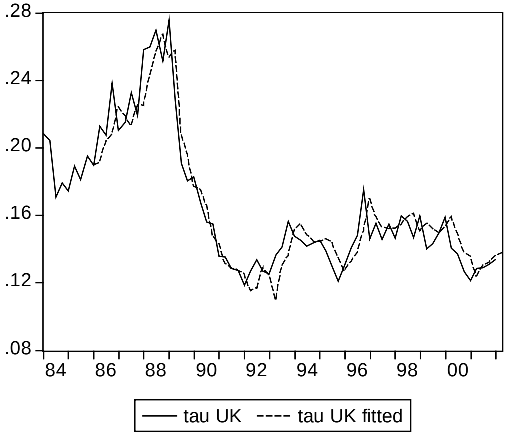
Table 6: Estimation result of housing investment ratio in UK

Parameters	Linear	Logistic
η_1	0.01	–2.70
	(0.007)	(0.049)
η_2	0.60	3.23
	(0.125)	(0.789)
η_3	0.33	3.05
	(0.126)	(0.813)
η_4	0.13	0.89
	(0.069)	(0.477)
η_5	–3.43	–35.01
	(1.335)	(9.458)
Adjusted R^2	0.890	0.897
LM-SC(2) *p* value	0.20	0.20
ARCH(1) *p* value	0.36	0.82
White HET *p* value	0.04	0.06
Ramsey RESET test *p* value	0.14	-

Standard errors are in brackets.

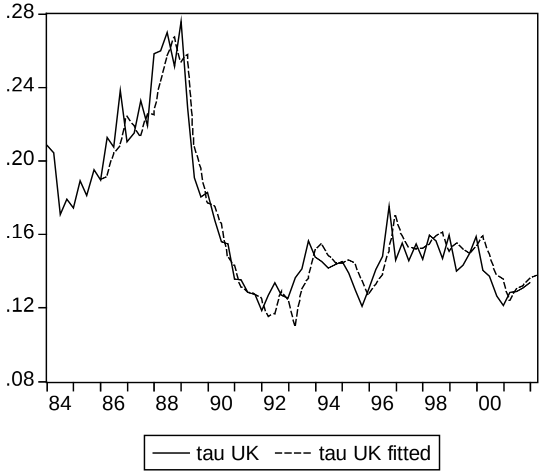
[16] If $x \to \infty$ then $(1+\mathrm{e}^{-x})^{-1} \to 1$ and if $x \to -\infty$ then $(1+\mathrm{e}^{-x})^{-1} \to 0$.

Figure 4a: Fitted value of the ratio of housing investment to gross savings (liner model)



τ is the rate of housing investment to gross savings.

Figure 4b: Fitted value of the ratio of housing investment to gross savings (logistic model)



τ is the rate of housing investment to gross savings.

To summarize these results we can conclude that the theoretical suggestions, namely the portfolio share of housing investment depends on the excess return and the variance of excess return, is underpinned empirically.

Summary

In this paper, we outlined an easily applicable micro-based model, which explains households' portfolio decisions in a stochastic environment. The model has several implications for households' behaviour. First, we found that house prices move in line with households' income. Second, house prices have higher volatility than income does. Third, the effect of expected return and the riskiness of an asset on portfolio choice depend on the covariance between consumption and expected return. If the return on an asset covaries positively with consumption, households expect a higher return on this asset. In addition, increasing expected return implies higher investment, whereas increasing riskiness results in more modest investment in this type of asset. The positive correlation between real asset and consumption also implies that the housing wealth cannot smooth consumption over time.

Although dwellings have several special characteristics, housing investment can be considered as a component of portfolio choice and we applied the changes in house prices as the main determinant of return on it. To test the implications of the model, we used UK data. First, we found that the ratio of house prices to disposable income has displayed a stable value in the past and empirical tests in the error correction model also accept the hypothesis that house prices and income are closely related. Second, we found a positive correlation between UK house prices and consumption, which implies a positive connection between higher excess return on dwellings and housing investment. The cross-correlation coefficient and the Granger causality tests suggest leading property of excess return related to housing investment, which can be explained by the long implementation time of house construction. Regressing the ratio of housing investment to gross savings on leads of excess return and the volatility of excess return, we found positive coefficients on the excess return and negative ones on volatility. These results empirically underpin our expectations based on the theoretical model.

