“Fire Sales” in Housing Market: Is the House-Search Process Similar to a Theme Park Visit?

Charles Ka Yui Leung *
Department of Economics and Finance, City University of Hong Kong, Kowloon Tong, Hong Kong; Phone: (852) 3442-9604; Email: kycleung@cityu.edu.hk

Jun Zhang
City University of Hong Kong

Three striking empirical regularities have been repeatedly reported: the positive correlation between housing prices and trading volume, and between housing price and time-on-the-market (TOM), and the existence of price dispersion. This short paper provides perhaps the first unifying framework which mimics these phenomena in a simple competitive search framework. In the equilibrium, sellers with heterogeneous waiting costs and buyers are endogenously segregated into different submarkets, each with distinct market tightness and prices. With endogenous search efforts, our model also reproduces the well-documented price-volume correlation. Directions for future research are also discussed.

Keywords

Housing market; Competitive search; Price dispersion; Trading volume; Time on the market

* Corresponding author
1. Introduction

Both casual observations and serious empirical research agree that the housing market is characterized by a strong decentralized pattern of exchange with severe search frictions. In sharp contrast to the predictions from traditional Walrasian settings, empirical "anomalies" such as price dispersion in the real estate market, nontrivial time-on-the-market (TOM) positively associated with housing prices, positive correlation between housing prices and trading volumes, etc., are repeatedly reported. This paper is among the first few efforts to develop a unifying competitive search framework with heterogeneous sellers to illustrate the behavior of the housing market reflected in the above empirical findings.

The modeling choice is indeed intuitive. The existence of price and rent dispersion naturally leads one to a search-theoretic setting (for instance, see Gabriel et al., 1992; Leung et al., 2006; Plazzi et al., 2008) to depict a decentralized pattern of exchange. Another necessary condition for price dispersion is the heterogeneity on the seller's and/or the buyer's side, which generates corresponding submarkets (Diamond, 1971). In most cases in reality, these submarkets are partially segregated since some of the sellers or buyers are free to flow between these submarkets. As a consequence, a competitive search framework may suit the issue better than traditional search-theoretic settings.

To simplify the exposition, we focus on one-side heterogeneity and assume that sellers are different in terms of their waiting costs. The assumption of heterogeneity in the waiting cost variables attempts to capture the differing financing costs, as well as the search efforts and costs among different house sellers. Some sellers are more pressed to sell the house since the financing costs of alternative funding recourses are high, for instance, when they are on the verge of bankruptcy. Other sellers may want to sell their houses as soon as possible since they will be relocated to another place, and the pecuniary and opportunity costs to deal with the housing selling procedure are considerably large compared with their gains/losses from selling the house. They are the "fire-sale" sellers in our model. Meanwhile, our model also contains other sellers who are willing to wait for better prices. On the buyer side, we include homogeneous waiting costs to reflect their lodging and search-related costs when they are looking for a suitable house to purchase.

There are various ways to deal with heterogeneity in housing search issues. For instance, one can consider identical and fully segregated submarkets to

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1 Adding other heterogeneities in the setting would not change our principal results, but will significantly complicate the algebra.

2 Needless to say, the model can be reformulated to have identical sellers and heterogeneous buyers. The principal results will not be changed.
investigate vacancy issues. As a result, the prices and TOMs are the same in all the submarkets.\textsuperscript{3} In contrast, we adopt a competitive search framework based on the seminal work in Moen (1997),\textsuperscript{4} to investigate heterogeneous and partially segregated submarkets, where buyers are free to enter either submarket. As a result, the price and the expected TOM in one submarket may differ from those in another submarket. Moreover, in cases with exogenous shocks, this framework allows us to illustrate how these submarkets interact with each other due to the flavor of the ex ante competitiveness embodied in this framework.

