Valuing Vulnerable Mortgage Insurance Contracts

With Capital Forbearance

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Abstract

This study extends Kau and Keenan (1995, 1996) by setting up a contingent-claim framework incorporating default risk of mortgage insurer and capital forbearance of regulators to derive a closed-form solution for mortgage insurance contracts. The research herein further investigates how critical policy parameters, such as capital requirements, prompt closure, and other policy-instrument parameters, relate to the value of mortgage insurance by evaluating the partial derivatives of the closed-form expression. The numerical analysis indicates that the default risk premium can be substantial in the presence of a catastrophic risk in the housing price. Moreover, forbearance threshold and time-to-delay effects could be more significant when the initial asset-liability ratio increases.

Key Words: Mortgage insurance, Default risk, Capital forbearance, Capital standards.
1. Introduction

Mortgage insurance (MI) plays an important role in the functioning of housing finance markets, because it transfers the risk exposures on borrowers’ defaults from lenders to mortgage insurers and facilitates the creation of secondary mortgage markets.¹ Private mortgage insurance, known as mortgage guaranty insurance, guarantees that, in the event of a default, a mortgage insurer will pay the mortgage lender for any loss resulting from a property foreclosure up to the 20% to 35% of the claim amount.² Mortgage insurers operate under state insurance laws and most states regulate the industry in three special ways: (1) A contingency reserve equal to one-half of all premiums received must be maintained for 10 years; (2) A 4% capital ratio applies to risk in force; (3) A monoline requirement forces a firm to write only mortgage insurance.

The U.S. mortgage guaranty industry is dominated by six insurance groups: MGIC Investment Corporation, Radian Group, Genworth Financial, PMI Group, American International Group and Old Republic International Corporation. Subsidiaries of them wrote 93% of the $4.4 billion of premiums in 2010. Due to the subprime mortgage crisis, the rising foreclosure rates of borrowers has resulted in capital scarcity for many mortgage insurers. These same companies also recorded $1.7 billion or 71% of the combined $2.4 billion losses in 2010. Two substantial groups, PMI and Old Republic, wrote 24.6% of 2010 earned premiums but were forced to stop writing new policies due to insufficient capital at the end of the third quarter of 2011. Two PMI subsidiaries were placed into receivership by the state insurance regulator. One, PMI Mortgage Insurance Company, recorded 11.6% of the total mortgage premiums earned in 2010. Huge losses,

¹ Statutory rules with GSEs require any loan with less than 20% down payment to purchase this coverage.
² See Canner and Passmore (1994) and MI Guide.
$2.4 billion in 2010, threaten to destroy the MI business. The concern is that the losses will continue to grow and, with limited growth in real estate sales requiring MI, there will be additional withdrawals from the market and or potential failures. As a consequence, default probabilities of mortgage insurers have become a critical factor in valuing MI contracts.

In response to the capital shortfalls of mortgage insurers, some forms of capital forbearance have been initiated at federal level. For instance, Freddie Mac announced that private mortgage insurers do not need to meet the increasing capital requirements when the credit ratings of mortgage insurers have been downgraded below AA. On March 19, 2008, the regulator of Fannie Mae and Freddie Mae, the Office of Federal Housing Enterprise Oversight (OFHEO), agreed to reduce the existing 30% OFHEO-directed capital requirement to 20%. The OFHEO estimates that this reduction should provide up to $200 billion of immediate liquidity to the mortgage-backed securities market. The Housing and Economic Recovery Act of 2008 also helped Fannie Mae and Freddie Mae by injecting capital into these two large U.S. suppliers of mortgage funding.³

New legislations also passed at most states to allow regulators to exercise more discretion in permitting mortgage insurers to continue with new business despite capital shortfalls below required levels. These actions also help mortgage insurers continue to write new insurance on a nationwide basis through their holding companies.

When mortgage insurers fail to meet the minimum capital requirements but are permitted to continue their operations, capital forbearance occurs. Capital forbearance has

³ The emergency loan of $85 billion to AIG in 2008 also helped its mortgage insurance unit, the fifth largest private mortgage insurer based on 2008 sales.
been well-documented in the banking literature as a major determinant of deposit insurance cost. As such, this study incorporates default risk of mortgage insurers and capital forbearance, which have not been considered in previous studies, into the pricing model of MI in order to investigate how critical policy parameters, such as capital requirements, prompt closure, and other policy-instrument parameters, affect the value of MI.

