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Efficiency and Justice in the Market for Cooperative Dwellings

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We exploit the fact that cooperative dwellings carry different mutual debt to examine whether such debt is perfectly reflected in sales prices. When mutual debt in housing cooperatives is paid down through the rent, the differences in mutual debt result in rent differences. We use implicit information on expenditures of serving mutual debt contained in rent differences as an alternative method for studying efficiency in the cooperative housing market. Our results indicate that differences in mutual debt are perfectly reflected in prices, but that rent differences are less useful for examining housing market efficiency.

Keywords

Market Efficiency, Mutual Debt, Rent, Co-operative Housing

1. Introduction

Some years ago, a court handled a case where two persons had purchased cooperative dwellings (co-ops). Prior to the transactions, the two had been informed about the mutual debt carried by each dwelling, but they later learned that the debt was substantially higher. The new knowledge induced them to request price reductions from the sellers. The sellers refused, and the two brought the cases to court. The court accepted that the plaintiffs had provided convincing evidence of being misinformed about the debt. Nevertheless, the court decided that the defendants were not obliged to offer a price reduction. The judge argued that what matters for the price of a co-op is the annual rent that the purchaser has to pay.[1] Down-payment of mutual debt and interest on that debt are important components of the rent, but the judge argued that the mutual debt carried by the dwelling, *as such*, is unimportant for the price for which it can be sold, cf. Tingrett (2007).

The statements of the plaintiffs and the judge bring us to the heart of the matter of this paper: (I) Is *the argument of the judge* that there is a direct link between the rent that the owner of a co-op has to pay and the price in a competitive market, valid? (II) Is *the argument of the plaintiffs* that the mutual debt carried by a co-op is directly reflected in the price for which it can be sold in a competitive market, correct? (III) Is it possible to reconcile the views of the plaintiffs and the judge - and the defendants? We provide answers to these questions. The results will tell us which side has valid arguments in court; in other words, the just price. The results will also inform us on whether price formation in a segment of the housing market is guided by pure economic rationality and efficiency. In this sense, our contribution belongs to the important vein of research initiated by Case and Shiller (1989), but in contrast to the tradition of Case-Shiller to focus on movements in housing prices over time, our contribution is basically a cross-section study.[2]

A model that incorporates the argument that market prices for co-ops are linked to the rent which holders of co-op housing units have to pay, is set out by Hjalmarsson and Hjalmarsson (2009). Schill et al. (2007) is an earlier

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[1] Our rent-concept is similar to the concept of association fees commonly used in the U.S. context, but note that our rent-concept includes interest on as well as installment of mutual debt.

[2] In addition to the seminal contribution of Case and Shiller (1989), this literature includes Case and Quigley (1991), Gatzlaff (1994), Berg and Lyhagen (1998), Malpezzi (1999), Hwang and Quigley (2004), and Røed Larsen and Weum (2008). Different aspects of housing market efficiency have been addressed by Gallin (2008), who examines the relationship between transaction prices and rents over time; Linneman (1986), who has conducted a cross-section analysis of deviations of transaction prices from the prices predicted by using an estimated hedonic price function; and Rosenthal (1999) who examines whether the relationship between transaction prices and costs of construction for new buildings is in accordance with economic efficiency.

contribution where rents play an important role in determining the intrinsic value of co-ops, but in the following, we adhere more to the paper of Hjalmarsson and Hjalmarsson (2009). They assume that prospective purchasers assess the present value of future rent-payments, and use this information to determine how much they are willing to pay for dwellings. Differences in present values of future rents are assumed to be discounted into the market price. Hereinafter, we refer to this as the RENT-approach. By using a dataset of Swedish co-ops, Hjalmarsson and Hjalmarsson (2009) find that differences in rents are not fully discounted into transaction prices. Hence, the view of the judge and the defendants that the rent of co-ops at the time of transaction provides a sufficient basis for assessing how prices of dwellings should be adjusted for future payment obligations, does not receive full support.

Robertsen and Theisen (2011) construct a model of co-op price formation where the price of a co-op is directly affected by the mutual debt that it carries, i.e. a model that incorporates the view of the plaintiffs. Hereinafter, we refer to this as the mutual debt (MUT) approach. By using a Norwegian dataset on transactions of co-ops and condominiums, they find that mutual debt is perfectly reflected in market prices. Hence, their results fully support the view of the two plaintiffs in that correct information about the mutual debt carried by a co-op is essential to determining the fair price that a buyer should be prepared to pay for the dwelling. Despite the use of similar approaches, different results are, however, obtained by Smith et al. (1984), who find that differences in the financial arrangements within a sample of US housing units are far from fully discounted into the prices of dwellings, and Kelly (1998) who concludes that mutual debt of co-ops in New-York City is excessively discounted into the prices.

Conflicting results on the impact of mutual debt on prices may be due to differences in the methods of analysis, the kind of data employed, and the markets from which the data are collected. In the present paper, we keep the sample and the market constant, and focus on differences in approach for analyzing the data. We use a new dataset on transactions of Norwegian co-ops, which in addition to transaction prices, includes data on mutual debt as well as the monthly rent paid by holders of co-op housing units. From the literature, it seems that such data are rare, but there is nothing in the way we analyze data that are confined to the Norwegian context. The great advantage is that our data make possible a systematic comparison of the RENT- and the MUT-approaches. Our unified theoretical model facilitates the comparison of these two approaches, and may provide evidence to the main issues that we are addressing; that is, the strengths and weaknesses of the RENT- and the MUT-approaches, and the conditions under which they should give similar results. Briefly stated, we find stronger support for the MUT-approach than for the RENT-approach.

In the next section, we set out a theoretical model of the relationship between mutual debt and the prices of co-ops. In Section 3, we modify the model to incorporate the approach of Hjalmarsson and Hjalmarsson (2009). In Section 4, we bring together the results from the two previous sections and discuss whether the two approaches may be reconciled. The econometric model is presented in Section 5, and data in Section 6. Section 7 contains the estimation results, and Section 8 concludes and provides some ideas for future research.

2. The Mutual Debt Approach

The purchaser of a co-op housing unit pays an equity price E_A^0 at the time of transaction $t=0$ for the unit.[3] The equity price is determined through competitive bidding, and must be personally financed by the purchaser in cash, by drawing on a savings account, or by loan from a credit institution.[4] In addition to the equity price, assume that the housing unit at $t=0$ carries an exogenous mutual debt, M^0, which is part of the mutual debt held by the housing cooperative. The mutual debt carried by the unit must be paid down over the years to come. Hence, the "initial payment obligation" affiliated with the unit considered is $\Pi^0 = E_A^0 + M^0$.

Holders of co-ops pay a monthly rent that includes two elements: capital expenditures and current expenditures. Capital expenditures include interest on and installments of mutual debt. Current expenditures contain operating expenditures, expenditures on current maintenance and repair, property taxes, and property insurance. Expenditures on current maintenance and repair encompass minor maintenance that must be done on a regular basis, like painting walls in the common halls, replacing a damaged window or door, etc. We assume that the current expenditures amount to a fixed yearly amount, b, per square meter of floor-space.[5]

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[3] Holders of Norwegian co-ops are formally shareholders in the co-operative.

[4] This is the case for second-hand sales, which is the focus of our paper. When a new housing cooperative is established by a housing association, the person who purchases a brand new co-op pays an equity price equal to the difference between the costs of construction and the mutual debt carried by the unit. If the housing cooperative is founded by a for-profit organization or firm, the seller obtains a loan that covers the mutual debt, and sells the dwellings at prices that correspond to what the market can bear. When a new housing cooperative is established, Norwegian law requires that at least 25 per cent of the costs are paid by the shareholders in the form of equity prices. The remaining 75 per cent (or less) constitutes the mutual debt. The law was changed in 2010. Prior to that, the equity price for new dwellings had to be at least 15 per cent of the total costs.

[5] This is the assumption made by Hjalmarsson and Hjalmarssom (2009), and also seems reasonable in the case of Norway. Comprehensive costs of maintenance/refurbishment are usually financed by mutual loans. Additional institutional facts on Norwegian housing co-operatives are provided by Robertsen and Theisen (2011).

Consider now a household who is about to purchase a co-op. Suppose that the choice is between two dwellings of exactly the same size, with other physical attributes also identical, and in the same location. Dwelling A carries a mutual debt, while there is no mutual debt on Dwelling B. How is the equilibrium equity price of Dwelling A related to its mutual debt and the equity price of Dwelling B? We examine this by means of a stylized model that builds on the user cost approach of inter alia McFadyen and Hobart (1978), and Poterba (1992).[6] Since the two dwellings that we consider by assumption are physically identical, current expenditures per year are, under our assumptions, the same (b multiplied by floor-space). Hence, we abstract from the current expenditures component in user costs. Similarly, for the case of the two physically identical dwellings, the remaining user-cost components, like depreciation, inflation, and risk-premium for homeownership, can also be abstracted from the current expenditures. By contrast, capital costs, which are related to the alternative cost of money invested and capital gains/losses, will be different for the two dwellings considered, and have to be accounted for in the following.

