The Chinese Housing Provident Fund

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The Chinese government has embarked upon an effort to reduce the number of tenants living in publicly owned housing. This is a significant challenge for any government, but may be especially so for a country where private homeownership is a new option. Out of concern that many of its citizens could not afford to purchase their housing units, the Chinese government created the Housing Provident Fund. This program, which is similar to housing fund programs in other countries such as Thailand and Singapore, combines a 401(k)-like savings and retirement account with subsidized mortgage rates and price discounts to provide a mechanism through which an employee could save for, and eventually complete, a housing purchase.

The housing provident fund (HPF) is very complex. It presents the participant with a number of options that can greatly affect their lifetime wealth. This paper presents a model of the HPF from the participating employee's perspective, and then uses simulations to examine how the fund would affect the employee's lifetime wealth under a variety of program parameters. From this information one can infer the optimal behavior for a rational program participant. It is also possible from the simulation results to infer which program parameters are the most likely to provide the best incentives to the employee to minimize the time that they stay in the program before they purchase their housing unit.

Keywords

provident fund, housing finance, China housing

Introduction

Since the early 1980's, China has maintained two separate housing systems. The Chinese government established the first system in the early 1950's. In that system, the government, government organizations, or state-owned enterprises directly own and allocate housing. In the second system, individuals privately own housing and trade it within a free market. During the past two decades the Chinese government has implemented a series of policies designed to privatize government owned housing.

Some of the difficulties that the Chinese government faces in this effort are similar to the difficulties that many Western governments face when privatizing public housing. One particular difficulty is that potential purchasers lack enough wealth to purchase the property outright, and they lack the income to make mortgage payments. To help alleviate this problem the Chinese government began a program under which it subsidizes savings for the purpose of purchasing formerly public housing. The purpose of this paper is to develop an economic model of the system, explore its comparative statics, and then illustrate its implications using a simulation model.

The outline of the paper is as follows. The next section describes the basic structure of the Chinese housing system with a particular focus on the operation of the public housing system. This section also discusses the difficulties that the Chinese government has had in privatizing public housing. The third section explains the details of the savings subsidy system implemented by the Chinese government. The fourth section presents a utility-based economic model of the system, while the fifth section presents simulation results that demonstrate the wealth effects the program would have on participants. Finally, the sixth section examines some policy implications of the model, summarizes and concludes the paper.

The Chinese Housing System

China has allowed the private development and ownership of housing since the early 1980's, although the cost of privately owned housing has kept the

majority of Chinese citizens from purchasing their own property.[1] Indeed, many of the privately developed housing units either sit empty, or are purchased by companies and then provided to employees at subsidized rental rates as part of their compensation package.[2,3] For most Chinese citizens living in cities their only real choice has been to live in state-owned housing.

Historically the rent that the Chinese government charges for public housing has been below the cost to maintain the property, much less a market rental rate.[4] As a result, housing was largely allocated through non-price methods. Usually the size and quality of the allocated housing was a function of the employee's occupational rank, although seniority and family size also played a role in the allocation. For example, in 1998, a Senior Staff member in Beijing[5] would be assigned a 75 square meter apartment; a Section Director, i.e., a manager with approximately 10 years work experience, would be assigned a 100 square meter apartment, and a Department Director, i.e. a senior manager with 20 years work experience, would be assigned a 150 square meter apartment.[6] In all cases the rent paid by the tenant was trivial. Rent and utilities accounted for 1.91% to 4.51% of household income from 1986 to 1997.[7]

During the 1980's and into the early 1990's, the government became increasingly unwilling and unable to continue the heavy subsidization of rents. The nation's central government mandated that city governments phase in considerable rent increases. In fact, the central government required rent to account for 15% of average household income by year 2000.[8] For example, in Shenyang rents were raised by approximately 30% annually from 0.15 Yuan per square meter of usable space per month to 1.98 Yuan per square meter in 2000. The Beijing city government raised its rent from 1.52 Yuan to 3.05 Yuan.[9] It should be noted, however, that in some cases the

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[1] One could argue that there are other reasons beyond income constraints for this lack of participation in the private sector housing market. Such reasons could include a lack of units, a lack of confidence in the willingness of the Chinese government to protect property rights, and a lack of economic sophistication on the part of potential housing owners. It is also very possible that the public housing is priced so far below an equilibrium market rent that it undercuts the private market. A thorough investigation of the reasons for this, however, is beyond the scope of this paper.

[2] According to the 1999 State Statistical Bureau Statistics, in 1997 and 1998 the vacancy rate for privately developed housing units was 51%.

[3] People's Daily, Overseas Edition, July 11, 1998.

[4] Gu and Colwell (1997).

[5] A senior staff member would normally be somebody with a College degree and at least five years of work experience.

[6] People's Daily, Overseas Edition, November 16, 1998.

[7] Statistical Yearbook of China, 1999

[8] The State Council's Decision on Further Urban Housing Reform, [1994] no. 43, July 18, 1994

[9] Beijing City Government Housing Reform Office, Document 2000-080

increased rents were then at least partially offset by direct monthly cash payments from the government. For example, in Beijing the near doubling of rents was tempered by monthly cash subsidies of between 90 and 150 Yuan per month. For a Senior Director living in a 150 square meter apartment, their rent may have increased from 228 Yuan per month to 457, but, after factoring in the 150 Yuan cash subsidy, their real rent increased by only 79 Yuan per month.[10]

In 1990 the Chinese government began a program that encouraged occupants of public housing to purchase their homes. During 1998, the government announced that it intended to completely end the old housing allocation system. Although the Chinese government argues that up to 70% of the urban public housing had been privatized by 2000,[11] the privatization program was not particularly successful in its early years. It was assumed that one reason for this lack of success was that Chinese citizens lacked the wealth to make a mortgage down payment and the income to make monthly payments.

It is probably unrealistic to believe that all residents of Chinese public housing will be able to eventually purchase their housing. Even in very affluent and developed countries such as the United States or Britain there are sizeable populations that lack the wealth or income to purchase their own housing and live in public housing. In a country such as China, where the social acceptability of wealth accumulation is not as strong, one might reasonably expect the percentage of the population living in public housing to be even higher. The Chinese government realizes, however, that one way to increase the home ownership rate is to increase the wealth level of those people that still live in publicly owned housing. To this end the Chinese government has created the housing provident fund, a savings program for housing purchases that also helps fund retirement for its members.

China is not alone in using a provident fund system to fund retirement and housing savings. A number of countries, including Singapore, India, and Thailand, have implemented such systems, with varying degrees of success. Although there are some common traits in each of these systems there are also significant differences among them in terms of their methods and goals. Some programs, such as the Chinese and Singapore plans, are compulsory for the majority of the population. Others, such as the Thai and Indian plans, are voluntary for large portions of the population. The Chinese and Indian

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[10] *Ibid.*

[11] People's Daily, Overseas Edition, March 30, 2000. We note that much of China's rural housing is still public, and it is not clear whether all of the housing that has been privatized has been sold to individuals, or if some of it has been transferred to the balance sheet of other state affiliated institutions.

programs pay a fixed rate of return on funds invested in the program; the Singapore and Thai plans allow the individual to invest in a variety of asset classes, including annuities, stocks, bonds, and gold.

Perhaps the most common trait among these plans, however, is their complexity. The programs require their members to make very sophisticated financial decisions, and endow their participants with a variety of choices. The next section describes the program in China in detail.

The Housing Provident Fund System

The housing provident fund was initially introduced as a pilot program in Shanghai in late 1991, and was extended nationwide in 1995. The program is open to employees of government agencies, state enterprises, universities, hospitals, and some semi-state companies. Recently the government has mandated that all eligible employees must join the program, although privately employed workers do not have to participate.