This is in contrast to earlier search-theoretic frameworks that explain price dispersion, such as Axell (1974), Butters (1977), Reinganum (1979), von zur Muehlen (1980), Burdett and Judd (1983), Diamond (1985), Rob (1985), Salop and Stiglitz (1985), Benabou (1988, 1992a,b, 1993), and Rauh (2001). They have focused on commodity markets with take-it-or-leave-it offers. In such markets, all the goods are alike, and sellers can easily adjust their inventory. As a consequence, TOM is not an important issue. In contrast, houses usually differ from each other in one way or another. Moreover, it takes a long time to build a house. Thus, inventory adjustment is much more difficult. Sellers have to sell what they have, and thus the tradeoff between the selling price and the speed of sales is crucial. Our continuous time framework also captures the fact that negotiation with buyers is more frequent than that in commodity markets.

In our model, sellers with higher waiting costs, i.e. those in the "fire-sale" situation, are willing to accept lower prices, which attract a larger number of potential buyers so that the house would be sold faster. As a consequence, the prices in the two submarkets would differ, and lower (higher) prices would be associated with shorter (longer) TOMs. These theoretical predictions are consistent with the empirical findings. For example, Merlo and Ortalo-Magné (2004) find that sellers post different prices to target various types of consumers, while the submarket with a higher listing price has a lower matching rate and a longer TOM. Leung, Leong and Wong (2006) find that price dispersion in the housing market is non-trivial. Moreover, the degree of price dispersion after controlling the traits of the house can be explained by the movements of the macroeconomic variables. In the context of commercial real estate, Plazzi, Torous and Valkanov (2008) also find significant rent dispersion. In this paper, the degree of dispersion is determined by the distribution of the seller's waiting costs, which is in practice affected by macroeconomic conditions. A positive correlation between TOM and transaction price is found in the work of Kang and Gardner (1989), Forgey et al. (1996), Leung, Leong and Chan (2002), and Anglin et al. (2003), among others.

\textsuperscript{3} For instance, see Wheaton (1990).

\textsuperscript{4} It is close to a directed search, or a directed matching framework that originated in Peters (1991) and Montgomery (1991). See also Becsi et al. (2005).
In addition, we can also demonstrate a positive relationship between housing prices and trading volume in the cases with shocks in the demand-side variables, such as the residential value of houses, and waiting costs and reservation values of buyers. This is in line with the traditional wisdom based on supply-demand analyses, as well as empirical findings in Fisher et al. (2003), and Leung, Lau and Leong (2002). Intuitively, in the cases with higher (lower) residential values, and lower (higher) reservation values, or lower (higher) waiting costs of buyers, the houses are relatively more (less) attractive. As a result, the housing price would rise (fall), and potential buyers would inflow to (outflow from) the town, which leads to a larger (smaller) transaction volume. In this regard, this paper provides an alternative search-theoretic explanation on the positive correlation between housing prices and trading volume, other than the down-payment explanation. More specifically, the down-payment effect model by Ortalo-Magne and Rady (2006) seems to capture the short-run dynamics while this paper focuses on the steady state relationship. It is consistent with the empirical finding of Leung, Lau and Leong (2002), which suggests that the short-run dynamics of the housing market is driven by a down-payment effect, where the longer-run relationship between housing price and trading volume is due to search friction.

The remainder of the paper is structured as follows: a baseline model with competitive search and heterogeneous waiting costs will be introduced in the next section. The results will be presented and discussed in order. In the concluding remarks, we will compare the competitive search model presented here with a theme park visit in intuitive ways. Future research directions will also be discussed.

2. A Baseline Model of Housing Price Dispersion

2.1. A Tale of Two Submarkets

This section outlines the formal model. The horizon of the model is infinite and time is continuous. There is a continuum of sellers who have different waiting costs, so that some of them are more pressed to sell the house than others. For simplicity, we focus on the case with only two types, impatient and patient sellers. The principal result, however, can be generalized to a more general setting. Furthermore, without loss of generality, the waiting cost for the impatient sellers, \( c^H \), is higher than that of patient ones, \( c^L \), \( c^H > c^L > 0 \). Let \( S^i \) be the population (or measure) for the sellers with a flow waiting cost of \( c^i \), \( i=H, L \). Similarly, we define \( B^H \) and \( B^L \) as the measures of buyers who focus on the two "submarkets", respectively. Notice that the "submarkets" need not

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be geographically separated. They simply represent the trading which involve type $i$ sellers, $i=H, L$. The impatient sellers, who are pressed to sell the house, may post this information on an advertisement, so that buyers can easily distinguish between the two types of sellers. For simplicity, we assume complete and perfect information between the agents. In the type-$i$ submarket, the buyer's waiting cost is $\kappa$.