For Dwelling B - which carries no mutual debt - the equity price is at any time $E_B^t = E_B^0$. The yearly alternative cost of the money invested in this dwelling is $r_p E_B^0$, where r_p is the real interest-rate. We assume that r_p is equal to the real interest rate on bank loans, which is also taken to be equal to the real interest rate on deposits. Hence, the present value of the yearly alternative costs of capital for Dwelling B, from the time of purchase (0) up to the time horizon (T) is $\sum_{t=0}^{T} d_t r_p E_B^0$, where $d_t = (1+r_p)^{-t}$.

For Dwelling A, which carries a mutual debt, the alternative cost of money invested in the dwelling amounts in year t to $\tilde{C}_A^t = r_P E_A^t + r_M M^t$, where r_M is the interest rate on the mutual debt, which in general may differ from r_p.[7] Under our assumptions, the sum of privately invested capital and the mutual debt is, however, constant over time $\left(E_A^t + M^t = \Pi^0\right)$.[8] Hence, the capital invested up to time t in a dwelling that carries mutual debt, is related to the equity price originally paid, E_A^0, and the initial and remaining debt, as

[6] McFadyen and Hobart (1978) distinguish six components of the user costs of housing: the alternative cost of money invested in the dwelling, depreciation, costs of maintenance and repair, property taxes, property insurance, and capital gains. Poterba (1992) includes in the user cost also a risk premium for homeownership, but since we consider only holders of co-ops here, we abstract from this element.

[7] In the US market, interest rates on mutual debt normally exceed interest rates on private loans, while Robertsen and Theisen (2011) argue that in the Norwegian market, it is the other way around.

[8] This follows from $\Pi^0 = E_A^0 + M^0 = E_A^t + M^t = \Pi^t$.

$E_A^t = E_A^0 + M^0 - M^t$. Substituting this into our expression for yearly alternative costs of money invested in dwelling A yields $\tilde{C}_A^t = r_P E_A^0 + r_P M^0 - (r_P - r_M) M^t$. Discounting $\tilde{C}_A^t$ for all years up to the time horizon T to the period of purchase, $t = 0$, yields the present value of the alternative cost of money invested:

$$\tilde{\tilde{C}}_A^t = \sum_{t=0}^{T} d_t \left(r_p E_A^0 + r_p M^0 - \left(r_p - r_M \right) M^t \right). \tag{1}$$

Assume now that Dwelling A is sold exactly at the time when its original mutual debt has been paid down. Since Dwellings A and B are physically identical, and both do not at $t = T$ carry any mutual debt, Dwelling A must, in equilibrium, be sold for the same price as Dwelling B, i.e. for E_B^0. At $t = T$ the holder of Dwelling A will therefore incur a capital loss $L^T = E_A^0 + M^0 - E_B^0$, for which the present value is $d_T \left(E_A^0 + M^0 - E_B^0 \right)$. Next, since the discounted user cost in equilibrium must be equal for the two dwellings, equilibrium requires:

$$\sum_{t=0}^{T} d_t \left(r_p E_A^0 + r_p M^0 - \left(r_p - r_M \right) M^t \right) + d_T (E_A^0 + M^0 - E_B^0) = \sum_{t=0}^{T} d_t r_p E_B^0 . \tag{2}$$

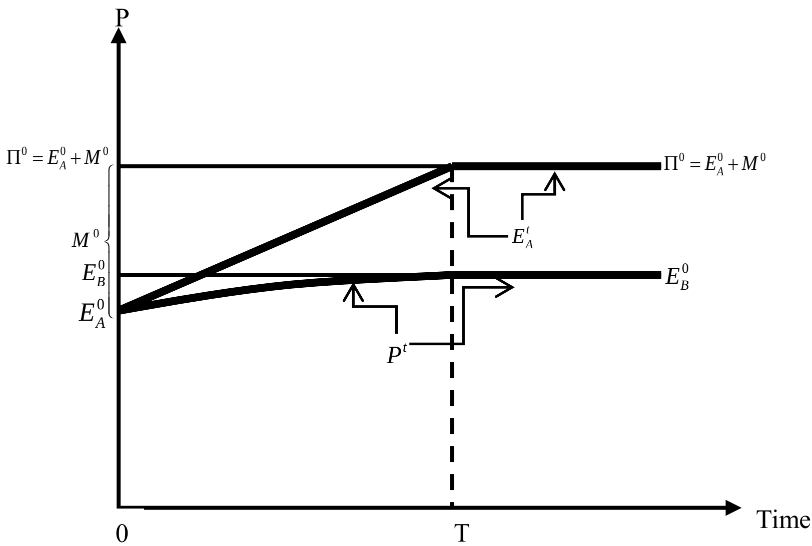
Eq. (2) solves for the equilibrium equity price of the dwelling with a mutual debt at $t = 0$:

$$E_A^0 = E_B^0 - M^0 + \left(r_p - r_M \right) \frac{\sum_{t=0}^{T} d_t M^t}{d_T + r_p \sum_{t=0}^{T} d_t} \quad . \tag{3}$$

The interpretation of Eq. (3) is simple: A person who at $t = 0$ purchases a dwelling with a mutual debt will pay the same as for a dwelling without such a debt, minus the mutual debt that rests on the dwelling, plus a term that captures what Robertsen and Theisen (2011) call the interest-discount-effect. For Dwelling A, the initial payment obligation, $\Pi^0 = E_A^0 + M^0$, will, if $r_P > r_M$, exceed that of Dwelling B. The reason is that the purchaser of Dwelling A has to pay for the benefit of a low interest rate on the mutual debt. The time-path for the equity price for the dwellings are shown in Figure 1, where the horizontal E_B^0-line represents the time-invariant equity price of Dwelling B. The straight upward-sloping line that starts from E_A^0 shows, for a person who at $t = 0$ acquires a dwelling with a mutual debt, how the amount of money invested in the dwelling increases as the mutual debt is paid down. At $t = T$, when the mutual debt is fully paid down, the line kinks and becomes horizontal at the level $\Pi^0 = E_A^0 + M^0$. The equity price that Dwelling A can be sold for at different points in time is illustrated by the curved line labelled P^t, which for

$r_P > r_M$ in all time periods $t < T$ lies below the kinked curve of total investments made in the dwelling that initially carries a mutual debt. This is due to the capital loss. For $t > T$, the P^t -line follows the E_B^0 -line.

Figure 1 Time-path of Mutual Debt, Money Invested, and Equilibrium Price



Let θ^t be the share of the initial mutual debt, M^0, that has *not* been paid down when period $t > 0$ comes. The time path of the mutual debt can then be fully characterized by the initial debt and the vector $\theta = \left(\theta^0\theta^1....\theta^T\right)$. At each point in time, θ^t may, however, be written as a function of down-payment rates, $\theta^t = \left(1-a^0\right)\left(1-a^1\right)....\left(1-a^{t-1}\right) = \Pi_{\zeta=0}^{\zeta=(t-1)}\left(1-a^\zeta\right)$, where a^t is the share of the remaining debt at the start of period t that is paid down at the end of period t. Substituting this expression for θ^t into $M^t = M^0\theta^t$ we get $M^t = M^0\Pi_{\zeta=0}^{\zeta=(t-1)}\left(1-a^\zeta\right)$, which upon substitution into Eq. (3) yields:

$$E_A^0 = E_B^0 + \underbrace{\left(-1+\left(r_p - r_M\right)\frac{\sum_{t=0}^{T} d_t \Pi_{\zeta=0}^{\zeta=(t-1)}\left(1-a^\zeta\right)}{d_T + r_p\sum_{t=0}^{T} d_t}\right)}_{\mu_{MUT}^M} M^0 = E_B^0 + \mu_{MUT}^M M^0, \quad (4)$$

where MUT indicates the mutual debt approach. By means of Eq. (4), one may examine the impact of all possible down-payment plans on the equity-price, but for the sake of simplicity, we focus here on two special cases defined by:

Assumption 1 (Constant down-payment rate): $a^t = a, \forall t, \ 0 < a < 1$.

Assumption 2 (Zero down-payment): $a^t = 0 \ \forall t$.

Assumption 1 implies that the mutual debt will never be completely paid down, and that $\Pi_{\zeta=0}^{\zeta=(t-1)}\left(1-a^{\zeta}\right)=\left(1-a\right)^t$, which simplifies Eq. (4). Assumption 2 amounts to no down-payment at all, and implies $\Pi_{\zeta=0}^{\zeta=(t-1)}\left(1-a^{\zeta}\right)=1$, which substantially simplifies Eq. (4).[9] Based on Eq. (4) and Assumptions 1 and 2, we obtain:

Proposition 1: (a) $r_M = r_P \Rightarrow \ \mu_{MUT}^{M} = -1$. *(b) Under Assumption 1, T=∞, which implies* $\mu_{MUT}^{M} = -1+\left(r_p - r_M\right)/\left(r_P + a\right)$. *(c) Under Assumption 2,* $\mu_{MUT}^{M} = -r_M / r_P$.