An employee that joins the program agrees to have between 4% and 10% of their salary deposited into a special account in a state owned bank. While this account only pays the risk-free interest rate, the participant's employer provides a one-for-one match for each Yuan that the employee deposits into the account. These employer-matching funds are deposited into the same account and they also earn the risk-free rate. The employee owns the account, and all money in it, although they can only use the funds for the purchase of housing.[12] Should the employee die, their funds become part of their estate. Once in the program, however, the employee must continue their monthly contributions to their HPF account until they retire, pass away, or are separated from their employer. The employee may not voluntarily withdraw from the program while they are still employed by the state or state-sponsored entity. Except for not being able to withdraw from the program, this savings component of the system is very similar to 401(k) or IRA retirement programs in the United States.

In addition to the employer-match, participants in the system also have the right to purchase the state-owned housing they occupy at a discount relative to the price the government would charge non-participants. Depending upon the employee's rank, job tenure, job performance, and other factors, the price of the property will range between 10% and 40% of the government's

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[12] Upon the retirement, the employee can withdraw the entire account and use it for other purposes. Upon the death of the employee, their heirs have the same access to the funds that a retired employee would have.

normal asking price. Interestingly, the program does allow the employee the opportunity to purchase housing that is better or larger than that to which their rank entitles them. In such a case the price the employee pays a discounted price for that portion of the unit to which they are entitled by rank, and the full price for the "extra" portion. For example, if a Section Director were entitled to a 100 square-meter unit but instead purchased a 150 square-meter unit, they would pay the discounted price on the first 100 square-meters of the unit and the full price on the remaining 50 square-meters.

If an employee is renting a unit and separates from their employer, they must either vacate the unit or pay a higher rental rate on it. If, however, they have purchased the unit, it is their property to keep. The employee may chose to continue living in the property, but they may also sell the property in a free market. While originally there was a five-year blackout period on reselling the property, that constraint was eliminated in 1997 in Shanghai, followed by almost all cities by 2001.

The government uses two additional financial subsidies to encourage participation in the system. First, participants in the program are able to acquire mortgages at below market rates from State-owned banks. Second, the government waives a 10% tax normally required on housing transactions; effectively further reducing the cost of a unit by 10%.[13]

A Utility-Based Model of Consumer Choice

The housing provident fund (HPF) system is very complex, and requires its participants to make some very sophisticated financial decisions. Indeed, in one sense the decision as to whether or not to join HPF is not trivial. Even though, as mentioned earlier, it is now mandatory for currently eligible employees to join, a worker just entering the workforce implicitly decides to join the HPF when they accept government employment. If they were to decide to accept only private employment they could avoid joining HPF. While modeling the worker's decision whether or not to take public employment is beyond the scope of this paper, we do use the idea that an individual could avoid HPF to establish our base economic case.[14]

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[13] *The Peoples Daily*, Overseas Edition, October 30, 1999.
[14] There is a well-established literature that examines housing consumption over the life cycle. Examples include Artle and Varaiya (1978), Kan (1999), and Kan (2000). That literature, however, generally is concerned with housing choice under where the primary constrain is a person's budget and preference set. This paper specifically focuses on the incentives that the HPF program provides, and so uses a more narrowly defined method for analyzing the problem. For example, Kan (1999) considers the problem of mobility and how it affects housing choice.

Even after an employee has joined HPF they still have a significant decision to make: when to actually purchase their housing. In the extreme case, an HPF participant could delay the purchase of housing indefinitely; in such a case they would essentially treat the program as a retirement fund.

The government must concern itself with these choices as well. Under the assumption that renters are unlikely to ever accumulate enough wealth to purchase their housing units without the HPF program convincing tenants to join is a necessary condition for eventually reducing or eliminating the government's ownership of housing. It is not, however, a sufficient condition, as the tenant must still decide exactly when to purchase the housing. During this interim period between the employee joining the HPF and purchasing their housing unit, the cost to the government is quite high given that the consumer continues to receive their rent subsidy and they also receive the savings subsidy. Clearly the government has an interest in reducing this time period.

The model begins with the assumption that the employee maximizes lifetime utility, and that the marginal benefit to the employee of the HPF program is the difference in their total expected utility under the program compared to their total expected utility if they were not in the program. That is, it compares their total expected utility under HPF with what their utility would be if they were able to simply live in public housing with subsidized rental rates for the rest of their lives.

Before building the model we must make a fundamental choice as to how we will proceed: we must decide whether we will work within a stochastic or deterministic framework. Although it is tempting to use a stochastic framework, we chose to work within a deterministic framework for a couple of reasons. First, the number of variables that we would have to treat as stochastic is prohibitively large.[15] Given that we are unlikely to find closed-form solutions for our model, we are limited to solving it through numerical methods and generally numerical methods work for two or three stochastic state variables, but become untenable for larger numbers. Second, we wish to build as complete a model as possible so that we can determine which variables are the most important factors in the model. If we were to treat a subset of parameters as stochastic while treating the rest as deterministic we would necessarily limit the scope of the model, and inappropriately shift the focus of the model onto just a few parameters.

We agree that mobility is an issue, but it is not one that directly relates to the HPF incentives that we wish to examine, and so we implicitly assume that the HPF participant is not mobile.

[15] At a minimum we would have to treat interest rates, house prices, employee income, market rents, and maintenance costs as stochastic.

The lifetime utility maximization problem is given by:

$$\text{Max}\left(\int_0^T U\left(\{H_t\},\{C_t\},B_T\right)\partial t\,\middle|\,\{P_{H,t}\},\{P_{C,t}\}\right)$$

$$\text{s.t. } \int_0^T (P_{H,t}H_t + P_{C,t}C_t)\partial t + B_T \leq \int_0^T y_t + W_0$$

(1)

where $U(.)$ is the employee's utility function, H_t is their housing consumption at time t, C_t is their non-housing consumption at time t, B_T is the amount of bequest left for their heirs upon death, $P_{H,t}$ is the housing price at time t, $P_{C,t}$ is the price of other goods at time t, y_t is the employee's income at time t, and W_0 is the employee's initial endowment.

Within this utility framework, the house price and mortgage interest rate subsidies that the HPF provides are equivalent to reducing the price of housing for the employee. Usually reducing the price of a good can lead to either price or substitution effects (or both). In this case, however, recall that the reduction in housing price is only on the unit that the consumer already occupies, not on any larger space, so, at least in the short run, there is effectively no substitution effect.[16] Instead the increased income can be used to increase the consumption of other goods or it can be used to increase their savings and eventually the bequest that they make to their heirs. The leaving of an inheritance is especially important in Chinese culture, and increasing the size of the bequest is given a heavy weighting in a consumer's utility function.

Since the consumer maximizes their expected utility, they will elect to redistribute their income to their highest marginal utility use. For tractability the model assumes that increasing marginal consumption provides no more marginal utility for the consumer than does increasing the marginal bequest (i.e. savings). This allows the model to express the benefits of the HPF relative to renting in terms of the present value of net savings, i.e. the increase in the present value of the future bequest.[17] Given this assumption, the decision whether or not to join the HPF becomes a rent versus buy decision, albeit a very complex one.

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[16] It is possible the consumer will elect to use the "saved" money to purchase additional housing units at the higher price, but this is really more of an income effect; they are changing their consumption bundle because they have additional money since the cost of housing they already occupy has changed, not because the price of the marginal unit of housing has changed.

[17] This future bequest amount will include any anticipated growth in the value of the housing itself. The consumer will have incentives to maintain and even increase the quality of the housing to maximize future appreciation in the property. This may well provide a positive externality relative to renting since renters have no incentive to maintain their property.