References

Aoki, Proudman and Vlieghe (2002). House Price, Consumption and Monetary Policy: a Financial Accelarator Approach, Working Paper No. 169, Bank of England.

Arrondel, L. and Lefebver, B. (2001). Households' portfolios behaviour in France: The role of housing, presented at the International Conference of the American Real Estate and Urban Economics Association at Berkeley.

Bank of England (2000). Economic models at the Bank of England, Bank of England.

Bernanke, B., Gertler, M. and Gilchrist, S. (2000). The financial accelerator in a quantitative Business framework, *Handbook of Macroeconomic*, (North Holland).

Case, K. E., Quigley, J. M. and Shiller, R. J. (2001). Comparing wealth effects: the stock market versus the housing market, Working Paper No. 8606, National Bureau of Economic Research.

Chen, M. and Patel, K. (1998). House price dynamics and Granger causality: An analysis of Taipei New Dwelling Market, *International Real Estate Review*, **1**, 1.

Cho, M. (1996). House price dynamics: A survey of theoretical and empirical issues, *Journal of Housing Research*, **7**, 2, 145-172.

Cochrane, J. H. (2001). *Asset Pricing*. Princeton University Press.

Davidoff, Thomas (2002). Labor income, housing prices and homeownership, mimeo, Department of Economics, MIT.

Fagan, G. Henry, J. and Mestr, R. (2001). An area-wide model (AWM) for the euro area, Working Paper No. 42, European Central Bank.

Girouard, N. and Blöndal, S. (2001). House price and economic activity, Economic Department Working Paper No. 279, OECD.

Hendry, D. and T. von Ungern Sternberg (1981). Liquidity and inflation effect on consumers, In A. Deaton (ed.) Essay in the theory and measurement of consumer behaviour. Cambridge University Press, Cambridge.

Hendry, D. F. (1997). *Dynamic Econometrics*. Oxford University Press, Oxford.

Hsueh, L. (2000). The relationship between housing price, tenure choice and saving behaviour in Taiwan, *International Real Estate Review*, **3**, 1.

Jakab, M. Z and Kovács, M.A (2002). Hungary in the NIGEM model, Working Paper No.2002/3, Magyar Nemzeti Bank.

Jakab, M. Z. and Vadas, G. (2001). Forecasting Hungarian Household consumption with econometric methods, Background Studies 2001/1, Magyar Nemzeti Bank.

Johansen, S. (1988). Statistical analysis of cointegration vector, *J. Econ. Dynam. Control*, **12**, 231-254.

Johansen, S. (1992). Determination of cointegration rank in presence of linear trend, *Oxford Bulletin Econ. Stat.*, **54**, 383-398.

Kiss, G (2002). The housing market and financial stability in the light of EU accession, Report on Financial Stability, December, MNB.

Malpezzi, S (1998). A simple error correction model of house price, The Centre for Urban Land Economics Research, University of Wisconsin.

Mayer, C.J. and Somerville, C.T. (1996). Unifying empirical and theoretical models of housing supply, Working Paper No. 96-12, Federal Reserve Bank of Boston.

Muellbauer, J. and Lattimore, R. (1995). The consumption function: a theoretical and empirical overview, Handbook of Applied Econometrics, Macroeconomics, Blackwell.

Muellbauer, J. and Murphy, A. (1997). Booms and bust in the UK housing market, *The Economic Journal*, **107** (November), 1701-1727.

Pain, N. and Westaway, P.F. (1994). Housing, consumption and borrowing: an econometric model of personal sector behaviour, National Institute of Economic and Social Research.

Pain, N. and Westaway, P.F. (1996). Modelling structural changes in the UK housing market: a comparison of alternative house price models, Discussion Paper no. 98, National Institute of Economic and Social Research.