In each submarket $i$, $i = H, L$, the number of successful matches in an infinitesimal period is governed by a random matching function, $M (B^i, S^i)$, which exhibits a constant return to scale in $B$ and $S$, with positive, but diminishing marginal returns in each argument. We define the market tightness $\theta^i = B^i / S^i$ in the sense that it is more difficult for a buyer to find a seller in a tighter market. For each submarket $i$, we can define $\eta^i$ as the flow matching rate for a buyer to find a seller in submarket $i$ such that

$$\eta^i = \frac{M (B^i, S^i)}{B^i} = M \left( 1, \frac{1}{\theta^i} \right)$$

(1)

Similarly, the flow matching rate for a seller to find a buyer, $\mu^i$, satisfies

$$\mu^i = \frac{M (B^i, S^i)}{S^i} = M \left( \theta^i, 1 \right) = \theta^i \eta^i$$

(2)

We can denote $M_j(.)$ as the first derivative of the matching function $M$ with respect to the $j$-th argument. Note that the assumed feature of the matching function suggests that $M_j(\theta, 1) = M_j(1, 1/\theta)$ since the first derivative of a constant-return-to-scale function must be homogeneous of degree zero. As a result, we use these two expressions interchangeably.

### 2.2. Housing Prices and Bellman Equations

In this model, sellers with different waiting costs could post different prices to differentiate each other. The actual price $P^i$ in the submarket $i$ would be determined by a Nash bargaining solution, which will be discussed in the next subsection. Let $\Pi^i$ denote the value for type-$i$ sellers, $V^i$ the value for type-$i$ buyers (who are still searching the market, but have not owned houses), for $i=H, L$, and $\Omega$ the value of a house owner, which is independent of the waiting cost level. Since the buyers are free to enter each of the two submarkets, the values of all types of buyers, $V^i$, are also the same as the reservation values (i.e. the value for outside options), $\bar{V}$, i.e. $V^i = \bar{V}$, for $i=H, L$. For simplicity, we assume that both $\bar{V}$ and $\Omega$ are exogenously determined.

As standard in the literature, we assume that buyers and sellers maximize the expected value of the sum of the periodic utility flow, which is constantly

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6 Needless to say, we can also define the market tightness from a seller perspective. The results in this paper will not change under this alternative definition of tightness.
discounted by the rate \( r \). Given the model structure outlined above, the dynamic optimization of the buyers and sellers can be summarized by the following Bellman equations, \( i=H, L \),

\[
\begin{align*}
    r\Pi^i &= -c^i + \mu^i \left( P^i - \Pi^i \right) \quad (3) \\
    rV^i &= -\kappa + \eta^i \left( \Omega - P^i - V^i \right) \quad (4)
\end{align*}
\]

From Equation (3), we can obtain:

\[
    \Pi^i = \frac{-c^i + \mu^i P^i}{r + \mu^i} = \frac{-c^i / \mu^i + P^i}{r / \mu^i + 1}, \quad (5)
\]

\( i=H, L \). Note that \( 1/\mu^i \) is actually the mean waiting time for the buyers. Hence, Equation (5) means that the value of the seller equals the discounted net gain from selling a house, while the net gain is the price net of the waiting cost during the waiting period.

Similarly, Equation (4) yields:

\[
    V^i = \frac{-\kappa + \eta^i \left( \Omega - P^i \right)}{r + \eta^i} \quad (6)
\]

\( i=H, L \). The intuition of (6) is analogous to that of Equation (5).