The model uses the following variables:

H_t = market house price at time t;

R_t = rent of the same house set by the government at time t;

M_t = housing maintenance cost at time t;

y_t = employee's income at time t;

T = expected time of house sale at maturity of the mortgage loan or lifetime of the participant, $T-\tau$ = mortgage term;

r^s = government-subsidized mortgage rate for HPF participants;

r^f = bank deposit rate, used to discount cash flows and estimate interest earned on the housing saving account;

g^H = house price growth rate;

g^M = maintenance cost growth rate;

g^R = rent growth rate;

g^Y = income growth rate;

F_t = balance in participant's HPF account at time t;

α = percentage of house price to be paid when purchasing housing under the program;

β = percentage of employee's income required to contribute to the housing fund;

D = down payment as percentage of purchasing price required in purchasing housing under the program;

DP = Cash amount of down payment;

τ = time of the housing purchase under the program;

ξ = the ratio of government's contribution to employee's contribution to the housing fund account;

W_0 = the initial wealth of the participant.

A Measure of Lifetime Utility

The model assumes that the employee's utility is monotonically increasing in net present wealth, and the employee's income from their job remains the same regardless of whether or not they join HPF.[18] The key to the model's analysis is that it treats renting as a pure cost to the employee, but recognizes that expenditures under HPF can have both a cost and an investment aspect to them. It is the investment aspect of the HPF program that complicates the measurement of the employee's utility. Any measure must take into account both the additional costs of the HPF program as well as the potential returns

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[18] This assumption allows us to avoid the consumption-smoothing problems like those discussed in Leung and Tse (2001) or Tse and Leung (2002). If we only maximized current utility then when the employee joined the program they might have an initial utility reduction, due to the cash flow lost by joining the program forcing them to reduce consumption. Since the consumer is maximizing net present wealth, however, the loss in immediate consumption is compensated by an increase in future consumption.

it provides. This model measures the employee's utility by measuring the net present value of the wealth that they have available to spend on goods other than the housing unit they would have had had they continued to rent.[19] By focusing on the other income that the employee has to spend, the model is able to isolate how the HPF program affects the employee's true cost or benefit of participating in the system.

The model defines the variable NHW_{Rent} as the wealth available for non-housing goods under the rental system:[20]

NHW_{Rent} = PV lifetime income – PV rental expenditures,

and the variable NHW_{HPF} as the wealth available for non-housing goods under the HPF system:

$$NHW_{HPF} = \text{PV lifetime income} - \text{PV HPF expenditures} \qquad (3)$$

Under HPF the expenditures can be either positive or negative. If they are positive, they represent a cash outflow to the employee and are a reduction in the income they have available to spend on non-housing goods. If they are negative they represent a cash inflow to the employee and increase the income that they have available to spend on non-housing goods. One source of such a negative expenditure, for example, would be capital gains on the purchased housing unit.

Given Eqs. (2) and (3), the model can calculate the net present benefit of the HPF program to the employee:

$$\text{Net present benefit} = NHW_{HPF} - NHW_{Rent} \qquad (4)$$

Since the model assumes that lifetime income is equal regardless of the employee's decision to join or not join HPF, some simple algebra reveals that the determinant of the participant's net marginal benefit is equal to the net present differences in the housing expenditures under renting and HPF:

$$\text{Net present benefit} = \text{PV rental expenditures} - \text{PV HPF expenditures} \qquad (5)$$

Equation (5) is the basic equation the model uses to examine the effects of joining HPF. The following sections examine the specific equations used to construct the various sub-components of this equation.

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[19] This wealth available to spend includes income they spend on any additional square footage of housing they elect to purchase under the HPF system.
[20] Again, these non-housing goods would include additional square footage purchased under HPF that they would otherwise be unable to purchase.

Lifetime Income

The model assumes that there are two potential sources of income available to the employee, regardless of their HPF status: their salary income and their initial wealth.[21] Working within a continuous time framework the present value of initial wealth and salary income is given by:

$$\text{Lifetime income } = \ W_0 + \int_{t=0}^{T} y_t e^{-r^f t} \partial t = W_0 + \frac{Y_0 \left(e^{\left(g^Y - r^f \right) T} - 1 \right)}{g^Y - r^f} \qquad (6)$$

Rental Expenditures

If the employee were not in the HPF system they would continue to pay the government-set rent for the remainder of their life. This does assume that there will always be rental housing available.

Working within a continuous-time model, this results in a lifetime present value of the housing expense as:

$$\text{Rental expenditures} = \int_{t=0}^{T} R_t e^{-r^f t} \partial t = \frac{R_0 (e^{(g^R - r^f)T} - 1)}{g^R - r^f} \qquad (7)$$

HPF Expenditures

Evaluating the lifetime expenditures of the consumer under the HPF program is much more complex. One must consider the combined rental and savings cash flows between the time when the employee enrolls in the program and when they actually purchase their house. One must consider mortgage and maintenance costs after the consumer purchases the house, as well as the sales value of the house, and the effect that these capital gains have on their bequest. It is possible to combine these into a single equation that is the HPF analog to Eq. (7): the net present expenditures of joining HPF:

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[21] Recall that any income generated by participating in HPF will be treated as a negative HPF expenditure.

$$\text{HPF expenditures } = \int_{t=0}^{\tau} (R_t + \beta y_t)\mathrm{e}^{-r^f t}\partial t + \int_{t=\tau}^{T} (\beta y_t + M_t)\mathrm{e}^{-r^f t}\partial t +$$

$$\int_{t=\tau}^{T} (\alpha H_\tau - DP)r^s \mathrm{e}^{-r^f t}\partial t - \left[H_T - (\alpha H_\tau - DP)\right]\mathrm{e}^{-r^f T} -$$

$$\left[F_\tau \mathrm{e}^{r^f (T-\tau)} + \left(\int_{t=\tau}^{T} \beta(1+\xi)y_t \mathrm{e}^{r^f (T-t)}\partial t \right)\right]\mathrm{e}^{-r^f T} \tag{8}$$

s.t. $\tau \geq \theta$

where each element has the following meaning:

- The first element ($\int_{t=0}^{\tau} (R_t + \beta y_t)\mathrm{e}^{-r^f t}\partial t$) is the rent and contribution to the housing fund account prior to the purchase of the house,

- The second element ($\int_{t=\tau}^{T} (\beta y_t + M_t)\mathrm{e}^{-r^f t}\partial t$) is the continuing contribution to the housing fund and the maintenance expense occurred after the purchase of the house,

- The third element ($\int_{t=\tau}^{T} (\alpha H_\tau - DP)r^s \mathrm{e}^{-r^f t}\partial t$) is the interest payments for the loan[22],

- The fourth element ($\left[H_T - (\alpha H_\tau - DP)\right]\mathrm{e}^{-r^f T}$) is the proceeds from selling the house less the repayment of the loan[23], and

- The last element ($\left[F_\tau \mathrm{e}^{r^f (T-\tau)} + \left(\int_{t=\tau}^{T} \beta(1+\xi)y_t \mathrm{e}^{r^f (T-t)}\partial t \right)\right]\mathrm{e}^{-r^f T}$) represents the refund of unused employee contribution to the housing fund account upon exiting of the program, either upon retirement or death.

There are a few constraints that must be imposed upon this equation. First, θ is the minimum time required to accumulate enough cash in the account to

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[22] For simplicity the model assumes an interest-only loan. One could easily incorporate an amortizing loan, but doing so adds nothing to the analysis and requires the addition of a new time parameter – the maturity date of the mortgage.

[23] If one is uncomfortable with the idea that the participant is "forced" to sell the house a time T, one can alternatively interpret this as the net capital gain (less the mortgage balance) earned on the house as of the participant's maturity horizon. We thank an anonymous referee for providing this interpretation.

be able to make the down payment. If this is based upon a percentage of the sales price, it is the time that makes the following statement true:

$$\alpha DH_\theta = \int_{t=0}^{\theta} y_t \beta(1+\xi)e^{r^f(\theta-t)}\partial t \tag{9}$$

Recently, the Chinese government has stopped requiring a specific down payment percentage, and has simply stated that after six months in the program HPF participants are eligible to purchase using whatever funds are in their account for their down payment. In such a case θ is simply set equal to six months.