Proof: Part (a) of Proposition 1 follows from Eq. (4). Parts (b, c) are proven in Appendix A.

Part 1 of Proposition 1 is valid for all possible down-payment plans, while Parts (b, c) are valid only in the special cases given by the two assumptions. From Proposition 1, it follows that $-r_M / r_P$ constitutes an upper (lower) bound on μ_{MUT}^{M} for $r_P > r_M \quad \left(r_P < r_M\right)$. Moreover, μ_{MUT}^{M} in Eq. (4) may be calculated, for any down-payment plan and interest rates. We exploit this in Section 5.

3. The Rent Approach

Consider now the case where mutual debt is not observed. Prospective buyers of dwellings – as well as researchers - can only observe the annual (monthly) rent. For this case, Hjalmarsson and Hjalmarsson (2009) use data on the rent paid by holders of co-ops to establish a proxy for capital expenditures. We make their RENT-approach comparable to that in the previous section by incorporating their main idea into the model in Section 2. In order to do so, consider first the rent-function. Since annual rent, R^t, by definition is the sum

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[9] In the sample of Schill et al. (2007), 98 per cent of the co-ops have “balloon-loans”, with zero down-payment. Hjalmarsson and Hjalmarsson (2009) argue that similar types of loans are also common in Sweden.

of capital expenditures and current expenditures, we can write the rent-function as:

$$R^t = \left(r_M + a^t\right)M^t + bF\,, \tag{5}$$

where F is the floor-space. Eq. (5) solves for the mutual debt, which when substituted into Eq. (3), gives:

$$E_A^0 = E_B^0 - \frac{R^0}{r_M + a^0} + b\frac{F}{r_M + a^0} + \left(r_p - r_M\right)\frac{\sum_{t=0}^{T} d_t \dfrac{R^t - bF}{r_M + a^t}}{d_T + r_p \sum_{t=0}^{T} d_t}\,. \tag{6}$$

By introducing the rent-profile, $R^t = \psi^t R^0$, where R^0 is the annual rent in year 0, and ψ^t is the rent in year t relative to the rent in year 0, and also writing $\hat{R}^0 = R^0 \big/ \left(r_M + a^0\right)$ and $\hat{F} = F \big/ \left(r_M + a^0\right)$, Eq. (6) takes the form:

$$E_A^0 = E_B^0 + \underbrace{\left(-1 + \left(r_p - r_M\right)\frac{\sum_{t=0}^{T} d_t \left(\dfrac{r_M + a^0}{r_M + a^t}\right)\psi^t}{d_T + r_p \sum_{t=0}^{T} d_t}\right)}_{\eta_{RENT}^{\hat{R}^0}} \hat{R}^0 + \underbrace{b\left(1 - \left(r_p - r_M\right)\frac{\sum_{t=0}^{T} d_t \left(\dfrac{r_M + a^0}{r_M + a^t}\right)}{d_T + r_p \sum_{t=0}^{T} d_t}\right)}_{\omega_{RENT}^{\hat{F}}} \hat{F}\,. \tag{7}$$

$$E_A^0 = E_B^0 + \eta_{RENT}^{\hat{R}^0} \hat{R}^0 + \omega_{RENT}^{\hat{F}} \hat{F}\,. \tag{8}$$

where the index RENT on the quasi-parameters $\eta_{RENT}^{\hat{R}^0}$ and $\omega_{RENT}^{\hat{F}}$ denotes the rent approach. Eq. (8) is the counterpart of Eq. (4) in the MUT-model, and expresses the equilibrium price of Dwelling A as a function of the price of Dwelling B, discounted annual rent, and discounted floor-space. Discounted floor-space enters the model because one has to extract the part of the annual rent that is unrelated to the mutual debt.

Hjalmarsson and Hjalmarsson (2009) base all of their analysis on Assumption 1, and their baseline alternative on the stronger Assumption 2. The simplicity of their RENT-model relies very much on these assumptions. Invoking Assumption 1, we obtain from Eq. (7):

$$\beta_{RENT}^{\hat{R}^0} = \left(-1 + \left(r_p - r_M\right)\frac{\sum_{t=0}^{T} d_t \psi^t}{d_T + r_p \sum_{t=0}^{T} d_t}\right),\quad \omega_{RENT}^{\hat{F}} = b\left(1 - \left(r_p - r_M\right)\frac{\sum_{t=0}^{T} d_t}{d_T + r_p \sum_{t=0}^{T} d_t}\right). \tag{9}$$

Dividing by $\sum_{t=0}^{\infty} d_t$ in the numerator as well as in the denominator of the two fractions in Eq. (9), and using the fact that Assumption 1 in effect implies $T = \infty$, we obtain:

$$\eta_{RENT}^{\hat{R}^0} = \left(-1 + \left(1 - \frac{r_M}{r_p}\right)\pi\right), \quad \omega_{RENT}^{\hat{F}} = b\left(\frac{r_M}{r_p}\right). \tag{10}$$

where $\pi = \sum_{t=0}^{\infty} d_t \psi^t \Big/ \sum_{t=0}^{\infty} d_t$. In order to examine the magnitude of ψ^t, notice that under Assumption 1, $M^t = M^0 \theta^t = M^0 (1-a)^t$, which yields $R^t = (r_M + a) M^0 (1-a)^t + bF$. From this, we obtain $\psi^t = R^t / R^0$ $= \left((r_M + a) M^0 (1-a)^t + bF\right) / \left((r_M + a) M^0 + bF\right)$. Since $0 < a < 1$, we must have $(1-a)^t < 1$, and it follows that $\psi^t \leq 1$, with strict inequality for t >0. We then obtain:

Proposition 2: (a) $r_M = r_P \Rightarrow \eta_{RENT}^{\hat{R}^0} = -1$, $\omega_{RENT}^{\hat{F}} = b$. *(b) Under Assumption 1,* $\eta_{RENT}^{\hat{R}^0} = -1 + (1 - r_M / r_p)\pi$ *, with* $\pi = \sum_{t=0}^{\infty} d_t \psi^t \Big/ \sum_{t=0}^{\infty} d_t$ *, and* $\omega_{RENT}^{\hat{F}} = b(r_M / r_p)$ *. (c) Under Assumption 2,* $\eta_{RENT}^{\hat{R}} = -r_M / r_P$ *,* $\omega_{RENT}^{\hat{F}} = b(r_M / r_p)$.

Proof: Part (a) of this proposition directly follows from setting $r_M = r_p$ in Eq. (9). Part (b) directly follows from Eq.(10). To prove Part (c), notice that $a^t = 0 \Rightarrow M^t = M^0$. This yields $R^t = r_M M^0 + bF$, $\forall t$, $\Rightarrow$ $\psi^t = R^t / R^0 = 1$, $\forall t$. Inserting $\psi^t = 1$ in $\pi = \sum_{t=0}^{\infty} d_t \psi^t \Big/ \sum_{t=0}^{\infty} d_t$ yields $\pi = 1$. For $\pi = 1$, the expression for $\eta_{RENT}^{\hat{R}^0}$ in Eq. (10) simplifies to that in Part (c) of Proposition 2.

In plain words, Proposition 2 says that future rent-payments are less than fully discounted in transaction prices if $r_M < r_p$, more than fully discounted if $r_M > r_p$, and exactly discounted if $r_M = r_p$. Parts (b) and (c) can be used to calculate the bound on how much discounting deviates from -1. In Part (b), i.e. under Assumption 1, the bound also depends on interest rates and the down-payment plan. In Part (c), i.e. under Assumption 2, the bound only depends on the relative interest rates.

4. Can the Mutual Debt Approach and the Rent Approach be Reconciled?

In order to determine whether the two approaches can be reconciled, let us first draw together Propositions 1 and 2. This yields:

Corollary 1: (a) For $r_M = r_P$, $\mu_{MUT}^{M} = \eta_{RENT}^{\hat{R}^0} = -1$. *(b) Under Assumption 1,* $\mu_{MUT}^{M} < \eta_{RENT}^{\hat{R}^0}$ *if* $\pi > \left(r_p + a\right)^{-1}$, *and* $\mu_{MUT}^{M} > \eta_{RENT}^{\hat{R}^0}$ *if* $\pi < \left(r_p + a\right)^{-1}$. *(c) Under Assumption 2,* $\mu_{MUT}^{M} = \eta_{RENT}^{\hat{R}} = -r_M / r_P$.

Part (a) of this corollary indicates that the difference between the two models disappears when interest rates on mutual debt and other loans are equal. The equity price in the MUT-model then drops by exactly one unit for each extra unit of mutual debt, and the equity price in the RENT-model drops by one unit for each unit increase in discounted annual rents. The reason is that capital costs in this case will be the same in the two models.