### 2.3. The Bargaining Process

The housing price is determined by a Nash bargaining solution with the bargaining power of the seller as \( \alpha \). This means that the seller and buyer will solve the following joint surplus maximization problem:

\[
    \max_{\mu^i} \left\{ \left( P^i - \Pi^i \right) \left( \Omega - P^i - V^i \right)^{-\alpha} \right\} \quad i=H, L.
\]

The solution is:

\[
    P^i = \left( 1 - \alpha \right) \Pi^i + \alpha \left( \Omega - V^i \right) \quad (7)
\]

Equation (7) says that the price is the weighted average of two objects: one is the value of the seller, \( \Pi^i \), and the other is the net gain from being a buyer (or a house-searcher) to a house owner between \( \left( \Omega - V^i \right) \). Thus, to solve for the house price, it is necessary to solve for the equilibrium values of the seller and buyer. From Equations (5), (6), and (7), we can solve \( \Pi^i, V^i \) and \( P^i, i=H, L \).

\[
\begin{align*}
    \Pi^i &= \frac{\alpha \mu^i \left( r \Omega + c^i + \kappa \right)}{r[\alpha \mu^i + (1 - \alpha) \eta^i]} - \frac{c^i}{r} \quad (8) \\
    V^i &= \frac{(1 - \alpha) \eta^i \left( r \Omega + c^i + \kappa \right)}{r[\alpha \mu^i + (1 - \alpha) \eta^i]} - \frac{\kappa}{r} \quad (9)
\end{align*}
\]
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\[ P^i = \frac{\alpha (r + \mu^i) (r \Omega + c^i + \kappa)}{r [r + \alpha \mu^i + (1 - \alpha) \eta^i]} - \frac{c^i}{r} \]  

(10)

2.4. Buyer’s Free Entry and the Price-TOM Relation

Since buyers are free to enter either submarket, the buyer’s values should be the same as the reservation values for outside options. Hence,

\[ V^H = V^L = \overline{V}. \]  

(11)

Note that the right-hand side of Equation (9) is increasing at \( c^i \), and decreasing at \( \theta^i \), \( i=H, L \). While \( c^H > c^L \) and (11), we have:

\[ \theta^H > \theta^L, \mu^H > \mu^L, \text{ and } \eta^H < \eta^L. \]  

(12)

Meanwhile, from Equation (6), we can show that, \( i=h,L \),

\[ P^i = \Omega - \overline{V} - \frac{r \overline{V} + \kappa}{\eta^i}, \]

which suggests that the housing price is decreasing at \( \theta^i \). Thus, the transaction prices of the two submarkets are indeed different, with the expected result that the submarket of impatient sellers (higher waiting cost) would sell at a lower price,

\[ P^H > P^L. \]  

(13)

Proposition 1 (Price-TOM Relation): In a competitive search framework with heterogeneous waiting costs for sellers and free entry for buyers, the submarket with a higher (lower) price must have a longer (shorter) expected TOM.

This result is intuitive. In cases where sellers are eager to sell the house, house price must be lower than usual. In observing a possibly low price, more buyers would crowd into that market segment, which leads to a higher probability of matching, and a shorter TOM. Empirically, Merlo and Ortalo-Magné (2004) find that sellers post different prices and the submarket with a higher listing price has a lower matching rate and a longer TOM. The empirical evidence in Kang and Gardner (1989), Forgey et al. (1996), Leung, Leong and Chan (2002), Anglin et al. (2003), among others, support Proposition 1.

2.5. Price Dispersion

The following proposition simply repeats (13).
Proposition 2 (Price Dispersion): In the current competitive search framework, housing prices would be different even for identical houses. Specifically, the seller with higher waiting costs would ask for a lower housing price, in an effort to reduce the waiting costs by selling the house faster.

The above results are in line with the empirical findings. Leung et al. (2006) find that price dispersion cannot be only attributed to randomness or econometric mis-specification as the degree of price dispersion systematically varies with some macroeconomic variables. In addition, Plazzi, Torous and Valkanov (2008) also find empirical rent dispersion in the commercial real estate market.