While θ is the minimum time that the employee must remain in the program in order to purchase their housing, the employee could stay in longer before making their purchase. In theory a person staying in the program for a sufficiently long period of time could accumulate more money in their account than is required to purchase their housing outright, i.e. without a mortgage. The model assumes, therefore, that upon the purchase of their housing the HPF participant will use all funds in the housing account, up to the price of the house, as their down payment. Thus the down payment (DP) made at a time $\tau>\theta$ is given by

$$DP = \min\left(\int_{t=0}^{\tau} y_t \beta(1+\xi)e^{r^f(\tau-t)}\partial t, \alpha H_\tau\right) \tag{10}$$

The balance of the housing fund after the down payment is made is

$$F_\tau = \max\left(0, \int_{t=0}^{\tau} y_t \beta(1+\xi)e^{r^f(\tau-t)}\partial t - \alpha H_\tau\right) \tag{11}$$

Marginal Benefit of Joining HPF

From Eq. (5) the differences in the marginal utility of the employee from joining HPF is the difference in the expenditures, thus the net marginal benefit is given by:

$$
\begin{aligned}
\text{Net marginal benefit } = {} & \int_{t=0}^{T} R_t e^{-r^f_t} \partial t - \int_{t=0}^{\tau} (R_t + \beta y_t) e^{-r^f_t} \partial t \\
& - \int_{t=\tau}^{T} (\beta y_t + M_t) e^{-r^f_t} \partial t \\
& - \int_{t=\tau}^{T} (\alpha H_\tau - DP) r^s e^{-r^f_t} \partial t \\
& + \left[H_T - (\alpha H_\tau - DP) \right] e^{-r^f_T} \\
& + \left[F_\tau e^{r^f (T-\tau)} + \left(\int_{t=\tau}^{T} \beta(1+\xi) y_t e^{r^f (T-t)} \partial t \right) \right] e^{-r^f_T}
\end{aligned}
$$
(12)

From Eq. (12), it is clear that the housing saving program is a collection of several different subsidy components. Implicit in Eq. (12) is the assumption that the HPF program and its subsidies will remain in place for the entire time horizon of the participant. One can reasonably question the ability and willingness of the Chinese government to maintain this program and its subsidies if the entire Chinese population were to elect to join the program. If the model were to allow for the potential that the program could disappear in the future because its popularity precluded the Chinese government from continuing its subsidies, this would alter the decisions of the participant with respect to when they would exercise their purchase option. Doing so, however, would require a general equilibrium model like that of Gervais (2002), and would not let us address the primary goal of this paper: to determine the incentives facing the participant under the program as it now stands.

The model can be used to assess the marginal contribution of each component with respect to the employee's decision. This also provides a measure of the relative effect of each of the various subsidy policies. Table 1 presents the signs of the various partial derivatives.

The partial derivatives in Table 1 provide insights as to how the program will affect the wealth of the participants. For those components with positive derivatives, increasing the value of the factor will increase the employee's marginal wealth under the program. For those factors with a negative first partial derivative, increasing that aspect of the program will decrease the participation in the program. Not surprisingly, the negative partials stem from costs that the participant must bear (i.e., housing price, mortgage amount, etc.) under the system.

Table 1: Comparative statics for net marginal benefit (NMB) from participating in program

$\dfrac{\partial NMB}{\partial H_0} < 0$	$\dfrac{\partial NMB}{\partial R_0} > 0$	$\dfrac{\partial NMB}{\partial D} \begin{cases} > 0 \\ < 0 \end{cases}$
$\dfrac{\partial NMB}{\partial \alpha} < 0$	$\dfrac{\partial NMB}{\partial G^y} > 0$	$\dfrac{\partial NMB}{\partial \tau} \begin{cases} > 0 \\ < 0 \end{cases}$
$\dfrac{\partial NMB}{\partial M_0} < 0$	$\dfrac{\partial NMB}{\partial \beta} > 0$	$\dfrac{\partial NMB}{\partial G^R} \begin{cases} > 0 \\ < 0 \end{cases}$
$\dfrac{\partial NMB}{\partial y_0} > 0$	$\dfrac{\partial NMB}{\partial \xi} > 0$	$\dfrac{\partial NMB}{\partial G^H} > 0$

These partial derivatives by themselves, however, do not provide enough information to fully analyze the benefits of the program from the perspective of either the participant or the government. Recall that a primary objective of the HPF program is to provide a mechanism through which the government can privatize public housing. A reason for this is that the rental rate is subsidized so heavily that rental income does not even cover maintenance costs. Presumably, the government wishes to incite the HPF participant so as to minimize the time between their joining and their purchase of the housing unit. There are also several partial derivatives with an indeterminate sign. For those factors, the impacts would depend upon the values of specific parameters. To address these issues, the next section numerically solves the marginal benefit equation for a variety of parameter values.

Numerical Examples

Some numerical examples may help illustrate the implications of the above model. In these examples we uses parameters that are representative of the Chinese economy and housing market and that best allow illustration of the effect that HPF participation has on the employee's utility. The model then conducts sensitivity analyses around these base conditions to illustrate the potential impacts of changes in either market conditions or the conditions of the program.

Table 2 lists the parameters the base case simulation uses. One can classify the parameters as fitting into four groups: those relating to the cost of renting, those relating to the cost of purchasing the housing unit and its financing, those relating to the employee's income and the structure of the program, and those relating to the timing of the employee's decisions.

Table 2: Base parameter values for simulations

Housing parameters	$H_0 = 175{,}000$
	$M_0 = 300$
	$g^H = 2\%$
	$g^M = 2\%$
	$r^s = 4.50\%$
	$\alpha = 33\%$
Rental parameters	$R_0 = 1000$
	$g^R = 2.0\%$
Income parameters	$y_0 = 12{,}000$
	$g^Y = 5\%$
	$\xi = 1$
	$r^f = 6.5\%$
	$\beta = 6\%$
	$W_0 = 0$
Horizon parameters	$T = 45$
	$\tau = 10$

There are six parameters relating to the cost of purchasing the housing unit. The first is the initial cost of the house at time 0, H_0. It has a base value of 175,000 Yuan. The base growth rate for housing costs, g^H, is set to 2%. The initial maintenance costs, M_0, are 300 Yuan per year, and their growth rate, g^M, is initially set to 2%. Both the value of H_0 and M_0 are set to values that are in line with published values for those parameters.[24] The two final parameters that directly affect the housing purchase cost are the percent of the market price that the employee must pay, α, and the subsidized mortgage rate, r^s. These parameters are set to 33% and 4.5% respectively. The value for α is set by the program but could vary according to the employee's rank and tenure with the enterprise, and the subsidized mortgage rate has been steadily in the 4.5% range over the past several years.

The two rental parameters are the initial rental rate R_0 and its growth rate g^R. These parameters are set to 1000 Yuan per year and 2%, respectively. The initial rental rate is consistent with that of a very junior employee. The model sets the growth rate in the rental rate to be equal to the growth rate in housing prices under the theory that in the long run rental price increases should closely mirror increases in housing prices. In some of the simulations the model relaxes this assumption and allows the rental growth rate to be considerably higher than that of the housing growth rate, to simulate the effect of the government increasing the rental rate at an artificially high speed as an incentive to participants to purchase their housing.

There are six income-related parameters in the model. The initial income level, y_0, is set to 12,000 Yuan per year, and the income growth rate, g^Y, is set to 5%. The risk-free rate, which is the rate paid on the HPF account, is set to

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[24] China Real Estate Development Statistics, Beijing, 2001.