Part (b) of Corollary 1 indicates that when the two interest rates differ, and mutual debt is paid down over time, there will not be a one-to-one relationship between an increase in the mutual debt or discounted annual rent and the drop in the equity price. Even in the simple case when the mutual debt each year is paid down by a constant percentage (Assumption 1), the magnitude of μ_{MUT}^{M} and $\eta_{RENT}^{\hat{R}^0}$ depends, in a complex way, on the interest rates and the down-payment schedule for the mutual debt. Perhaps needless to say, in the real world, with even more complex down-payment schemes, the two approaches will not lead to results that are easily comparable. For any given down-payment plan, it is, however, possible to calculate the magnitude of μ_{MUT}^{M} and $\eta_{RENT}^{\hat{R}^0}$.

Part (c) of Corollary 1 indicates that when the holders of co-ops do not pay down the mutual debt over time, the difference between the two approaches disappears. The key coefficients, μ_{MUT}^{M} and $\eta_{RENT}^{\hat{R}^0}$, will then be equal in the two models, and to the ratio between the interest rate on mutual debt and private debt, with a negative sign. Hence, in this case, there will not be a one-to-one correspondence between, on the one hand, the equity price, and on the other hand, the mutual debt or the discounted future rents.

From the discussion above, it follows from Part (a) of Corollary 1 that if one is convinced that (1) the two interest rates are identical, and (2) individuals in the housing market discount perfectly, there should not be any reason to estimate the coefficients by using econometric methods. Hjalmarsson and Hjalmarsson (2009) question, however, the assumption that actors in the market for co-ops discount perfectly. Hence, they estimate the RENT-model and test whether this is the case. For this case, the RENT-model is equally well suited as the MUT-model.

If mutual debt is not paid down over time, as assumed in Part (c) of Corollary 1, and if we as researchers can be confident (1) on what the actual interest rates are, and (2) that individuals in the housing market discount perfectly, there should not be any reason to estimate the coefficients by using econometric methods. Usually it is, however, far from straightforward to determine which interest rates (short-, medium-, or long-term) that should be used, and whether actors discount properly. There is then, the rational for an econometric examination, but it should not matter whether we estimate the MUT- or the RENT-model. In both cases, the estimated coefficients can be interpreted as the implicit relative interest rates used by market actors in their discounting.

In the case covered by Part (b) of Corollary 1, when the two interest rates differ and down-payment plans are complex, the situation is so complex that it may be warranted to test whether actors in the market for co-ops account for the differences in financial arrangements in an economically rational manner. Robertsen and Theisen (2011) have considered this case.

5. Econometric Model and Estimation

We will estimate the relationships derived in Sections 2 and 3 from data on dwellings sold during a two-year period. In addition to the financial attributes measured by the mutual debt in the MUT-model, and by the discounted annual rent supplemented by the discounted floor-space in the RENT-model, we account for differences between dwellings by using a hedonic function, $X_i\beta_j$, where X_i is a vector of non-financial housing attributes, and β_j $(j = MUT, RENT)$ are the parameter vectors.

In estimating hedonic functions for house prices, it is common to use non-linear functional forms, like a semi-logarithmic, double-logarithmic, or Box-Cox transformation. In our case, there is, however, a particular rationale for choosing a functional form that implies a linear relationship between the sales price and mutual debt in the MUT-model, and between price, discounted rent and discounted floor-space in the RENT-model. The rationale for this is that the intrinsic value of a co-op that carries a mutual debt is the sum of the equity price term and the term that captures the effect of mutual debt on price. The future costs of servicing a mutual debt are in fact a *deferred payment*. Hence, at $t = 0$, the intrinsic value of the dwelling is in the MUT-model $P_i + \mu_{MUT}^{M} M_i^0$, and in the RENT-model $P_i + \eta_{RENT}^{\hat{R}} \hat{R}_i + \omega_{RENT}^{\hat{F}} \hat{F}_i$. The terms $\mu_{MUT}^{M} M_i^0$ and

$\eta_{RENT}^{\hat{R}}\hat{R}_i + \omega_{RENT}^{\hat{F}}\hat{F}_i$ should not be modelled as interacting with the elements of the X_i -vector. This is avoided by choosing a linear specification. Hence, we specify the counterparts of Eqs. (4) and (8) as:

$$P_i = \alpha_{MUT} + X_i\beta_{MUT} + \mu_{MUT}^{M}M_i^0 + Z_i\gamma_{MUT} + D_i\lambda_{MUT} + \varepsilon_{i,\mathrm{MUT}}\,. \tag{11}$$

$$P_i = \alpha_{RENT} + X_i\beta_{RENT} + \eta_{RENT}^{\hat{R}}\hat{R}_i + \omega_{RENT}^{\hat{F}}\hat{F}_i + Z_i\gamma_{RENT} + D_i\lambda_{RENT} + \varepsilon_{i,RENT}\,. \tag{12}$$

The left-hand-side variable, P_i , in Eqs. (11) and (12), is the equity price for which dwelling i is sold. On the right hand side, we have included a vector of zip-code dummies $Z_i = \left(Z_i^2 ... Z_i^S\right)$, and a vector of time-period dummies $D_i = \left(D_i^2 .... D_i^{24}\right)$. Furthermore, α_j is a constant term, $\gamma_j = \left(\gamma_j^2 ...... \gamma_j^{24}\right)$, and $\lambda_j = \left(\lambda_j^2 .... \lambda_j^S\right)$ are the parameter vectors $\left(j = MUT, RENT\right)$. Finally, $\varepsilon_{i,j}$ are the stochastic error terms. Conditional on j to be equal to either MUT or RENT, $\varepsilon_{i,j}$ is assumed to be identically normally distributed with zero expectation and a constant variance.

In order to accommodate a possible non-linearity in the relationships between the sales price and other independent variables than mutual debt and rent, we will, in Section 7, experiment with different ways of including floor-space and age. For the age of dwellings, we will try out both a set of dummy variables similar to those used by Hjalmarsson and Hjalmarsson (2009), and a spline function like the one used by Robertsen and Theisen (2011). Similarly, we will, in several of the estimated relationships, use a spline function for floor-space.[10]

Our main interest is in the coefficient affiliated with mutual debt in Eq.(11), and discounted annual rent in Eq. (12). Consider first Eq. (11), which corresponds to Eq. (4), from which it is evident that the time-period over which mutual debt is supposed to be paid down in general will differ between dwellings. Hence, in general, T should be indexed by dwelling, i.e. as T_i . When the T_i s differ between dwellings, the magnitude of the parameters that describe the down-payment plan will also depend on T_i . Consequently, the parameter μ_{MUT}^{M} in Eq. (11) will, in general, be a function of T_i . For instance, with $r_M = 0.02$,and $r_P = 0.03$, we obtain, for the case when mutual debt for a new dwelling is paid down as an annuity loan over 30 years, $\mu_{MUT}^{M} = -0.89$. If there are very few years left of the down-payment period, μ_{MUT}^{M} is close to -1.

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[10] Robertsen and Theisen (2011) provide arguments for using spline functions to model the impact of floor-space and age of dwelling on the price of dwellings.

To conclude, for a most typical financial arrangement for the mutual debt of Norwegian co-ops – an annuity loan that is paid down over 30 years, and interest rates $r_M = 0.02$, and $r_P = 0.03$ – one should when estimating Eq. (4) expect a magnitude of the mutual debt coefficient close to the mid-point, $\mu_{MUT}^{M} \approx -0.94$ of the interval $\mu_{MUT}^{M} \in (-1, -0.89)$.

Through additional numeric calculations, it can be shown that μ_{MUT}^{M} is a slightly convex function of T_i. Moreover, the magnitude of μ_{MUT}^{M} is only slightly affected by the level of r_P, as long as the interest rate difference $(r_P - r_M)$ is kept constant, and the variation of r_P is within a reasonable range of 2-3 percentage points on both sides of the reference level. Since μ_{MUT}^{M} is slightly convex and decreasing in T_i, for a given interest-rate difference, the linear approximation $\mu_{MUT}^{M} = \sigma_{MUT}^{M} + \rho_{MUT}^{M} T_i$, where $\sigma_{MUT}^{M} < 0$, and $\rho_{MUT}^{M} < 0$ are the parameters, provides a fairly good approximation of the true relationship between the length of the down-payment period and the magnitude of μ_{MUT}^{M}. In Section 7, we test whether ρ_{MUT}^{M} is different from zero.

For the Hjalmarsson-model, we have carried out calculations of $\eta_{RENT}^{\hat{R}}$. For an annuity loan with $r_M = 0.02$, and $r_P = 0.03$, we obtain $\eta_{RENT}^{\hat{R}} = -0.99$ for $T_i = 1$, and $\eta_{RENT}^{\hat{R}} = -0.83$ for $T_i = 30$. Hence, we expect the estimate of $\eta_{RENT}^{\hat{R}}$ in Section 7 to lie close to the mid-point, $\eta_{RENT}^{\hat{R}} \approx -0.91$, of the interval $\eta_{RENT}^{\hat{R}} \in (-0.99, -0.83)$. Moreover, through additional calculations, we find that $\eta_{RENT}^{\hat{R}}$ is a slightly convex function of T_i. Hence, the linear approximation $\eta_{RENT}^{\hat{R}} = \sigma_{RENT}^{\hat{R}} + \rho_{RENT}^{\hat{R}} T_i$, where $\sigma_{RENT}^{\hat{R}} < 0$, and $\rho_{RENT}^{\hat{R}} < 0$ are the parameters, provides a fairly good approximation of the true relationship between the length of the down-payment period and the magnitude of $\eta_{RENT}^{\hat{R}}$.