A volume of previous studies on MI contracts, such as Schwartz and Torous (1992), Dennis et al. (1997), Bardhan et al. (2006), and Chen et al. (2010), model the unconditional probability of default exogenously with a constant interest rate. Separately, Kau et al. (1992, 1993, 1995) and Kau and Keenan (1995, 1996) improve the literature by using a structural approach with two state variables - interest rate and housing price - to be more realistic about the market environment and to model defaults endogenously as a put option. This study herein extends Kau and Keenan (1995, 1996) by setting up an option-pricing framework to value MI contracts with the considerations of insurer’s default risk and capital forbearance. Our MI valuation problem is further complicated by a number of practical considerations. The liabilities facing these mortgage insurers are mainly the premiums of MI contracts, which are interest rate and housing price sensitive. This fact makes the considerations of interest rate and housing price risks particularly important in modeling an insurer’s liability. This paper uses the square-root process of Cox et al. (1985) to describe the stochastic process of the interest rate and adopts a jump-diffusion model to describe housing price dynamics so as to reflect the catastrophic events of natural and financial disasters (such as the subprime mortgage crisis).

Our model and numerical analyses show that the intensity and severity of

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4 See Kane (1986), Nagarajan and Sealey (1995), etc.
catastrophic events upon housing prices have a significant impact on the default risk premiums of MI contracts. The higher the mortgage insurer’s initial asset-liability structure is, the lower the default risk of the mortgage insurer and the higher the value of the MI will be. Not surprisingly, a mortgage insurer’s interest rate risk management practice will affect the value of MI. We also demonstrate that a lower forbearance threshold increases the impact of delay on MI, and that a longer time delay amplifies the impact of forbearance threshold on MI.

The next section presents the model of interest rate, housing price, liability, and asset dynamics of a mortgage insurer. Section 3 specifies the payoffs of MI for various scenarios and derives the closed-form formula of MI contracts. Section 4 provides the numerical analyses and discussion of the results. The last section draws conclusions about our findings and their implications.

2. A model for the mortgage insurer

This study adopts a structural approach to calculate the default probability of mortgage insurers that issue MI contracts. Structural models provide a link between a firm’s credit quality and its economic and financial conditions, such as its assets and capital structure. Hence, defaults are endogenously generated within the model. Our structural model specifically follows from Cummins (1988), Duan et al. (1995), Duan and Yu (2005), and Lee and Yu (2002, 2007) by encompassing the dynamics of interest rates and liabilities. This structural model allows a mortgage insurer suffering from the default risk of borrowers to bear a higher default risk of its own.

Since the mortgage insurer’s asset-liability structure, interest rate, and housing price
specifications are important factors in determining the values of MI, this section begins by specifying the interest rate process, the borrower’s housing price process, and the mortgage insurer’s assets and liability dynamics. We then show their corresponding processes under the risk-neutralized pricing measure.

2.1 The mortgage insurer’s liability process

The literature typically models the liability process by a lognormal diffusion process, such as Cummins (1988). This modeling fails to explicitly take into account the impact of stochastic interest rates and housing prices. This shortcoming is particularly important for modeling a mortgage insurer’s liability value, because falling house prices and rising interest rates are the precipitating factors for the catastrophic nature of MI. Following Duan et al. (1995) and Lee and Yu (2007), we model the mortgage insurer’s total liability value as consisting of two other risk components: interest rate and housing price risks, which are captured by the Wiener components of the interest rate and housing price processes. Hence, the liability value of the mortgage insurer is governed by the following process:

\[
\frac{dL(t)}{L(t)} = \mu_L dt + \phi_{L,r} dr(t) + \phi_{L,H} \frac{dH(t)}{H(t)} + \sigma_L dW_{L,t}
\]  

(1)

where \( \mu_L \) is the instantaneous drift of the liability value at time \( t \); \( W_{L,t} \) is the Wiener process that pertains to idiosyncratic shocks to the capital market; \( \phi_{L,r} \) denotes the instantaneous interest rate elasticity of the mortgage insurer’s liability, and \( \phi_{L,H} \) represents housing price elasticity of the mortgage insurer’s liability. Moreover,

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5 One may refer to Duan et al. (1995) and Lee and Yu (2007) for a similar detailed derivation.
\[ \sigma_L = \sigma_L^* \Psi, \quad \Psi = \sqrt{1 - \rho_{Lr}^2 - \rho_{LH}^2}, \] where \( \sigma_L^* \) is the volatility of small shocks, and \( \rho_{Lr} \) (\( \rho_{LH} \)) represents the correlation coefficient of the interest rate (housing price) and the liability value. From Equation (1), the total liability risk can be expressed as \[ \sqrt{\phi_{Lr}^2 \nu^2 + \phi_{LH}^2 \sigma_H^2 + \Psi^2}. \] In other words, the total instantaneous variance of a mortgage insurer’s liabilities can be decomposed into three components: interest rate risk, housing price risk, and credit risk.