6.5%, which is consistent with the Chinese risk-free rate in the recent past. One can see from this that the government does heavily subsidize the HPF mortgage rate, r^s, which is 4.5%. The employer match of HPF employee's funds contributions, ξ, is set to 1 and the mandatory contribution from the employee is 6%. Both of these parameters are the values the program currently uses. The initial wealth of the participant, W_0, is set to 0. Since this parameter appears on both sides of Eq. (5), it simply washes out and one can set it to 0 without loss of generality.

The final two parameters relate to the horizon of the employee. The parameter T is the time before the employee exits the HPF program in its entirety and liquidates all of their assets. The model assumes this happens upon the retirement of the employee. The base value for T is 45 years, which corresponds to the horizon for a 20 year old beginning their career. The second horizon parameter is τ, the date when the employee purchases their housing unit. Although this parameter has a base value of 10 years, in each of the simulations the model examines a variety of values for τ.

When considering these parameter values, one must keep in mind that the purpose of these simulations is not to precisely replicate the Chinese economy or to provide a robust estimate of the value of the HPF program to a particular employee. Rather, they are conducted to illustrate the relative effects of each parameter on employee wealth, and to do so using values that are of the appropriate scale for the HPF program and for the Chinese economy.

Simulation Results

Base Case

Using the base parameter set, the net present benefit for the HPF participant is 24,175 Yuan, which is slightly more than 2 times the initial base salary, Y_0, of 12,000 Yuan. One value, of course, presents only a very limited view of the model. To present a more complete view, Tables 3-6 demonstrate the net present benefit to the participant under a variety of parameter combinations and with differing times until the participant purchases their house.

Table 3: **Net present benefit of HPF in Yuan for various values of τ, r^f, g^H, and g^Y**

Other parameter values: T=45 years, y_0=12,000 Yuan, H_0=175,000, r^s=4.5%, M_0=300, g^M=3%, α=33%, β=6%, ζ=1, R_0=1000, g^R=2%.

Panel A, τ=0.5 years

						r^f				
	4%			6%			8%		
		g^H			g^H			g^H	
g^Y	2%	6%	10%	2%	6%	10%	2%	6%	10%
2%	49,892	407,898	2,580,119	14,611	159,807	1,042,600	969	59,707	418,323
6%	81,075	439,081	2,611,302	31,985	177,180	1,059,974	11,136	69,873	428,490
10%	195,087	553,092	2,725,314	90,477	235,672	1,118,465	42,315	101,053	459,669

Panel B, τ=10 years

						r^f				
	.04			.06			.08		
		g^H			g^H			g^H	
g^Y	2%	6%	10%	2%	6%	10%	2%	6%	10%
2%	56,433	390,270	2,525,834	19,967	151,178	1,012,694	5,452	55,658	401,240
6%	87,887	421,724	2,557,288	36,787	167,998	1,029,514	14,709	64,915	410,496
10%	202,255	536,092	2,671,656	94,556	225,766	1,087,283	44,704	94,910	440,491

Panel C, τ= 20 years

						r^f				
	4%			6%			8%		
		g^H			g^H			g^H	
g^Y	2%	6%	10%	2%	6%	10%	2%	6%	10%
2%	61,850	369,940	2,429,536	24,163	144,611	971,270	8,525	54,661	384,447
6%	93,954	402,045	2,461,641	40,349	160,243	986,902	18,460	62,196	391,982
10%	208,226	517,658	2,577,254	98,842	215,761	1,042,420	49,642	88,962	418,748

Panel D, τ=30 years

						r^f				
	4%			6%			8%		
		g^H			g^H			g^H	
g^Y	2%	6%	10%	2%	6%	10%	2%	6%	10%
2%	65,797	347,442	2,263,056	28,936	139,488	909,067	14,552	55,358	363,587
6%	96,980	380,131	2,295,744	46,311	154,186	923,766	24,722	61,963	369,950
10%	210,991	496,738	2,413,252	104,805	210,521	976,277	55,905	93,145	392,967

Panel E, τ=45 years

						r^f				
	4%			6%			8%		
		g^H			g^H			g^H	
g^Y	2%	6%	10%	2%	6%	10%	2%	6%	10%
2%	69,033	309,751	1,766,012	34,405	132,274	724,345	19,073	58,863	299,582
6%	100,216	340,934	1,797,194	51,781	149,650	741,721	29,243	69,033	309,751
10%	214,227	454,945	1,911,205	110,274	208,143	800,214	60,425	100,216	340,934

The Effects of Housing and Income Growth Rates

Table 3 presents the result of varying four parameters, τ, r^f, g^H, and g^Y, from their base parameter values. These parameters are the ones that most directly affect the employee's income, the cash they accumulate in their housing fund, and the house value. Table 3 consists of five panels, each with a different value of τ. In Panel A, τ is set to six months, meaning the employee purchases the house as soon as they are eligible to do so under the program. In Panel B, τ is set to 10 years, in Panel C it is set to 20 years, in Panel D it is 30 years, and in Panel E it is set to the maximum at 45 years. Panel E corresponds to the situation where the participant purchases their house at the moment they retire; they essentially treat the HPF as a retirement account. They never purchase the house to live in, but rather only purchase it and immediately resell it upon their exiting the HPF program. Since the government will sell the housing units at the price αH_T, and presumably the employee can resell it immediately at H_T, the model assumes they act rationally and exploit the arbitrage.

Table 3 varies the other parameters within each panel. The risk free rate, r^f, is set to 4%, 6%, and 8%. The housing growth rate, g^H, is set to 2%, 6%, and 10%, and the income growth rate, g^Y, is set to 2%, 6%, and 10%.

It is clear from Table 3 that the parameter that is most influential to the participant's net present benefit is the housing growth rate, g^H. For example, in Panel A, with r^f set to 4% and g^Y set to 2%, changing g^H from 2% to 10% results in the participant's wealth increase changing from 49,892 Yuan to 2,580,119 Yuan.[25] Similarly, if r^f is set to 8%, and g^Y is held at 2%, changing g^H from 2% to 10%, results in the participant's wealth increase changing from 969 Yuan to 418,323 Yuan. Panels A, B, C, and D of Table 3 demonstrate that increasing the housing growth rate monotonically increases the participant's wealth.

Increasing the risk-free rate has two effects on a HPF participant. The first effect is that it increases their earnings on the HPF account, thus reducing the size of the mortgage they must use for a given τ value. The second effect is that all of the cash flows are now discounted at a higher rate, which reduces the present value of the future benefits of the HPF. Given that some of the most valuable benefits of HPF occur very far in the future, this effect can be significant. The results in Table 3 show that holding everything else constant, increasing r^f will reduce the wealth of the employee. For example,

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[25] At first glance the net present benefit of 2,580,119 Yuan is startling. One must keep in mind, however, that nearly all of this value comes from the increase in the house value. Note that under a continuous compounding rate of 10%, a house that has an initial value of 175,000 Yuan would be worth 9,554,676 Yuan after 40 years.

in Panel A, with r^f equal to 4% and g^Y and g^H set equal to 2%, increasing r^f from 4% to 8% results in a decrease in the net present benefit of joining HPF from 49,892 Yuan to 969 Yuan. Similarly, with g^Y and g^H each set at 10%, increasing r^f from 4% to 8% results in a decrease in the net present benefit of joining HPF from 2,725,314 Yuan to 459,669 Yuan.