2.6. Comparative Statics and Price-Volume Correlation

On top of the two major propositions, we can also derive the comparative-statics results, which are summarized by the following table:

Table 1 Comparative Statics for the Model with a Costly Search Effort

<table>
<thead>
<tr>
<th>( i = H, L )</th>
<th>( \delta )</th>
<th>( \kappa )</th>
<th>( \Omega )</th>
<th>( \alpha )</th>
<th>( \bar{V} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta^i )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \mu^i )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \eta^i )</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( \rho^i )</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( \Pi^i )</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( M (B^i, S^i) )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Price-Volume Co-movement</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Note that the trading volume for a given period is proportional to the matching rate, \( M (B^i, S^i) \), and therefore, we can deduce the price-volume co-movements from the table above. While the intuitions are in fact straightforward, it may be instructive to present the explanations in a more systematic manner.

1. When sellers are more eager to sell their houses, in the case that their waiting costs \( (\delta^i) \) are higher, house prices would be lower. As a result, more buyers would be attracted to the economical supply of housing, thus leading to a higher buyer-seller ratio (market tightness), higher selling rate and lower buying rate. In this case, a lower price would be associated with a higher trading volume.

2. On the other hand, given the outside values of the buyer (for instance, potential buyers may prefer to have leisure time instead of involvement in house-searching), a higher level of buyer's waiting cost \( (\kappa) \) may discourage them from entering the market at all, which results in a lower market
tightness (i.e. a lower buyer-seller ratio). House price would be lower, as compensation for the greater difficulties in house-searching, so that the "returns" from house searching will match the alternatives. Thus, we would observe a lower price and a smaller trading volume (and hence a positive price-trading volume correlation).

3. When the benefits from owning a house and living in the economy ($\Omega$) are greater, more buyers would be attracted by the economic potentials, and the market tightness is higher. With more demand, the house price is driven higher. Thus, a higher price is associated with higher trading volume (and hence a positive price-trading volume correlation).

4. If the seller's bargaining power ($\alpha$) is relatively larger, then the house price is higher. Meanwhile, some buyers would leave the market, which leads to lower market tightness and a lower trading volume.

5. While the buyer's entry value ($\bar{V}$) is higher, some buyers will not participate in house searching activities, which results in a lower market tightness, and thus a lower price (and hence a positive price-trading volume correlation).

Some economists insist on framing the house market as if it has a downward sloping demand curve and an upward sloping supply curve. We can then rephrase the result in the price-volume co-movements in Proposition 3:

**Proposition 3 (Comparative Statics and Price-Volume Correlation):** In the baseline model with a fixed entry value for the buyers and fixed number of sellers, housing prices and the trading volumes would move in the same direction as if the system has been hit by a demand shock, such as changes in the buyer's waiting costs, the values for owning a house, or the buyer's entry values. On the other hand, housing prices and trading volumes would move in the opposite directions as if the system is hit by a supply shock, such as changes in the seller's waiting costs, or the seller's bargaining power.

Empirically, Fisher et al. (2003), and Leung, Lau and Leong (2002), among others, find strong contemporary co-movements between housing prices and trading volumes, while Leung, Lau and Leong (2002) also find that price would lead the trading volume by 24-48 months in the monthly data.

### 3. Conclusion

To a certain extent, the idea behind this paper is analogous to a theme park visit. In the popular theme parks, visitors do not know ex ante whether it will be very crowded or not; if they would need to wait in a long queue before they can enjoy some of the rides or the haunted house. They can choose to buy a more expensive "VIP-pass" and save some waiting time, or buy the cheaper
normal pass and hope that they will not need to wait that long ex post. Normally, people with a higher "waiting cost" such as tourists would prefer the more expensive options. For others, they will prefer to wait. Thus, time-on-the-queue (TOQ) will be negatively related to the price of the "pass".

Similarly, some sellers in this paper have higher waiting costs than others, and prefer to sell their houses in a submarket with higher "liquidity." Unlike the theme park visit, however, the supply side of the housing market is endogenous. The buyers will take the strategies of the sellers as given and then self-select into different submarkets. Moreover, while pass-purchasing is certain, house-purchasing is not. Even within each submarket, there is a random matching process among potential buyers and sellers. Moreover, while the price of the pass is given, the housing price in each submarket will be determined through a Nash bargaining process, which will in turn, depend on the market-tightness of the corresponding submarket.