In Eq. (12), we have to discount annual rent and floor-space before estimation in order to obtain the two variables $\hat{R}_i$ and $\hat{F}_i$. It seems reasonable to assume that this discounting should be made on the basis of long-term interest rates. Hjalmarsson and Hjalmarsson (2009) use five-year average interest rates, and with transaction data collected over a period of more than 3.5 years, their interest-rate data exhibit sufficient variation to identify $\omega_{RENT}^{\hat{F}}$. In the present paper, we will only use data for a two-year period. In this period, interest rates fluctuated strongly, but rather than introducing assumptions about the relationship between short-term and long-term interest rates, we consider two extreme alternatives: In Alternative I, we assume myopic interest-rate perceptions, i.e. that short-term rates are perceived as corresponding to long-term rates. In Alternative II, we assume that the long-term interest rates are equal to the average interest rates over the period. Under the latter assumption, discounted floor-space will be perfectly correlated with floor-space. Hence, on the assumption that short-run variations in interest rates do not affect the perceived long-term rates, $\omega_{RENT}^{\hat{F}}$ cannot be identified. Since $\omega_{RENT}^{\hat{F}}$ is not a parameter of primary interest, this does not cause particular problems. It is crucial, however, to examine whether our assumptions about the relationship between short-run and long-run interest rates affect the estimation of $\eta_{RENT}^{\hat{R}}$.

One final caveat should be added: Annual rent is not included in our version of the MUT-model. Under our assumption that current costs per square meter are constant across dwellings, this is theoretically correct. The impact of current costs on dwellings with different floor-space will be captured by the coefficient of the floor-space-variable, and should not be added as a separate variable in the MUT-model. We return to this at the end of Section 7.

In Section 7, we estimate Eqs. (11) and (12) by using the ordinary least squares (OLS) method. Equipped with the estimation results, we test two main hypotheses. First, we test $H_0 : \mu_{MUT}^{M} = \tilde{\bar{\mu}}_{MUT}^{M}$ against the alternative $H_1 : \mu_{MUT}^{M} \neq \tilde{\bar{\mu}}_{MUT}^{M}$, where $\tilde{\bar{\mu}}_{MUT}^{M} = -0.94$ is the midpoint of the interval found in previous calculations. Secondly, we test $H_0 : \eta_{RENT}^{\hat{R}} = \tilde{\bar{\eta}}_{RENT}^{\hat{R}}$ against $H_1 : \eta_{RENT}^{\hat{R}} \neq \tilde{\bar{\eta}}_{RENT}^{\hat{R}}$, where $\tilde{\bar{\eta}}_{RENT}^{\hat{R}} = -0.91$ is the midpoint of the interval obtained from the calculations of the expression for $\eta_{RENT}^{\hat{R}}$ in Eq. (8). Based on these results, we will discuss the relationship between the estimated parameters μ_{MUT}^{M} and $\eta_{RENT}^{\hat{R}}$, and the appropriateness of the RENT- versus the MUT-model through the estimates of these coefficients.

6. Data and Descriptive Statistics

We use data on cooperative dwellings in Kristiansand, the fifth largest city in Norway, with a population of 85,000 in 2010. According to Statistics Norway (2001), 77% of the housing units in both Kristiansand and in the country as a whole are owned by their occupants. Of those living in their own dwellings, about 20% of the households in Kristiansand live in housing cooperatives, while the corresponding figure for Norway as a whole is 18%. Furthermore, 21% of the housing units in Kristiansand, and 19% in the country as a whole, are in blocks of apartments. Hence, the housing market in Kristiansand is in many respects representative of the Norwegian housing market.

We extract transaction-data for co-ops in Kristiansand from January 1st 2009 to December 31st 2010, from the complete register of property transactions in the data-base of Eiendomsverdi (property value). Secondly, we obtain supplementary information on housing attributes of the dwellings from the web-page Finn.no, which is linked to the data-base of Eiendomsverdi.[11] Thirdly, the housing association active in the Kristiansand market complements the data. The result is a dataset of 1092 dwellings. After exclusion of 42 cases with item-non-response for price, floor-space, age-of-dwelling, mutual debt, or annual rent, and 6 observations of dubious quality, we are left with a sample of 1044 co-ops.[12]

Table 1 contains the descriptive statistics for the basic variables used in the empirical analysis, and some aggregate information on the dummy variables of location and month of transaction. Exact variable definitions are provided in Appendix B. The span in sales price, floor-space and age is substantial. The oldest units in our sample are 65 years old, built in 1945, the year when the first Norwegian housing cooperatives were established. A share of 6% of the dwellings in the sample carry no mutual debt, 32% carry a mutual debt in the interval between 0-100 000 NOK, 48% between 100 000-500 000 NOK, 6% between 500 000 - 1 000 000 NOK, and 8% between 1 000 000-3 000 000 NOK.[13] The minimum equity ratio $\left(E_i^0 \big/ \left(E_i^0 + M_i^0\right)\right)$ in the sample is 0.17. Finally, about 90% of the dwellings in our sample are apartments in block buildings, while 10% are in non-block buildings, which are mainly row-houses.

Table 2 contains the correlation matrix for the main independent variables. Most of these are weakly correlated. Notice, however, that mutual debt and rent – as expected - are highly correlated. Moreover, as well mutual debt as annual rent is highly correlated with the age of the dwellings. As we shall see in Section 7, this gives rise to some estimation problems, but these can be handled by making use of prior information on how age of dwelling impacts price.

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[11] Finn.no is a website where prospects for almost all properties for sale in Norway are posted, see http://www.finn.no/finn/realestate/homes/browse1.

[12] A few of the transactions may not be the result of competitive bidding processes, mainly because of sales within the family and transfer of property-rights by inheritance. According to the Norwegian tax laws, all transactions due to inheritance have to be made at market prices, but we cannot rule out that the price in some cases may be slightly lower than would have been the result of competitive bidding. We have, however, no way of detecting non-market transactions. The number of such transactions is likely to be small, though. The 6 observations of dubious quality include 5 units for which we suspect that information on mutual debt may not have been accurately perceived by the purchasers. One dwelling registered as a self-owned housing unit is also excluded. Presumably, this unit has recently been converted into a self-owned dwelling (condominium).

[13] On March 16th 2015, 1 US\$ ≈ 8.21 NOK, 1 Euro ≈ 8.62 NOK.

Table 1 Descriptive Statistics

Variable	Mean	Standard dev.	Minimum	Maximum
Price	1 504 824	448 507	55 000	3 600 000
Mutual	278 746	391 339	-23 323	2 700 000
Rent	37 953	17 481	11 700	162 372
Interest	1.1440	1.4340	-1.05	2.32
Space	70.9406	21.1085	23	174
Noblock	0.0987	0.2983	0	1
Age	37.2625	16.4592	0	65
Floor	2.8870	2.3313	1	11
Lift	0.2395	0.4270	0	1
Kitchen	2.7146	0.8209	2	4
Bath	0.6188	0.8217	0	2
Period	-	-	0.0220	0.0651
Location	-	-	0.0019	0.1753

7. Empirical Results

7.1 The MUT-Model

Estimation results for the MUT-model are shown in Table 3. Consider first briefly, Specification MUT_A. This explains 69 per cent of the variation in the dependent variable; all the estimated coefficients carry the expected sign, and most of them are significant at standard levels of statistical significance. The absolute magnitude of the estimated mutual-debt-coefficient is, however, much smaller than expected. Moreover, the coefficients affiliated with the age-dummies exhibit an unreasonable pattern, indicating that dwellings between 10 and 50 years are lower priced than those that are more than 50 years old. Hence, let us consider a few alternative specifications.

In Specification MUT_B, we have replaced the age-dummies in Specification MUT_A by a spline-function for age, and the impact of floor-space is also modeled by means of a spline function. This gives a mutual-debt-coefficient with a slightly larger negative magnitude than in Specification MUT_A. The two age-variables and mutual debt are, however, highly correlated, and a variance inflation factor (VIF) test indicates that the standard deviations are inflated. We are therefore concerned that high correlation between mutual debt and age in Specification MUT_B may give poor identification of the coefficients for both of these variables.