### 2.2 The instantaneous interest rate process

The instantaneous interest rate is assumed to follow the square-root process of Cox et al. (1985). This setting avoids the negative interest rate that appears in Vasicek’s model (Vasicek, 1977). Thus, the instantaneous interest rate process can be written as follows:

\[ dr(t) = \eta(\theta - r(t))dt + \nu \sqrt{r_t} dZ_{r,t}, \]

where \( \eta \) denotes the mean-reverting force measurement; \( \theta \) presents the long-run mean of the interest rate; and \( \nu \) is the volatility parameter for the interest rate.

It is standard to use a risk-neutral pricing technique to price a derivative contract such as MI contracts. According to Girsanov’s theorem:

\[ \eta^* = \eta + \lambda_r, \quad \theta^* = \frac{\eta \theta}{\eta + \lambda_r}, \quad dZ_{r,t}^Q = dZ_{r,t} + \frac{\lambda_r \sqrt{r(t)}}{\nu} dt, \]

where term \( \lambda_r \) is the market price of interest rate risk and is a constant under the assumption of Cox et al. (1985). Moreover, \( Z_{r,t}^Q \) is a Wiener process under \( Q \). The dynamics of the interest rate process under the risk-neutralized pricing measure \( Q \) can now be described as:
\[ dr(t) = \eta^* (\theta - r(t)) dt + v \sqrt{r(t)} \, dZ^0_{r,t} . \]  

(3)

2.3 The housing price process

We follow Kau and Keenan (1996) and describe the housing price dynamics for the borrower at time \( t \) as the combination of a Wiener process and a compound Poisson process in the following:

\[
\frac{dH(t)}{H(t)} = m_H dt + s_H dW_{H,t} + \sum_{\tau=0}^{N(t)} (Y_n - 1) - l k dt ,
\]

(4)

where \( H(t) \) is the housing price at time \( t \), \( m_H \) is the expected growth rate of housing price, and \( \sigma_H \) is the constant volatility of the Wiener component of the housing price process. Term \( \{W_H(t): t > 0\} \) is a standard Wiener process.

The role of \( W_H(t) \) is the unanticipated instantaneous change in housing price, which reflects normal events, but it may not work so well for abnormal events (such as catastrophic events, government policy, or a mortgage crisis). Here, \( \{Y_n: n = 1, 2, \ldots\} \) is a series of identically distributed non-negative random variables representing the \( n^{th} \) jump size. The jumps in housing prices correspond to the arrival of abnormal information, and this paper assumes that jump size is lognormally distributed with mean \( \theta \) and variance \( \theta^2 \). The last term, \( \lambda \kappa dt \), compensates for the jump process, where \( \kappa = E(Y - 1) = \exp(\theta + \frac{1}{2} \theta^2) \), i.e., the expected percentage changes in housing price if an abnormal event occurs, while \( \{N(t): t > 0\} \) represents the total number of jumps during a time interval of \((0, t]\) and is based on a Poisson process with a intensity parameter \( \lambda \). Furthermore, all three sources of randomness - standard Wiener process
\( W_H(t) \), Poisson process \( N(t) \), and jump size \( Y_n \) - are assumed to be independent.

By Girsanov’s theorem, \( dW_{H,t}^Q = dW_{H,t} - \lambda_H dt \), where term \( \lambda_H \) is the market price of housing price risk. Assuming that the jump risk is non-systematic and diversifiable (i.e. a risk premium to the jump risk is unnecessary), we now show the dynamic process of housing price under the risk-neutral probability measure \( Q \):

\[
\frac{dH(t)}{H(t)} = r(t)dt + s_H dW_{H,t}^Q + \sum_{n=0}^{N(t)} (Y_n - 1) - l k dt.
\] (5)

Based on the risk-neutralized results for the interest rate in Equation (3) and housing price in Equation (5), the dynamics of a mortgage insurer’s liabilities under the risk-neutralized measure can be arranged as follows:

\[
\frac{dL(t)}{L(t)} = r(t)dt + \phi_L r(t) dW_{L,t}^Q + \phi_{L,H} dW_{H,t}^Q + \sigma_L dW_{L,t}^Q,
\] (6)

where \( W_{L,t}^Q \) is a Wiener process for the risk-neutral probability measure \( Q \) and is independent of \( Z_{r,t}^Q \) and \( W_{H,t}^Q \).