Perhaps more importantly, this paper differs from the theme park visit example in that there are three stylized facts for this paper to mimic, namely, the existence of price dispersion, positive correlation between the market price and trading volume, and that between the transaction price and TOM. The empirical "anomalies" found against the Walrasian predictions can be explained within our competitive search framework. The free-entry assumption implies a positive correlation between housing prices and TOM. With the introduction of costly search efforts, buyers with higher waiting costs are more eager to purchase a house, and hence put forth more search efforts. The increase in search intensity would lead to higher trading volumes. It also holds in the case with a positive shock in the waiting costs. In addition, we show that price dispersion can easily exist even with perfect information and perfect competition in the ex ante sense, as long as the trades are decentralized.

The current model, of course, can be further improved. For instance, in this model, both the reservation values of a house buyer (or house searcher) and the values of a house owner are exogenously determined. Future work should endogenize these values in a more general model. The model also implicitly assumes that there is some "commitment" mechanism on the seller's side. Recall that in the model, sellers with different waiting costs self-select into different "sub-markets," say, by advertisements. We also assume that once a match between a potential seller and potential buyer is made, the price will be determined by Nash bargaining. In that case, sellers with a higher waiting cost ("Fire Sale") would sell at a lower price. This attracts more potential buyers to that sub-market. Hence, from the seller's point of view, the "Fire Sale sub-market" will have a higher matching rate. However, patient sellers (sellers with lower waiting costs) may find it profitable to enter that sub-market, and pretend that it is a Fire Sale. Thus, this model demands sellers to truthfully
reveal their situations and commit to “stay” in that sub-market. Future research should relax such an assumption.\footnote{Among others, see Tse and Leung (2011) for some earlier efforts on this issue.}

Future work can also be extended in other directions. For instance, "middlemen" are missing in this analysis. Previous partial equilibrium analyses (such as Yavas, 1994, 1995) show that the introduction of an intermediary may affect the equilibrium configuration, and efficiency under some conditions. Second, Zenou (2009) has studied the location of various types of workers in a search theoretic framework. It will be interesting to extend the analysis here to a model with both a house market and a labor market search.\footnote{See Coulson and Fisher (2007) for some related attempts along this line.}

Third, the search friction in the housing market may influence the asset portfolio, as illustrated by Anglin and Gao (2010). It would be interesting to explore the general equilibrium implications for such a consideration. Moreover, while this paper focuses on heterogeneity in waiting costs, future research may explore the situation jointly with financial constraints and search frictions. Another possibility for future research is to merge the current housing market model with a conventional neoclassical framework, as in Lagos and Wright (2005). These directions are indeed being pursued.

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Appendix

Proof for the Comparative Statics

From (8) to (11), we know that the equilibrium is determined by the following equations, $i=H, L$,

$$V^H = V^L = \bar{V} = \frac{(1-\alpha)\eta^i(r\Omega + c^i + \kappa)}{r + \alpha\mu^i + (1-\alpha)\eta^i} - \frac{\kappa}{r}$$

$$p^i = \frac{\alpha (r + \mu^i)(r\Omega + c^i + \kappa)}{r + \alpha\mu^i + (1-\alpha)\eta^i} - \frac{c^i}{r}$$

$$\Pi^i = \frac{\alpha\mu^i (r\Omega + c^i + \kappa)}{r + \alpha\mu^i + (1-\alpha)\eta^i} - \frac{c^i}{r}$$

Notice that there is no direct dependence of market $i$ variables (i.e. $P^i, \Pi^i$) on the market $j$ variables (i.e. $P^j, \Pi^j$), $i,j=H, L$, and $i \neq j$. In this sense, the two sub-markets are “segmented.” Thus, we can first solve the market tightness, and then the values and prices, and worry less on the cross-market effects. Consequently, the effects from exogenous variables, including $c^H, c^L, \kappa, \Omega, \alpha$ can be figured out.