Since the root of the problem lies in the nature of the data, an obvious solution would be to acquire additional data. Such a procedure is used by Robertsen and Theisen (2011), who in addition to co-ops, include condominiums, with zero mutual debt, in their sample. Here, we have chosen another avenue, namely, to acquire additional data in a "condensed form". That is, we have used the estimated coefficients in the spline-function for age in Robertsen and Theisen (2011) as prior information in the estimation in Specification MUT_C in Table 3. Details on this approach are provided in Appendix B. The results for Specification MUT_C in Table 3 show that the magnitude of the mutual

Table 2 Correlation Matrix of the Main Independent Variables

	Mutual	Rent	Interest	Space	Noblock	Age	Floor	Lift	Kitchen	Bath
Mutual	1.00									
Rent	0.75	1.00								
Interest	- 0.00	0.01	1.00							
Space	0.16	0.37	- 0.05	1.00						
Noblock	- 0.07	0.04	0.00	0.46	1.00					
Age	- 0.70	- 0.62	0.01	- 0.21	0.01	1.00				
Floor	- 0.15	- 0.15	- 0.01	- 0.26	- 0.26	0.21	1.00			
Lift	0.15	0.10	0.02	- 0.11	- 0.18	- 0.19	0.35	1.00		
Kitchen	0.44	0.30	0.09	0.05	- 0.03	- 0.37	- 0.06	0.07	1.00	
Bath	0.48	0.35	0.05	0.03	- 0.03	- 0.41	- 0.05	0.10	0.63	1.00

debt coefficient is substantially larger negative than in Specifications MUT_A and MUT_B. In addition, the standard deviation affiliated with the mutual-debt coefficient in Specification MUT_C is substantially smaller than in the previous specifications. Finally, the overall fit of Specification MUT_C is clearly superior to MUT_B. Hence, it seems that incorporation of prior information on the impact of age has substantially improved the results, thus making Specification MUT_C our preferred specification.

As a robustness-check, we have included results from re-estimating Specification MUT_C with alternative prior age-coefficients in Appendix C. This exercise reveals that a 10 per cent increase in the prior age-coefficients changes the estimated mutual debt coefficient from -0.872 to -0.897, while a 10 per cent reduction in the prior age-coefficients gives a change in the estimated mutual debt coefficient from -0.877 to -0.846. From this, we conclude that the estimated mutual debt coefficient in MUT_C is reasonably robust.

In Specification MUT_D, we have added the variable MRemain to the explanatory variables in Specification MUT_C, thus capturing the possible impact of the remaining years before the mutual debt is paid down. From Table 3, it is evident that the impact of this variable is not significant at standard levels of statistical significance. The reason may be that our measure of the remaining years before the mutual debt is paid down is too crude, but there is little that we can do about this.

In Specification MUT_E, we have added the variable MInterest to the explanatory variables in Specification MUT_C. This variable has, however, no statistically significant impact on the price. We interpret this as evidence that short-term variations in interest rates do not affect the *relative* prices of dwellings. That is, the actors in the co-op housing market seem to base their discounting of future obligations to pay down mutual debt on *long-term interest rates* that presumably change slowly over time.

To conclude, the results above imply that Specification MUT_C is our preferred specification, with an estimate of the mutual-debt-parameter that amounts to $\mu_{MUT}^{M} \approx -0.87$. This estimate is robust and precise, and the 5% confidence interval of this parameter includes the "expected" magnitude $\tilde{\bar{\mu}}_{MUT}^{M} \approx -0.94$ obtained through calculations based on the theoretical model. Hence, the null hypothesis $H_0 : \mu_{MUT}^{M} = \tilde{\bar{\mu}}_{MUT}^{M}$ cannot be rejected. We therefore conclude that mutual debt seems to be (almost) perfectly reflected in the transaction prices.

Table 3 Estimation Results (OLS) for the Mutual Debt Model. N=1044

	MUT_A	MUT_B	MUT_C	MUT_D	MUT_E
Mutual	-.7480***	-.7754***	-.8716***	-.8266***	-.8525***
	(.0728)	(.0750)	(.0562)	(.0890)	(.0667)
MInterest					-.0163
					(.0290)
MRemain				-.0017	
				(.0034)	
Space	13921***	24907***	26714***	26565***	26794***
	(623)	(1892)	(1936)	(1940)	(1938)
Space50		-12861***	-15249***	-15148***	-15380***
		(2329)	(2345)	(2332)	(2350)
SNblock	-798*	-245	-133	-132	-116
	(350)	(351)	(359)	(360)	(360)
Age		-16340***			
		(3874)			
Age25		20908***			
		(4622)			
Age10-19	-248320*				
	(98028)				
Age20-29	-222279***				
	(81170)				
Age30-39	-282523***				
	(80844)				
Age40-49	-248485***				
	(80868)				
Age50-59	-147782				
	(86388)				
Age60+	-55436				
	(105019)				
Floor	1834	5673	9812**	9702**	9731***
	(3902)	(3685)	(3705)	(3759)	(3705)
Lift	71973***	70421***	45069	47098*	46031
	(24533)	(24087)	(23893)	(24673)	(24289)
Kitchen	56389***	54012***	51205***	51728***	51100***
	(10907)	(10601)	(10807)	(10834)	(10796)
Bath	38878***	37059***	31852***	31744**	31842**
	(12030)	(11673)	(12301)	(12294)	(12308)
Constant	575195	180522	332244	331395	323266
Periods	Yes	Yes	Yes	Yes	Yes
Zip-codes	Yes	Yes	Yes	Yes	Yes
R-sq. adj.	.6776	.6897	.7590	.7588	.7591
F	43.15	48.31	70.90	69.37	69.46

Note: Period-specific interest rates. Robust standard errors below coefficients. * indicates statistical significance at 5 per cent level, ** at 1 per cent level, and *** at 0.1 per cent level.

7.2 The RENT-Model

Let us now turn to the RENT-model, for which results are shown in Table 4. All of these results are obtained by estimating Eq. (12), and using $a^0 = 0.03$ when calculating $\hat{R}$ and $\hat{F}$. In their baseline equation, Hjalmarsson and Hjalmarsson (2009) assume that $a^0 = 0$, but for the Norwegian case, we find $a^0 = 0.03$ to be more realistic. We return below, however, to the role of the parameter a^0.

Table 4 Estimation Results (OLS) for the Rent Model. N=1044

	Period-specific Interest Rates			Average Interest Rates	
	RENT_A	RENT_B	RENT_C	RENT_D	RENT_E
PVRent	-.5014***	-.7471***	-.7404***	-.4932***	-.7481***
	(.0750)	(.0657)	(.0663)	(.0849)	(.0743)
PVSpace	260	408			
	(148)	(156)			
Space	23936***	23607***	31523***	28567***	31096***
	(3268)	(3547)	(2188)	(2024)	(2198)
Space50	-16558***	-18490***	-18894***	-16389***	-18438***
	(2334)	(2519)	(2572)	(2392)	(2586)
SNblock	-282	75	141	-231	167
	(354)	(358)	(371)	(360)	(368)
Age	5055			5605	
	(3833)			(3871)	
Age25	-5373			-5710	
	(4563)			(4529)	
Floor	10616**	19611***	20328***	10664**	20166***
	(4056)	(4368)	(4396)	(4003)	(4344)
Lift	60969*	-5056	-5301	59967*	-9299
	(27543)	(28938)	(29170)	(27574)	(29062)
Kitchen	39918***	16391	18891	41973***	18861
	(11057)	(11954)	(11812)	(10907)	(11695)
Bath	28770*	-1977	-4830	28047*	-4125
	(11935)	(14070)	(14069)	(11880)	(14103)
Constant	-252172	496417	314607	-285123	463386
Periods	Yes	Yes	Yes	Yes	Yes
Zip-codes	Yes	Yes	Yes	Yes	Yes
R-sq. adj.	.6320	.6802	.6776	.6283	.6746
F	36.82	47.22	47.64	36.98	47.01

Note: Robust standard errors below coefficients. * indicates statistical significance at 5 per cent level, ** at 1 per cent level, and *** at 0.1 per cent level.

In Specifications RENT_A, RENT_B and RENT_C in Table 4, period-specific interest rates are used for calculating $\hat{R}$ and $\hat{F}$. The estimated PVRent-coefficient in Specification RENT_A carries a negative sign, and is precisely estimated, but the magnitude is only -0.50, compared to the expected -0.91.

Moreover, the age-coefficients are not statistically significant, and carry unexpected signs, a result that is most likely due to the co-linearity between age and rent. Therefore, consider Specification RENT_B, where we have used prior information for the impact parameters of age of dwellings, in exactly the same way as in the preferred specification for the MUT-model. The fit of Specification RENT_B is much better than for RENT_A. Moreover, the estimate $\eta_{RENT}^{\hat{R}} = -.74$ is much closer to the expected -.91, and the standard deviation of this parameter is also smaller than in RENT_A. Notice also that the impact on the estimate of $\eta_{RENT}^{\hat{R}}$ of using prior information for the impact of age is much stronger in the RENT-model than in the MUT-model. A robustness-check that is reported in Appendix C shows that the estimate for the impact parameter $\eta_{RENT}^{\hat{R}}$ in RENT_B is quite robust towards variations in the priors for the age-coefficients. Hence, the problems due to the fact that age and rent are correlated in our sample may be remedied by using prior information on the impact of age. Nevertheless, the $\eta_{RENT}^{\hat{R}}$ -estimate in Specification RENT_B substantially falls short of the expected -0.91. Moreover, the VIF-test reveals that in Specification RENT_B, there are problems related to co-linearity, in particular for the floor space and discounted-floor-space variables. Hence, we have estimated Specification RENT_C, which is equal to Specification RENT_B, except that in RENT_C, we have dropped discounted floor-space. This has little impact on other coefficients than that of the plain floor-space-variable. Most importantly, the estimate of the PVRent-coefficient is minimally affected.