### 2.4 The mortgage insurer’s asset process

We assume that the asset dynamics of the mortgage insurer follow a lognormal diffusion process and consider the impact of stochastic interest rates on the asset value as follows:

\[
\frac{dA(t)}{A(t)} = \mu_A dt + \phi_{A,r} d\langle r \rangle + \sigma_A dW_{A,t}^Q,
\] (7)

where \( \mu_A \) is the instantaneous drift of the asset value at time \( t \); \( W_{A,t} \) is the Wiener process that denotes the credit risk on the assets of the mortgage insurer; \( \phi_{A,r} \) is the
instantaneous interest rate elasticity of the mortgage insurer’s asset; \( \sigma_A = \sigma^*_A \gamma \), and 
\[ \gamma = \sqrt{1 - \rho_{Ar}^2} \], where \( \sigma^*_A \) is the volatility of the credit risk; and \( \rho_{Ar} \) is the correlation coefficient of the interest rate and the asset value. In a risk-neutral world, the asset of the mortgage insurer can be risk neutralized as follows:

\[
\frac{dA(t)}{A(t)} = r(t)dt + \phi_{Ar} \sqrt{r(t)}dZ_{r,d} + \sigma_A dW_{Ar}^Q, \tag{8}
\]

where term \( W_{Ar}^Q \) is the Wiener process under the risk-neutral measure and is independent with \( Z_{r,d}^Q \).

3. Valuation of mortgage insurance

Once the risk-neutral processes of interest rate, housing price, liability, and asset dynamics are known, one can value the MI by discounting expected payoffs in the risk-neutral world. This section specifies the payoffs from MI under alternative considerations. We first present the base case, in which mortgage insurers will not default, and then look into the MI with counterparty default risk. Finally, we evaluate how regulatory actions and capital forbearance affect the values of MI.

3.1 Payoffs from mortgage insurance

At time \( t = 0 \), i.e., at origination, the lender issues a \( T - \) month mortgage, secured by the underlying housing, for the amount of \( B(0) = L_v H(0) \). Let \( L_v \) be the initial loan-to-value ratio and \( H(0) \) is the initial housing price. We assume that the mortgage loan has a fixed interest rate \( c \) (risk-free interest rate plus a spread, \( r(t) + \text{spread} \)) and
that installments \( y \) are paid monthly. Hence, with no prepayment or default prior to time \( t \), the loan balance \( B(t) \) at time \( 0 \leq t \leq T \) is given by the following expression:

\[
B(t) = \frac{y}{c} \left( 1 - \frac{1}{(1+c)^{T-t}} \right).
\]  

(9)

At time \( t = 0 \), the mortgage insurer writes a MI that promises to compensate the lender only when the borrower defaults. The following section presents the payoffs from MI under alternative considerations.

### 3.1.1 Default-free Mortgage Insurer

In the case where the mortgage insurer will not default, we follow Kau and Keenan (1995, 1996) and model MI as a portfolio of American put options. Thus, the payoffs from a MI contract at time \( t \), \( M_{ND}(t) \), is given as follows:

\[
M_{ND}(t) = \begin{cases} 
L_R B(t) & \text{if } H(t) < (1-L_R)B(t) \\
(B(t) - H(t)) & \text{if } (1-L_R)B(t) \leq H(t) < B(t) \\
0 & \text{if } H(t) > B(t) 
\end{cases}
\]  

(10)

where \( L_R \) denotes the coverage ratio. Equation (10) implies that if the housing price \( H(t) \) exceeds the remaining loan balance \( B(t) \) at time \( 0 \leq t \leq T \), then it implies that the borrower would not default at time \( 0 \leq t \leq T \) and thus the loss of the mortgage insurer is zero. On the other hand, if \( (1-L_R)B(t) \leq H(t) < B(t) \) at time \( 0 \leq t \leq T \), then it shows that the housing price is not sufficient for a full repayment of the loan balance, and the possibility that the borrower defaults increases such that the mortgage insurer covers the remaining loan balance up to some maximum coverage \( L_R \) in return for receiving the defaulted housing \( H(t) \).
3.1.2 Vulnerable Mortgage Insurer