Observe that the buyer’s value is increasing in $c^i$, and decreasing in $\theta^i$, $i=H, L$. For instance, when $c^H$ increases, $\theta^H$ would be larger, but $P^H$ and $\Pi^H$ would decline. Since

$$P^i = \Omega - \bar{V} - \frac{r\bar{V} + \kappa}{\eta^i}$$

and

$$\Pi^i = \frac{\alpha\mu^i (r\Omega + c^i + \kappa)}{r + \alpha\mu^i + (1-\alpha)\eta^i} - \frac{c^i}{r}$$

$$= \frac{\alpha (r + \mu^i)(r\Omega + c^i + \kappa)}{r + \alpha\mu^i + (1-\alpha)\eta^i} - \frac{c^i}{r}$$

$$= \Omega - \bar{V} - \frac{r\bar{V} + \kappa}{\eta^i} - \frac{1}{(1-\alpha)\eta^i} (1-\alpha)\eta^i (r\Omega + c^i + \kappa)$$

$$= \Omega - \bar{V} - \frac{r\bar{V} + \kappa}{\eta^i} - \frac{\alpha (\kappa + r\bar{V})}{(1-\alpha)\eta^i}$$

$$= \Omega - \bar{V} - \frac{r\bar{V} + \kappa}{(1-\alpha)\eta^i}$$
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Recall that both \( V \) and \( \Omega \) are exogenously determined. Thus, a change in \( c_i \), will only go through a change in \( \eta_i \), \( i=H, L \). The variables in the other sub-market, \( \theta^H \), \( P^L \), \( \Pi^L \) would not change. In this case, a lower price would drive higher trading volume. Similarly, when \( c_L \) increases, \( \theta_L \) would increase, but \( P_L \), \( \Pi_L \) would be lower. The variables in the other submarket, \( \theta^H \), \( P^H \), \( \Pi^H \), would not change, and we also have higher trading volume accompanied with lower prices. This is in line with responses to supply shocks.

Note that the buyer’s value is decreasing in both \( \kappa \) and \( \theta_i \), \( i=H, L \). Thus, if the buyer’s waiting cost, \( \kappa \), is larger, \( \theta_i \) would be smaller. Note that

\[
V = \frac{(1-\alpha)\eta^i (r \Omega + c^i + \kappa)}{r [r + \alpha \mu^i + (1-\alpha)\eta^i]} - \frac{\kappa}{r}
\]

\[
= \frac{(1-\alpha)\eta^i (r \Omega + c^i)}{r [r + \alpha \mu^i + (1-\alpha)\eta^i]} - \frac{\kappa (r + \alpha \mu^i)}{r [r + \alpha \mu^i + (1-\alpha)\eta^i]}
\]

Notice that the first term in the expression above is directly independent of \( \kappa \). As \( r + \alpha \mu^i \), \( r + \alpha \mu^i + (1-\alpha)\eta^i \) are all positive, \( V \) is decreasing in \( \kappa \).

Furthermore,

\[
\frac{rV + \kappa}{(1-\alpha)\eta^i} = \frac{r \Omega + c^i + \kappa}{r + \alpha \mu^i + (1-\alpha)\eta^i}
\]

\[
= \frac{r \Omega + c^i}{r + \alpha \mu^i + (1-\alpha)\eta^i} + \frac{r}{r + \alpha \mu^i} \frac{(1-\alpha)\eta^i (r \Omega + c^i)}{r [r + \alpha \mu^i + (1-\alpha)\eta^i]} - \frac{r}{r + \alpha \mu^i} V
\]

Thus, we can re-write

\[
P^i = \Omega - V - \frac{rV + \kappa}{\eta^i} = \Omega - V - (1-\alpha) \frac{r(\Omega - V) + c^i}{r + \alpha \mu^i}
\]

\[
P'^i = \Omega - V - \frac{rV + \kappa}{(1-\alpha)\eta^i} = \Omega - V - \frac{r(\Omega - V) + c^i}{r + \alpha \mu^i}
\]