Specifications RENT_D and RENT_E in Table 4 show the results from estimating Eq. (12) with $\hat{R}$ calculated from the average interest rate over the entire two-year period from which we collect our transaction data. The parameter a^0 is still pegged at 0.03. Recall also from Section 5 that the impact parameter of $\hat{F}$ cannot be identified without variation in interest rates. Hence, PVSpace is excluded from Specifications RENT_D and RENT_E. In these specifications, the plain floor-space variable captures the combined impact of floor-space and PVSpace. Hence, we see that the estimated coefficient of the floor-space variable in Specifications RENT_D and RENT_E is substantially larger than the corresponding coefficient in Specifications RENT_A and B. Of greater interest is, however, that the estimates of $\eta_{RENT}^{\hat{R}}$ in Specifications RENT_D and RENT_A are virtually identical, and that the same is the case in Specifications RENT_E and RENT_C. Hence, we conclude that it seems to matter very little for the estimate of $\eta_{RENT}^{\hat{R}}$ whether we use period-specific or average interest rates for calculating variables $\hat{R}$ and $\hat{F}$.

In order to more closely examine the role of Assumption 1, we have re-estimated the preferred specification, RENT_B, for different magnitudes of the parameter a^0. The results in Table 5 reveal that the prior choice of a^0 has a decisive impact on the estimation results for $\eta_{RENT}^{\hat{R}}$. A larger magnitude of a^0 means a more strongly negative estimated $\eta_{RENT}^{\hat{R}}$. The problem is that correct prior information on the magnitude of a^0 is not easily available to us as researchers. Moreover, even if it should be possible to obtain valid information on the average magnitude of a^0, the problem remains that the magnitude of this parameter may differ between dwellings. Such information is in general not available to researchers. For these reasons, we will argue that the RENT-model should only be used when it is known that the interest rate on mutual debt and private debt is the same. As stated in Part (a) of Corollary 1, one is then on firm ground. The paper of Hjalmarsson and Hjalmarsson (2009) is based on the assumption that the two interest rates are identical.

Table 5 Results (OLS) for the RENT-Model, Specification RENT_B, with Different Magnitudes of a^0. Based on Stable Interest Rates over All Periods. N = 1044

	Annual rate of down-payment					
	$a^0 = 0.00$	$a^0 = 0.01$	$a^0 = 0.02$	$a^0 = 0.03$	$a^0 = 0.04$	$a^0 = 0.05$
PVRent	-.285*** (.024)	-.445*** (.038)	-.598*** (.052)	-.747*** (.066)	-.893*** (.079)	-1.037*** (.093)

Note: R-sq. = 0.689 for all alternatives. Robust standard errors below coefficients. * indicates statistical significance at 5 per cent level, ** at 1 per cent level, and *** at 0.1 per cent level.

7.3 A Test of the MUT-Model vs. the RENT-Model

The MUT- and the RENT-models are non-nested models. We use the J-test suggested by Davidson and MacKinnon (1981) for testing the maintained hypothesis that the MUT-model is the valid model against the alternative that the RENT-model is the valid model, and vice versa. In order to carry out these tests, we run a regression where the fitted value of the dependent variable in Specification RENT_B is included in addition to the independent variables of the MUT-model, with the impact-parameter α^{RENT} for the fitted value. If the MUT-model is the valid model, we expect $\hat{\alpha}^{RENT} = 0$. The estimation result is, however, $\hat{\alpha}^{RENT} = 0.255$, with a t-statistic $t = 4.40$, which implies that we cannot reject the hypothesis that the RENT-model is valid. Similarly, testing the maintained hypothesis that the RENT-model is the valid model against the alternative that the MUT-model is valid, we obtain $\hat{\alpha}^{MUT} = 0.860$, with $t = 18.69$. Hence, we cannot reject that the MUT-model is valid. In other words, based on a J-test, neither of these models can be rejected. In our case, this result is not very surprising - it simply reflects the fact that both models are built on sound economic reasoning. On the other hand, this does not mean that both models are equally satisfactory.

In order to further examine the relative merits of two rival models like the MUT- and the RENT-models, Davidson and MacKinnon (1981) suggest the use of a compound model that contains the whole set of independent variables in Specifications MUT_C and RENT_B. Although such a model may not be straightforward to *theoretically* rationalize under the assumptions that our models build on, it may nevertheless be instructive to examine it as part of an *empirical exercise*. Selected estimation results for the compound model are shown in Table 6, along with the results for the preferred specifications of the MUT- and the RENT-models. The fact that the adjusted R-sq. for the COMP-model is only marginally higher than that for MUT_C, and that the Akaike and Schwarz information criteria (AIC and BIC) are only marginally larger for MUT_C than for the COMP-model, indicate that very little is lost by excluding PVRent and PVSpace from the compound model, i.e. by using the MUT-model. On the other hand, in comparing the estimation results for COMP with RENT_B, we see a substantial drop in the R-sq. adjusted, and a distinct increase in the AIC and BIC. These results indicate that a substantial loss is incurred if mutual debt is excluded from the compound model. Hence, we conclude that the MUT-model in general performs best in explaining equity prices for co-ops. That is, the MUT-model outperforms the RENT-model when it comes to predicting transaction prices. The main goal of the present paper is, however, not to predict prices, but determine as exactly as possible, how future payment obligations are reflected in transaction prices. Our prior expectations are $\mu_{MUT}^{M} \approx -0.94$ and $\eta_{RENT}^{\hat{R}} = -0.91$. As seen from the results in Table 6, at this point, the MUT-model clearly outperforms not only the RENT-model, but also the compound model. Hence, we conclude that the MUT-model is the best model of those considered here.

Table 6 Selected Estimation Results (OLS) for the Compound Model and Specifications MUT_C and RENT_B. N=1044

	COMP	MUT_C	RENT_B
Mutual	-.7501(.0762)***	-.8716(.0562)***	-
PVRent	-.1904(.0790)*	-	-.7471(.0657)***
PVSpace	113(141)	-	408(156)*
Other variables	Yes	Yes	Yes
R-sq. adjusted	.7632	.7590	.6802
F	69.59	70.90	47.22
AIC	28980	28997	29293
BIC	29228	29234	29536

Note: Robust standard errors in parentheses. * indicates statistical significance at 5 per cent level, ** at 1 per cent level, and *** at 0.1 per cent level.

8. Concluding Remarks

We have developed two versions of a theoretical model to determine equilibrium equity prices for co-ops. We have demonstrated that the equilibrium price of a co-op may be alternatively expressed as a function of mutual debt - in the MUT- model - or as a function of discounted annual rent and discounted floor-space - in the RENT-model -, in both cases supplemented by a set of control variables that captures the impact of non-financial housing attributes. Both the MUT- and the RENT-models firmly build on the user cost approach for studying housing prices. In our theoretical examination, we have demonstrated that the MUT- and the RENT-approaches lead to the same results in two special cases, namely, if interest rates on mutual debt and other loans are equal, or if there is no down-payment of mutual debt. Both of these conditions are, taken separately, sufficient conditions for obtaining identical results in the two models. On the other hand, in general, it seems reasonable to assume that interest rates on mutual debt and private debt are different, and that mutual debt is paid down over time. The MUT- and the RENT-models will then give different results. In the Norwegian case, with a lower interest rate on mutual debt than private debt, and strictly positive installments of mutual debt, both versions of the model contain an interest discount effect, thus implying that mutual debt is not fully discounted into the prices in the MUT-model, and that debt-related differences in rents are not fully discounted into prices in the RENT-model.

From our theoretical models, we are able to establish a numerical hypothesis about the impact parameters of key interest. In confronting the estimation results with the hypothesis, we conclude that mutual debt seems to be efficiently mirrored in transaction prices for co-ops. The estimation results based on the RENT-approach indicate, however, that rent-differences in our sample are not perfectly discounted in transaction prices. Moreover, recall that the results in the RENT-model are critically dependent on correct prior information on the rate of the down-payment for mutual loans. Such prior information is hard to obtain. From these results, we also conclude that the two plaintiffs in the court case that introduced our research questions have valid arguments. In fact, at the end of the day, this is also realized by the defendants, who the day before the case was scheduled for the appeal-court, resigned and came to a non-court agreement with the plaintiffs.

We can also conclude that the estimation results for the MUT-model support the maintained hypothesis that price formation in the market for co-ops is efficient. Based on the RENT-model, however, it is in our case, not possible to draw a definite conclusion on whether the market for co-ops functions according to the efficient market hypothesis.