Given that each time interval is a month, the borrower must make payments each month and the borrower has the opportunity to default at those times. When the mortgage insurer itself is default-risky, the payoffs from MI at time $t$, $M_D(t)$, are given as follows:

\[
M_D(t) = \begin{cases} 
L_R B(t) & \text{if } H(t) < (1-L_R)B(t) \text{ and } A(t) \geq L(t) \\
\frac{L_R B(t) A(t)}{L(t)} & \text{if } H(t) < (1-L_R)B(t) \text{ and } A(t) < L(t) \\
(B(t) - H(t)) & \text{if } (1-L_R)B(t) \leq H(t) < B(t) \text{ and } A(t) \geq L(t) \\
\frac{(B(t) - H(t)) A(t)}{L(t)} & \text{if } (1-L_R)B(t) \leq H(t) < B(t) \text{ and } A(t) < L(t) \\
0 & \text{otherwise}
\end{cases}
\]  

The first and third terms of Equation (11) represent the respective loss of the mortgage insurer when the borrower defaults at time $0 \leq t \leq T$ and when the mortgage insurer does not default during the remaining life of the MI; namely, during the remaining life of the MI, the value of the mortgage insurer’s total asset $A(t)$ is greater than the expected default trigger $L(t)$, and thus the mortgage insurer does not default. The second and fourth terms of this equation present recovery values of the MI when the mortgage insurer defaults at time $0 \leq t \leq T$, i.e., $A(t)$ is less than $L(t)$, and when the borrower also defaults at time $0 \leq t \leq T$ during the remaining life of the MI, respectively. The recovery value is therefore equal to the nominal claim of the MI multiplied by the recovery rate (asset-liability ratio), $A(t)/L(t)$. Hence, the set-up of Equation (11) indicates that our MI contract embeds a portfolio of vulnerable American puts that may be exercised not only when the mortgage borrowers default, but also when the contract is
forced to be terminated by the default of mortgage insurers.

### 3.1.3 Vulnerable Mortgage Insurer with capital forbearance

In the previous section we assume an insolvent mortgage insurer will liquidate its assets to meet its obligations. The MI contract will be terminated and the insurer will not be in operation. In reality, however, undercapitalized mortgage insurers are not closed immediately and are still permitted to operate. This phenomenon resembles the capital forbearance observed in the banking industry.

We model capital forbearance along the line of Ronn and Verma (1986) and Duan and Yu (1994). At the time $0 \leq t \leq T$, a mortgage insurer cannot be taken over unless its asset value falls below the forbearance threshold, $\rho L(t)$, where $\rho$ is the forbearance parameter and is taken to be less than or equal to one. Even if the mortgage insurer’s asset value cannot meet the capital standard, as long as it does not fall below the forbearance threshold, then the mortgage insurer will not face an immediate intervention and can extend its operations until $t + \tau$.

The cash flow of the vulnerable MI with capital forbearance at time $t$, $M_C(t)$, can hence be characterized as:

$$
M_C(t) = \begin{cases} 
L_R B(t) & \text{if } H(t) < (1-L_R)B(t) \text{ and } A(t) \geq qL(t) \\
F(t) & \text{if } H(t) < (1-L_R)B(t) \text{ and } \rho L(t) \leq A(t) < qL(t) \\
\frac{L_R B(t) A(t)}{L(t)} & \text{if } H(t) < (1-L_R)B(t) \text{ and } A(t) < \rho L(t) \\
(B(t) - H(t)) & \text{if } (1-L_R)B(t) \leq H(t) < B(t) \text{ and } A(t) \geq qL(t) \\
G(t) & \text{if } (1-L_R)B(t) \leq H(t) < B(t) \text{ and } \rho L(t) \leq A(t) < qL(t) \\
\frac{(B(t) - H(t)) A(t)}{L(t)} & \text{if } (1-L_R)B(t) \leq H(t) < B(t) \text{ and } A(t) < \rho L(t) \\
0 & \text{otherwise} 
\end{cases}
$$