Thus, both \( P^i \) and \( \Pi^i \) are lower due to a smaller \( \theta^i \), \( i=H, L \), in the case of a higher value of \( \kappa \).
Now, let us consider the effects from a higher $\Omega$. Note that the buyer’s value is increasing in $\Omega$ and decreasing in $\theta_i, i=H, L$. Hence, a higher $\Omega$ would attract more immigrants, and raise $\theta_i, i=H, L$. Since

$$\frac{r\bar{V} + \kappa}{(1-\alpha)\eta} = \frac{r\Omega + c^i + \kappa}{r + \alpha\mu^i + (1-\alpha)\eta},$$

we have

$$P^i = \Omega - \bar{V} - \frac{r\bar{V} + \kappa}{\eta} = \Omega - \bar{V} - \frac{(1-\alpha)\left(\frac{\Omega - \bar{V}}{\eta} + c^i\right)}{r + \alpha\mu^i}$$

$$= \left(\Omega - \bar{V}\right)\left(\frac{r + \alpha\mu^i}{r + \alpha\mu^i} - \frac{(1-\alpha)c^i}{r + \alpha\mu^i}\right).$$

Since

$$\bar{V} = \frac{(1-\alpha)\eta\left(r\Omega + c^i\right)}{r[r + \alpha\mu^i + (1-\alpha)\eta^i]} - \frac{\kappa}{r} \frac{(r + \alpha\mu^i)}{r + \alpha\mu^i + (1-\alpha)\eta^i},$$

Therefore,

$$\left(\Omega - \bar{V}\right) = \left(\Omega\right)\left[\frac{r + \alpha\mu^i}{r + \alpha\mu^i + (1-\alpha)\eta^i}\right] + \frac{\kappa}{r} \frac{(r + \alpha\mu^i) - (1-\alpha)c^i}{r[r + \alpha\mu^i + (1-\alpha)\eta^i]}. $$

Thus, $\left(\Omega - \bar{V}\right)$ is increasing in $\Omega$, and hence, so is $P^i, i=H, L$.

An alternative way to see the positive correlation between $\Omega$, and $P^i, i=H, L$ is to notice that

$$P^i = \alpha\left(\frac{r_{\Omega} + \mu^i}{1-\alpha}\right) - \frac{c^i}{r}.$$

Since $\mu^i = \theta_i \eta^i$, an increase in $\theta^i$ (due to an increase in $\Omega$) would lead to an increase in $P^i, i=H, L$.

Similarly,

$$\Pi^i = \alpha\left(\frac{\mu^i}{1-\alpha}\right) - \frac{c^i}{r}.$$

Therefore, an increase in $\theta^i$ (due to an increase in $\Omega$) would lead to an increase in $\Pi^i, i=H, L$. 
Now, we want to investigate the implication of a change in the bargaining power $\alpha$. Observe that buyer’s value is decreasing in both $\alpha$ and $\theta^i$. So the market tightness would be smaller when the seller’s bargaining power is larger. Note also that

$$P^i = \Omega - \overline{V} - \frac{r\overline{V} + \kappa}{\eta^i}$$

$$\Pi^i = \Omega - \overline{V} + (r\overline{V} + \kappa) \frac{1}{(1-\alpha)} \left( \frac{-1}{\eta^i} \right).$$

When $\alpha$ increases, $\eta^i$ will also increase, and hence the house price $P^i$ will increase as well. At the same time, both $1/(1-\alpha)$ and $-1/\eta^i$ increase with $\alpha$. Thus $\Pi^i$ increases, $i=H, L$.

If the buyer’s entry value ($\overline{V}$) is higher, market tightness must be lower. Both house price and seller’s values would decline, since both $P^i$ and $\Pi^i$ are decreasing in $\overline{V}$.

$$P^i = \Omega - \overline{V} - (1-\alpha) \frac{r(\Omega - \overline{V}) + c^i}{r + \alpha \mu^i}$$

$$= \Omega - \frac{(r + \alpha \mu^i)\overline{V} + (1-\alpha)(r\Omega + c^i - r\overline{V})}{r + \alpha \mu^i}$$

$$= \Omega - \frac{(r + \mu^i)\overline{V} + (1-\alpha)(r\Omega + c^i)}{r + \alpha \mu^i}$$

and

$$\Pi^i = \Omega - \overline{V} - \frac{r(\Omega - \overline{V}) + c^i}{r + \alpha \mu^i} = \Omega - \frac{r\Omega + \alpha \mu^i \overline{V} + c^i}{r + \alpha \mu^i}$$

Q.E.D.