Poterba, J.M. (1984). Tax subsidies to owner-occupied housing: an asset market approach, *Quarterly Journal of Economics*, **99** (November), 729-752.

Westaway, P.F. (1992). A simulation model of consumer spending and housing, Discussion Paper no. 15, National Institute of Economic and Social Research.

Xiao Di, Z. (2001). The role of housing as a component of household wealth, Joint Center For Housing Studies, No. W01-6, Harvard University.

Appendix

A.1 Parameter restriction in VECM

Consider the following arbitrary VAR for p lags on the vector of N variables $\boldsymbol{z}_t$

$$\boldsymbol{z}_t = \sum_{k=1}^{p} \boldsymbol{D}_k \boldsymbol{z}_{t-k} + \delta + \varepsilon_t \quad \varepsilon_t \sim \mathrm{IN}_{\mathrm{N}}(\mathbf{0}, \Sigma) \tag{22}$$

$\boldsymbol{D}_j$ is an $N\times N$ matrix. δ is $N\times 1$ intercept vector and $\boldsymbol{\varepsilon}_t$ is a vector of errors with zero mean and constant covariance matrix Σ. Independently of whether the variable $\boldsymbol{z}_t$ is I(0) or I(1), the VAR in Eq. (22) can be reparameterised as a VECM (see Johansen (1988, 1992) and Hendry (1995))

$$\Delta \boldsymbol{z}_t = \sum_{k=1}^{p-1} \boldsymbol{\Gamma}_k \Delta \boldsymbol{z}_{t-k} + \boldsymbol{\Pi} \boldsymbol{z}_{t-1} + \boldsymbol{\delta} + \boldsymbol{\varepsilon}_t \,,\ \boldsymbol{\varepsilon}_t \sim \mathrm{IN}_{\mathrm{N}}(\mathbf{0}, \boldsymbol{\Sigma}) \tag{23}$$

where $\boldsymbol{\Gamma}_k = -\sum_{i=j+1}^{p} \boldsymbol{D}_i$ ($j \in (1, 2, ..., p-1)$) is $N{\times}N$ matrix of short-run parameters, $\boldsymbol{\Pi} = -(\boldsymbol{I}_N - \sum_{i=1}^{p} \boldsymbol{D}_i)$ is $N{\times}N$ matrix of long-run parameters. Although Eqs. (22) and (23) are equivalent of each other, Eq. (23) is more attractive due to the interpretation of its static long-run solution, namely $E(\boldsymbol{\Pi z}_t + \boldsymbol{\delta}) = 0$ means the equilibrium of the system.

In the static long-run solution, $E(\boldsymbol{\Pi z}_t + \boldsymbol{\delta}) = 0$, $E(\boldsymbol{\varepsilon}_t) = 0$ and $\Delta \boldsymbol{z}_t = \Delta \boldsymbol{z}_{t-k}$ for every k, thus

$$\Delta \boldsymbol{z} = \sum_{k=1}^{p-1} \boldsymbol{\Gamma}_k \Delta \boldsymbol{z} \tag{24}$$

rearrange it,

$$(\boldsymbol{I} - \sum_{k=1}^{p-1} \boldsymbol{\Gamma}_k) \Delta \boldsymbol{z} = 0 \tag{25}$$

supposing that $\Delta \boldsymbol{z} \neq 0$ yields

$$\boldsymbol{I} - \sum_{k=1}^{p-1} \boldsymbol{\Gamma}_k = 0 \tag{26}$$

which implies:

$$1 = \sum_{k=1}^{p-1} \sum_{j=1}^{N} \boldsymbol{\Gamma}_{k,i,j} \quad \text{for every } i \text{ where } i \in (1, 2, ..., N) \tag{27}$$

where $\boldsymbol{\Gamma}_{k,i,j}$ denotes the i-th (row) j-th (column) element of the k-th $\boldsymbol{\Gamma}$ matrix, thus i denotes the i-th equation in VECM. In other words, Eq. (27) means the sum of short-run parameters should equal one in every equation of VECM.