One parameter that the employee controls is τ, so it is interesting to examine how various values of τ can affect the employee's net present benefit. This is a way of examining the employee's optimal holding period for their option to purchase the house.[26] Reading vertically across panels allows one to examine this issue. For example, increasing τ from 0.5 year to 45 years, given values for r^f, g^Y, and g^H of 4%, 2%, and 2%, respectively, results in the employee's net present benefit increasing from 49,892 Yuan to 69,033 Yuan. Given that the growth rate in housing is so low, the employee is better off delaying their purchase of housing for as long as possible and continuing to earn the higher risk free rate of return. On the other hand when g^H is greater than r^f, the employee is better off purchasing the housing as soon as possible. This can be seen by increasing τ from 0.5 year to 45 years when r^f, g^Y, and g^H have values of 4%, 10%, and 10%, respectively. In this case the net present benefit is 2,580,119 Yuan when τ is 0.5 years and only 1,911,205 Yuan when τ is 45 years. This basic pattern – delay purchasing when g^H is low and purchase as soon as possible when g^H is high - repeats regardless of whether g^Y is high or low.[27]

Perhaps the most intriguing result in Table 3 is when g^H is set to be slightly less than r^f, i.e. when g^H=6% and r^f=8%. Regardless of the g^Y value, the HPF participant's Net Present Benefit is roughly equal when τ is either set to .5 years or 45 years, but follows a U-shaped pattern for τ values of 10, 20 or 30 years. The risk free rate is not large enough to allow it to dominate the growth rate housing in the early years: it takes many decades for the risk-free rate's higher value to overcome the initial advantage that immediate purchase provides due to the subsidized house purchase price. This means that for employees with a shorter time horizon (i.e. older employees) that immediate purchase is optimal, but that employees with a longer time horizon (i.e. younger employees) will be indifferent between immediate purchasing their housing and delaying their purchase for a very long time.

One implication that can be drawn from Table 3 is that the employee's expectations regarding the future growth rate of housing may be more

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[26] One must solve for the optimal holding period numerically.

[27] We note that when analyzing these results we place to constraints on the employee – a minimum purchase time (τ=0.5 years) and a maximum purchase horizon (τ=45 years), so that when we discuss an optimal time to purchase we necessarily mean a constrained optimum. This causes many of our results to be corner solutions.

important in their decision as to when they purchase their housing than is their perception as to the growth rate in their salary. In other words, the optimal behavior of the employee is determined by the relative attractiveness of housing as an investment, not by its affordability in terms of the employee's income.[28]

The Effects of the Income Match and the House Price Discount

The HPF program contains various discounts and subsidies for participants. Two of the most important such benefits are the matching of the employee's contribution by their employer and the discount on the house price they get for participating in the HPF. Table 4 examines the relative effect of these benefits on the employee's net present benefit, and how that might affect the employee's decision as to when to make their purchase.

Specifically, Table 4 examines the effects that four parameters, τ, T, α, and ξ, have on the employee's net present benefit. Recall that α is the percentage of the housing unit's "market" price that the employee must pay under HPF, and ξ is the rate at which the government matches the employee's HPF contributions. These two parameters provide mechanisms through which the government can reduce the cost of the housing and increase the employee's wealth. The effect of α on the employee's wealth ends at time τ, when the participant purchases their housing unit; the effect of ξ, however, depends on both τ and T. It depends upon τ because ξ and τ jointly determine the amount of cash the employee will have to make a down payment.[29] It depends upon T since the employee must continue to make contributions to HPF even after they purchase the housing unit. Table 4 consists of three panels that are characterized by differing time horizons T, for the investor. Within each panel are various net present benefit values for combinations of τ, α, and ξ.

The most striking feature of the results in Table 4 is that for many parameter combinations the net present benefit is negative, indicating that it is a net wealth loss for the employee to join HPF. Presumably the employee would not join HPF if they had the option of not joining. This does raise the issue of whether the Chinese government's decision to mandate participation in the program was in the best interest of all participants.

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[28] There is obviously a lower limit to this, since the homeowner must be able to make the basic mortgage payments.
[29] For a given set of values of r^l, y_0, and g^Y,

Table 4: Net present benefit of joining HPF in Yuan for various values of T, τ, α, and ξ

Other parameter values: y_0=12,000 Yuan, g^Y=.05, r^l=6.5%, H_0=175,000, g^H=2%, r^s=4.5%, M_0=300, g^M=3%, β=.06, R_0=1000, g^R=2%.

Panel A, T=10 years

	α								
	0.33			0.66			1		
	ξ			ξ			ξ		
τ	0	0.5	1	0	0.5	1	0	0.5	1
0.5	68,364	71,682	74,999	19,903	23,221	26,538	–30,026	–26,709	–23,391
5	71,614	74,809	78,004	29,437	32,632	35,827	–14,018	–10,823	–7,628
10	74,762	78,105	81,448	37,939	41,282	44,625	0	3,343	6,686

Panel B, T=25 years

	α								
	0.33			0.66			1		
	ξ			ξ			ξ		
τ	0	0.5	1	0	0.5	1	0	0.5	1
0.5	24,260	31,721	39,182	–18,365	–10,904	–3,443	–62,282	–54,821	–47,360
5	27,617	34,734	41,851	–8,175	–1,059	6,058	–45,052	–37,935	–30,819
10	30,635	37,499	44,364	868	7,733	14,597	–29,800	–22,936	–16,071
25	38,066	45,571	53,076	19,317	26,822	34,327	0	7,505	15,010

Panel C, T=45 years

	α								
	0.33			0.66			1		
	ξ			ξ			ξ		
τ	0	0.5	1	0	0.5	1	0	0.5	1
0.5	–3,903	7,825	19,553	–43,958	–32,230	–20,502	–85,226	–73,498	–61,770
5	–500	10,786	22,073	–33,479	–22,193	–10,907	–67,458	–56,171	–44,882
10	2,461	13,318	24,176	–24,197	–13,339	–2,482	–51,662	–40,805	–29,947
25	8,184	19,127	30,908	–6,368	–3,732	13,833	–21,361	–11,267	–1,160
35	12,311	24,091	35,872	989	12,137	23,917	–9,516	822	11,600
45	15,476	27,257	39,037	7,854	19,634	31,414	0	11,780	23,560

So what causes this result? Clearly the net present benefit is most likely to be negative when the house price discount is small. Consider that if there is no discount, i.e. if α=1, then the HPF participant must take on a mortgage that is significantly larger than their rental payments. Further, because in the base case the housing growth rate is considerably lower than the risk-free rate, they are not earning much on their housing investment. The HPF participant must also take on the additional cost of maintenance for the property. Note, however, that the net present benefit can only be negative if the HPF participant purchases their house prior to time T. By waiting until time T the participant avoids the maintenance costs and the additional cost of the debt service, their money earns the risk-free rate, and they get the benefit of capturing the price discount if there is any.

While the net present benefit is increasing with respect to both α and ξ,

Table 4 demonstrates that α has by far the biggest effect. The reason for this is because the house value, and hence any significant discount from it, far outweighs the employee's income, including any match the government may provide. The scales on which these two parameters operate are on different orders of magnitude. To see this, realize that if y_0 is 12,000, the initial year's match by the government will be 720 Yuan. A discount on the house price, however, could be much more valuable. Consider that a discount of 67% of the house price, which is the difference between setting α to 1 and setting it to 0.33, would be 117,250 Yuan if the house price stayed constant at 175,000 Yuan. If the house price is increasing, the benefit would be even greater.

By comparing values across time horizons, T, one sees that net present benefit is decreasing in T. Further, the employee can maximize their gain (or minimize their losses) by delaying their purchase of the property as long as possible. This is a function of the values of the base parameters g^H and r^f, which are 2% and 6.5% respectively. As Table 3 demonstrated if g^H is less than r^f, then in general the participant can increase their wealth by delaying their purchase of the house and investing in the risk-free asset since its return is higher. If g^H were higher than r^f, the opposite results would hold. It is also the case that if g^H is sufficiently high, then the net present benefit will be non-negative

The Effect of Raising Rental Rates

As documented earlier in this paper, one incentive that the Chinese government has used to reduce the number of people living in public housing has been to aggressively raise the rent on these properties. The purpose of Table 5 is to examine how various rent growth rates, g^R, affect the HPF participant's net marginal benefit as well as the timing of their purchase.