In this paper, we have used a Norwegian sample of co-ops. In this market, the interest rates on mutual debt are in general lower than on private debt. It would be desirable to extend the analysis to other countries where the ratio of interest rates on mutual debt to private debt is different. If different relative interest rates are reflected in different estimates of the impact parameters of mutual debt and rent, in the way that our theoretical model predicts, they will provide strong evidence that differences in financial arrangements are efficiently reflected in market prices for co-ops.

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Appendix A Proof of Proposition 1

To prove Part (b), we start from Eq. (4), where:

$$\mu_{MUT}^{M} = -1 + \left(r_p - r_M\right) \frac{\sum_{t=0}^{T} d_t \Pi_{\zeta=0}^{\zeta=(t-1)} \left(1 - a^{\zeta}\right)}{d_T + r_p \sum_{t=0}^{T} d_t}$$

$$= -1 + \left(1 - \frac{r_M}{r_p}\right) \frac{r_p \sum_{t=0}^{T} d_t \Pi_{\zeta=0}^{\zeta=(t-1)} \left(1 - a^{\zeta}\right) / \sum_{t=0}^{T} d_t}{\left(d_T / \sum_{t=0}^{T} d_t\right) + r_p}$$

Under Assumption 1, $\Pi_{\zeta=0}^{\zeta=(t-1)} \left(1 - a^{\zeta}\right) = \left(1 - a\right)^t$, which inserted into the above expression, gives:

$$\mu_{MUT}^{M} = -1 + \left(1 - \frac{r_M}{r_p}\right) \frac{r_p \sum_{t=0}^{T} \left(\left(1-a\right)/\left(1+r_P\right)\right)^t \Big/ \sum_{t=0}^{T} \left(1+r_P\right)^{-t}}{\left(1+r_P\right)^{-T} \Big/ \sum_{t=0}^{T} \left(1+r_P\right)^{-t} + r_p}$$

Since under Assumption 1, $T = \infty$, we now obtain:

$$\mu_{MUT}^{M} = -1 + \left(1 - \frac{r_M}{r_p}\right) \frac{r_p \left(1 - \left(\left(1-a\right)/\left(1+r_P\right)\right)\right)^{-1} \Big/ \left(\left(1+r_P\right)/r_p\right)}{\left(1+r_P\right)^{-T} \Big/ \left(\left(1+r_P\right)/r_p\right) + r_p}$$

$$\mu_{MUT}^{M} = -1 + \left(1 - \frac{r_M}{r_p}\right) \frac{r_p r_p \left(1 - \left(\left(1-a\right)/\left(1+r_P\right)\right)\right)^{-1} \Big/ \left(1+r_P\right)}{r_p \left(1+r_P\right)^{-T} \Big/ \left(1+r_P\right) + r_p}$$

$$\mu_{MUT}^{M} = -1 + \left(1 - \frac{r_M}{r_p}\right) \frac{r_p \Big/ \left(1 - \left(\left(1-a\right)/\left(1+r_P\right)\right)\right)\left(1+r_P\right)}{1 \Big/ \left(1+r_P\right)^T \left(1+r_P\right) + 1}$$

$$\mu_{MUT}^{M} = -1 + \left(1 - \frac{r_M}{r_p}\right) \frac{r_p \Big/ \left(\left(1+r_P\right) - \left(1-a\right)\right)}{1 \Big/ \left(1+r_P\right)^T \left(1+r_P\right) + 1} = -1 + \left(1 - \frac{r_M}{r_p}\right) \frac{r_p \Big/ \left(r_P + a\right)}{1 + 1 \Big/ \left(1+r_P\right)^{T+1}}$$

$$\lim_{T\to\infty} \mu_{MUT}^{M} = \lim_{T\to\infty} \left\{ -1 + \left(1 - \frac{r_M}{r_p}\right) \frac{r_p \Big/ \left(r_P + a\right)}{1 + 1 \Big/ \left(1+r_P\right)^{T+1}} \right\} = -1 + \left(1 - \frac{r_M}{r_p}\right) \lim_{T\to\infty} \left\{ \frac{r_p \Big/ \left(r_P + a\right)}{1 + 1 \Big/ \left(1+r_P\right)^{T+1}} \right\}$$

$$\lim_{T\to\infty} \mu_{MUT}^{M} = -1 + \left(1 - \frac{r_M}{r_p}\right) \left(\frac{r_p}{\left(r_P + a\right)}\right) \lim_{T\to\infty} \left\{ \frac{1}{1 + 1 \Big/ \left(1+r_P\right)^{T+1}} \right\} = -1 + \frac{\left(r_p - r_M\right)}{\left(r_P + a\right)}$$

Hence, under Assumption 1, $\mu_{MUT}^{M} = -1 + \left[\left(r_p - r_M\right) \Big/ \left(r_P + a\right)\right]$.

Part (c) of Proposition 1 is proven by setting $a = 0$ in the above expression. This yields:

$$\mu_{MUT}^{M} = -1 + \left[\left(r_p - r_M\right) \Big/ r_P\right] = -r_M / r_P \ .$$

Appendix B Variable Definitions

Table B1 Variable Definitions

Variable Name	Definition
Price	Transaction price of dwelling, measured in Norwegian crowns (NOK).
Mutual	Mutual debt that rests on the dwelling at the time when the transaction takes place, measured in NOK.
Interest	The difference between the interest rate on loans from private banks and the interest rate on loans from state housing banks.
MInterest	MInterest = Mutual x Interest.
MRemain	MRemain = Mutual x Remain, where Remain = (30-Age) if Age < 30, and Remain = 0 if Age $\geq$ 30.
Rent	Annual rent in NOK
PVRent	Rent divided by the sum of the interest rate for mutual debt and the amortization rate.
Space	Size of the dwelling measured in m^2 .
Space50	Space50 = (Space – 50) if Space > 50, otherwise 0.
PVSpace	Space divided by the sum of the interest rate on mutual debt and the amortization rate.
Age	Years since the building of the dwelling was constructed.
Age25	Age25 = (Age – 25) if Age > 25, otherwise zero.
Age Ω	Dummy variable equal to 1 if the dwelling is in age class Ω , otherwise 0. Age-classes are Ω = 10-19, 20-29, 30-39, 40-49, 50-59, or more than 60.
Noblock	Dummy variable equal to 1 if the dwelling is not in a block building, otherwise 0.
SNblock	Product of Space and Noblock.
Floor	The floor on which the dwelling is located.
Lift	Dummy variable equal to 1 if there is a lift in the building, otherwise 0.
Kitchen	Indicator variable equal to 2 if kitchen of modest quality (more than 10 years since new/renovated), 3 if kitchen of medium quality (5-10 years since new/renovated), 4 if kitchen of high quality (new or renovated less than 5 years ago). Researcher's inspection of pictures, etc. in sales prospects was also used to categorize the quality of kitchens.
Bath	Indicator variable equal to 0 if bathroom of modest quality (more than 10 years since new/renovated), 1 if bathroom of medium quality (5-10 years since new/renovated), 2 if bathroom of high quality (new or renovated less than 5 years ago). Researcher's inspection of pictures, etc. in sales prospects was also used in categorizing the quality of bathrooms.
Period	Dummy variables for month that the transaction took place.
Zip-codes	Dummy variables for zip-code where the dwelling is located.

Prior information on the age-coefficients has been obtained from Robertsen and Theisen (2011). They estimate $\beta_{MUT}^{Age} = -12253$, and $\beta_{MUT}^{Age24} = 10874$, both measured in Norwegian crowns (NOK), from a sample of housing transactions in 2004. The average price of the co-ops in their sample is 878 805 NOK. In our sample, the average price is 1 504 824 NOK. From this, we obtain the following coefficients to be used as prior information in our estimations:

$$\hat{\beta}^{AGE} = -12253*(1504824/878805) = -20981,$$
$$\hat{\beta}^{Age24} = 10874*(1504824/878805) = 18620.$$

These coefficients are used to modify the dependent variable. That is, the plain equity-price is replaced by $\hat{P}_i = P_i + \hat{\beta}^{AGE} * AGE + \beta^{AGE24} * \mathrm{I} * (AGE - 24)$, where I is an indicator variable that takes the magnitude of 1 if the age of the dwelling is more than 24, and 0 if the age is less. Next, $\hat{P}_i$ is regressed on all independent variables except for age.

Appendix C Additional Estimation Results

Table C1 Selected Estimation Results (OLS) for the Impact of Mutual Debt and Rent. N = 1044

	Prior Estimate of Coefficients for Two Age-Variables		
	10% Lower Than Main Alternative	**Main Alternative**	**10% Higher Than Main Alternative**
Mutual debt	-.846***	-.872***	-.897***
	(.055)	(.056)	(.057)
PVrent	-.727***	-.747***	-.767***
	(.065)	(.066)	(.067)

Note: Results for mutual debt obtained by estimating Specification MUT_C. Results for PV-rent obtained by estimating Specification RENT_B. Robust standard errors below coefficients. *** indicates statistical significance at 0.1 per cent level.