Specifically, Table 5 examines the effect of four parameters, T, τ, g^H, and g^R, on the employee's net present benefit. Once again, the table consists of three panels, Panels A, B, and C, where the participants time horizon (T) is set to 10, 25 or 45 years, respectively. Within each panel, g^H and g^Y each take on values of 2%, 6%, and 10%, and τ varies from a minimum of 0.5 years to a maximum equal to T.

The results in Table 5 display some of the same basic characteristics of Table 3. In general higher g^H values dramatically increase the participant's net present benefit. Once again, for higher values of g^H the participant's optimal strategy, regardless of the value of g^R, is to reduce τ as much as

possible by purchasing the housing unit immediately upon attaining eligibility. Although this is slightly more beneficial at higher g^R values, the primary benefit comes not from avoiding the increased rent, but rather from gaining the high housing growth rate. For all situations where g^H is 6% or greater, it is always optimal, regardless of the value of g^R, to purchase the housing unit when $\tau=0.5$ year.

Table 5: Net present benefit of HPF in Yuan for various values of T,τ, g^R, and g^H

Other parameter values: y_0=12,000 Yuan, g^Y=5%, r^f=6.5%, H_0=175,000, ζ=1.0, α=0.33, r^s=4.5%, M_0=300, g^M=3%, β=6%, R_0=1000

Panel A, *T*=10 years

	g^H								
	2%			6%			10%		
	g^R			g^R			g^R		
τ	2%	6%	10%	2%	6%	10%	2%	6%	10%
0.5	74,999	76,696	78,910	128,900	130,597	132,811	209,773	211,470	213,684
5	78,004	79,245	80,938	123,546	124,787	126,480	194,012	195,253	196,946
10	81,448	81,448	81,448	118,212	118,212	118,212	173,072	173,072	173,072

Panel B, *T*=25 years

	g^H								
	2%			6%			10%		
	g^R			g^R			g^R		
τ	2%	6%	10%	2%	6%	10%	2%	6%	10%
0.5	39,182	47,670	64,133	135,944	144,432	160,894	400,432	408,920	425,382
5	41,851	49,883	65,824	131,549	139,581	155,522	387,236	395,268	411,209
10	44,364	51,155	65,403	127,347	134,138	148,386	370,873	377,664	391,912
25	53,076	53,076	53,076	118,483	118,483	118,483	296,278	296,278	296,278

Panel C, *T*=45 years

	g^H								
	2%			6%			10%		
	g^R			g^R			g^R		
τ	2%	6%	10%	2%	6%	10%	2%	6%	10%
0.5	19,553	40,556	109,704	135,385	156,388	225,536	840,199	861,202	930,350
5	22,073	42,620	111,247	131,412	151,959	220,586	828,133	848,681	917,307
10	24,176	43,482	110,416	127,706	147,013	213,946	813,787	833,093	900,027
25	30,908	43,422	96,108	120,022	132,536	185,222	757,692	770,207	822,892
35	35,872	42,393	74,964	117,352	123,873	156,444	696,661	703,182	735,751
45	39,037	39,037	39,037	117,187	117,187	117,187	589,965	589,965	589,965

The only case where g^R makes a marginal difference in the participant's optimal τ value is when the housing growth rate is at 2%. Recall from Table 3 that when the housing growth rate is sufficiently low it becomes optimal for the HPF participant to delay their purchase of the property as long as possible. This allows the participant to earn the risk-free rate as long as possible before they invest in the lower-returning house. Indeed, this is the exact pattern seen in Table 5 when g^H is 2% and g^R is also set at 2%. If, however, the rental growth rate is raised to either 6% or 10%, and the

participant's time horizon is sufficiently long, i.e., either 25 or 45 years, it can be the case that the additional rental cost could outweigh the benefit of earning the risk-free rate, making the optimal τ value shorter than T.

To see this, consider the Panel C, when T=45 years, and the case where g^H=2%, and g^R=10%. As one increases τ from 0.5 years to 5 years, the net present benefit increases from 109,704 Yuan to 111,247 Yuan. Increasing τ from 5 years to 10 years, however, decreases the net present benefit to 110,416 Yuan, and this decrease continues as τ increases until when it reaches 45 years, when the net present benefit is only 39,037. Thus, the optimal purchase time for the HPF participant would be at five years. If g^R were only 2%, however, the optimal purchase time would be at 45 years. Thus, in a low housing growth rate environment, the government could alter the participant's optimal behavior by increasing the rental rate on the property.

Figure 1 illustrates a similar situation. This plot essentially presents expanded data from the third data column of Panel B. The figure is a plot of the net present benefit to the participant when τ is set equal to values of between 0.5 years and 25 years, for g^H=2% and g^R=10%. Note that in this case the net present value initially increases as τ increases and then in year 7 declines and declines every year thereafter. In this situation, the optimal value of τ for the participant would be 7 years.

Figure 1: Net present benefit of HPF as a function of τ when T=25, g^H=2%, and g^R=10%
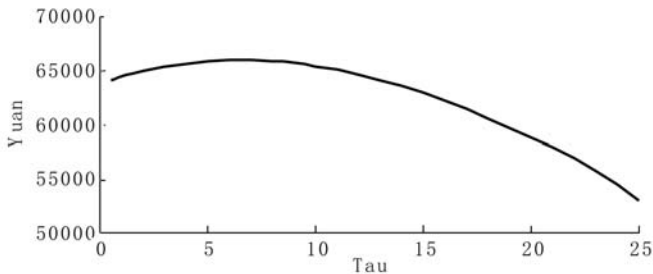
Other Parameter values: H_0=175,000, M_0=300; g^M=3% , y_0=12,000, g^Y=5%, r^S=4.5%, r^f=6.5%, α=.33, R_0=1000, ζ=1, β=6%

One effect at first counter-intuitive is that when τ is equal to T, changing g^R does not affect the participant's net present benefit. To understand this one must recall that net present benefit is the difference between the employee's position if they rented and were not a participant in HPF minus their position given that they are in HPF, as outlined in Eq. (9). If τ=T, the employee's rental cash expenditures are exactly the same in both cases, and so changes

in the rental growth rate do not affect the participant's net present benefit.

The Effect of the Mortgage Subsidy and the Maintenance Growth Rate

A primary stated motivation for the privatization of public housing has been the rapid increase in government-born maintenance costs. This begs the question as to why an individual would be willing to take on these costs if the government finds them so onerous. One potential answer to that question is that it may be the case that an owner-occupant of the property can maintain the house more efficiently than can the government. Also, the physical deterioration of an owner occupied property tends to be much milder than a renter occupied property since the owner has an incentive not to abuse the property.

If maintenance costs significantly detract from the attractiveness of the program one option available to the government would be to provide an additional subsidy to the homeowner. This subsidy could either be direct or indirect. From the perspective of this model a direct subsidy would show up simply as a lower growth rate in maintenance costs. One way of providing an indirect subsidy would be to subsidize the mortgage rate. In fact, the mortgage rate is already subsidized, having averaged 4.5% over the past several years when the risk free rate has been closer to 6.5%.[30] A benefit of subsidizing the mortgage rate − as opposed to other subsidies such as increasing ξ − is that the subsidy would only start after the HPF participant purchased their housing unit.

Table 6 examines how g^M and r^s affect the net present benefit of the HPF participant for various combinations of T and τ. The table consists of Panels A, B, and C. As in Tables 4 and 5 the panels correspond to different T values, but in this case T takes on values of 10, 25, and 45 years, respectively. Within each panel τ is set to a range of values from 0.5 years to the value of T, g^M is set to values of 2%, 6%, and 10%, and r^s is set to values of 2.5%, 6.5%, and 10.5%. The values of r^s were selected to correspond to three states: a heavily subsidized rate (2.5%), a marginally subsidized rate (6.5%, which is the same as the risk-free rate), and a non-subsidized rate (10.5%).

While it is the case that increasing g^M monotonically reduces the net present benefit to the HPF participant, the magnitude of this reduction varies depending upon the participant's time horizon T and the expected time of house purchase, τ. In Panel A, where T is set to 10 years, the effect of g^M is very small. In Panel C, where T is set to 45 years, when τ is 0.5 years the effects of increasing g^M are large. In the case where r^s is 6.5%, increasing g^M

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[30] China Real Estate Development Statistics, Beijing 2001

from 2% to 10% reduces the net present benefit to the HPF participant by 27,046 Yuan, taking them from a positive net benefit of 3,351 Yuan to a negative net benefit of –23,695 Yuan. If τ is set to 35 years, however, the same change in g^M results in only a decrease of only 21,728 Yuan in net present benefit. The reason for this is, of course, that as τ increases, the participant bears the maintenance cost for a shorter period of time.

Table 6: Net present benefit of HPF in Yuan for various values of T, τ, r^s, and g^M

Other parameter values: y_0=12,000 Yuan, g^Y=.05, r^f=6.5%, H_0=175,000, g^H=2%, ζ=1.0, α=0.33, M_0=300, β=6%, R_0=1000, g^R=2%

Panel A, T=10 years

	r^s								
	2.5%			6.5%			10.5%		
	g^M			g^M			g^M		
τ	2%	6%	10%	2%	6%	10%	2%	6%	10%
0.5	82,902	82,393	81,729	67,097	66,588	65,923	51,291	50,782	50,118
5	81,349	80,977	80,469	74,659	74,287	73,779	67,970	67,597	67,089
10	81,448	81,448	81,448	81,448	81,448	81,448	81,448	81,448	81,448

Panel B, T=25 years

	r^s								
	2.5%			6.5%			10.5%		
	g^M			g^M			g^M		
τ	2%	6%	10%	2%	6%	10%	2%	6%	10%
0.5	52,846	50,300	45,361	25,518	22,972	18,033	–1,809	–4,356	–9,294
5	50,620	48,210	43,428	33,081	30,672	25,889	15,542	13,133	8,350
10	48,858	46,820	42,546	39,870	37,832	33,558	30,882	28,844	24,570
25	53,076	53,076	53,076	53,076	53,076	53,076	53,076	53,076	53,076

Panel C, T=45 years

	r^s								
	2.5%			6.5%			10.5%		
	g^M			g^M			g^M		
τ	2%	6%	10%	2%	6%	10%	2%	6%	10%
0.5	35,755	29,454	8,710	3,351	–2,950	–23,695	–29,054	–35,355	–56,099
5	33,232	27,068	6,480	10,319	4,749	–15,893	–11,405	–17,570	–38,158
10	30,650	24,858	4,778	17,702	11,910	–8,170	4,754	–1,038	–21,118
25	30,908	27,153	11,347	30,908	27,153	11,347	30,908	27,153	11,347
35	35,872	33,915	24,144	35,872	33,915	24,144	35,872	33,915	24,144
45	39,307	39,307	39,307	39,307	39,307	39,307	39,307	39,307	39,307

The mortgage rate, r^s, also plays a major role in the value of the HPF program to the participant. As in the case of g^M, the magnitude of the effect of r^s on the net present benefit to the participant is a function of both T and τ. For example, when r^s is equal to 10.5%, increasing the participant's time horizon from 10 to 45 years, while holding τ constant at .5 years, and with g^M at only 2%, results in a decline of 80,285 Yuan, from 51,291 Yuan to -29,054 Yuan. There are a couple of reasons why increasing T results in such

a large decline in the participant's net benefit. One obvious reason is because with the increased T value, the participant pays the mortgage rate for a longer time, but it is also the case that with the longer time horizon the participant bears the maintenance cost for a longer time. If the mortgage rate were subsidized, however, increasing T will still decrease the value of the net present benefit, but it will be a significantly smaller amount. For example, with r^s at 2.5%, and with g^M at 2%, the net present benefit declines by 27,456 Yuan – roughly one third of the decline when r^s is set to 10.5%.

There are a couple of surprising results in Table 6 that deserve mentioning. The first is that when τ and T are set equal to each other neither g^M nor r^s affect the participant's net present benefit. As noted earlier, when τ and T are equal the model assumes that the HPF participant purchases and immediately resells the property, thus they never bear maintenance costs, nor do they receive a mortgage subsidy. The second surprising and somewhat anomalous result is that, given the large discount on the house (i.e. the low α value), the low housing growth rate (g^H), and the relatively high growth in income (g^Y) assumed in this Table, the HPF participant would accumulate enough cash in their HPF account after about 24 years to purchase their house without using a mortgage. This is why in Panel C, when τ greater than or equal to 25 years, the HPF benefit is not sensitive to the mortgage rate.

Conclusions

The Housing Provident Fund is quite complex and presents the potential homeowner with a number of options that could greatly affect their lifetime wealth. This paper presents an economic model of the HPF from the participating employee's perspective. In particular it derives the lifetime wealth change for a participant in the program relative to their wealth if they had continued to rent public housing. The paper then solves this model under a variety of parameter values.

From these results one can infer the optimal behavior for a rational program participant. This allows one to draw further inferences about which key parameters are the ones that best incentive program participants to minimize the time that they stay in the program prior to purchasing their unit. Given that a stated goal of the government is to privatize housing as quickly as possible, identifying these key parameters is important.

The results from Tables 3-6 demonstrate that the parameter that most strongly affects participant's wealth is the future growth rate of housing. When this parameter is high, relative to the risk-free rate, the participant's optimal strategy is to purchase their housing as soon as possible; conversely,

when it is low, their optimal strategy is to delay their purchase as long as possible. Unfortunately, this is also one parameter that it is virtually impossible for any government to control.

There are, however, other parameters that are much easier for the government to control, with the easiest perhaps being the growth rate in rents for public housing. While it is clear from Table 5 that increasing the growth rate in rents will, at the margin, tend to reduce the optimal time for a participant to be in the program before purchasing their housing, the magnitude of this effect is small. From a policy standpoint the difficulty is that even at high rental growth rates, the option to delay purchasing can still be valuable.

Whereas increasing the rental growth rate increases the attractiveness of purchasing by reducing the value of renting, increasing the discount offered on the house price, i.e. decreasing the parameter α, increases the attractiveness of purchasing by increasing the payoff to the participant. Unfortunately, both parameters have the same shortcoming; altering their values can increase the value of joining HPF, but they do not necessarily minimize the optimal time-until-purchase of the participant.

Perhaps the most effective parameter that the government can easily set and that provides an incentive to the participant to minimize the time before they purchase their housing is the mortgage interest rate subsidy. In many cases reducing the mortgage rate to below the risk-free rate minimizes the optimal time to purchase the housing unit, while in other cases it greatly reduces the benefit of delaying the purchase.

The Housing Provident Fund is one tool that the Chinese government has developed in its efforts to privatize housing that has previously been publicly owned. The program has had some success over the past decade in assisting in the privatization of the Chinese urban housing stock. Of course those citizens that were most likely to privatize have already done so. The challenge for HPF is to motivate those participants that are in the system but have not yet purchased their housing. This paper demonstrates that by careful selection of certain program parameters, the government can set up incentives for HPF participants to minimize the time until they purchase their housing.

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