(12)
where parameter $q$ reflects the capital standard set by the regulatory authority, which is the lower bound of the mortgage insurer’s asset value. Generally, a mortgage insurer must operate within a 25-to-1 ratio of risk to capital, which means it sets aside $1$ of capital for every $25$ of risk it insures. Insured risk is defined as the percentage of each loan covered by an insurance policy. This capital standard can be translated into $q = (25 + 1)/25 = 1.04$ in our model. Furthermore, $F(t)$ and $G(t)$ are the values from the extended operation under forbearance, and their cash flows at the time of $(t+\tau)$ can be described as:

$$F(t+\tau) = \begin{cases} L_R B(t+\tau) & \text{if } A(t+\tau) \geq L(t+\tau), \\ \frac{L_R B(t+\tau) A(t+\tau)}{L(t+\tau)} & \text{otherwise.} \end{cases}$$

$$G(t+\tau) = \begin{cases} (B(t+\tau) - H(t+\tau)) & \text{if } A(t+\tau) \geq L(t+\tau), \\ \frac{(B(t+\tau) - H(t+\tau)) A(t+\tau)}{L(t+\tau)} & \text{otherwise.} \end{cases}$$

### 3.2 Valuation of mortgage insurance contracts

To understand the effects of capital requirement and capital forbearance on MI premiums, in this section we consider a single-period case of MI (single payment loans, i.e., $t = T$) with the following simplifications. The interest rate simplifies to a constant, and the borrower’s housing price and the mortgage insurer’s liability are independent. Based on these conditions, we intend to ascertain the exact relationship of the MI premium under capital standard and forbearance.

Based on Equation (10), using Merton’s (1976) pricing results for European put options with a constant interest rate, the closed-form formula of MI can be represented by
\( DL_1 \) as follows:

\[
DL_1 = \sum_{m=0}^{\infty} \frac{\text{exp}(-l(1+1)T)(1+1)^m}{m!} \left[ P(K_1) - P(K_2) \right],
\]

where \( P(K_i) = K_j \text{exp}(-r_m T) \mathcal{N}(-d_{2m}(K_j)) - H(0)N(-d_{1m}(K_j)), i = 1, 2, \)

\[
d_{1m,2m}(K_j) = \frac{\ln(H(0)/K_j) + (r_m - \frac{s_m^2}{2})T}{\sqrt{s_m^2 T}}, \quad K_1 = B(T), \quad K_2 = (1 - L_R)B(T),
\]

\[
r_m = r - l + \frac{mq + 0.5mq^2}{T}, \quad s_m^2 = s_m^2 + \frac{mq^2}{T}.
\]

Here, \( DL_1 \) can be duplicated by a long position in a European put option with a strike price \( K_1 \) and a short position in a European put option with a strike price \( K_2 \), both with their time to maturity equal to time \( T \). If \( l = 0 \), then it means that no abnormal shock event occurs, and so the volatility of the housing price process only captures normal volatility.

To derive the closed-form formula of the MI in response to considering insurer’s default risk and capital forbearance, we directly assume that the asset-debt ratio of the mortgage insurance company \( A(t)/L(t) \) follows a geometric Brownian motion, which replaces the set-up of \( A(t) \) and \( L(t) \) in Equations (5) and (7). Thus, the dynamics of the asset-debt ratio of the MI company under risk-neutrality are given as:

\[
\frac{dR(t)}{R(t)} = r_t dt + \sigma_R dW^Q_{R,t},
\]

where \( \sigma_R \) denotes the constant volatility of the instantaneous rate of return of \( R(t) \), which represents the uncertainty sources of both the mortgage insurer’s total asset value and total liability value. Hence, when insurer’s default risk and capital forbearance are
involved, the pricing formula of the MI, \( DL_2 \), can be described as:

\[
DL_2 =\left[ DL_2 \left( \frac{R(0)}{R(t)} \right) + N(b_1)\right] - \left[ R(0)\left( N(d_1(K_3), a_2, m) - N(d_1(K_3), b_2, m) \right) \right] + N(d_2(K_3), a_1, m) - N(d_2(K_3), b_1, m).
\]  

\( (15) \)

where

\[
d_{i,2}(K_3) = \frac{\ln(1/R(0)) + \frac{1}{2} s_R^2 (T - t)}{\sqrt{s_R^2 (T + t)}} - m = \sqrt{\frac{T}{T + t}}
\]

\[
a_{i,2} = \frac{\ln(q/R(0)) m}{\sqrt{s_R^2 T}} \frac{1}{2} s_R^2 T, b_{i,2} = \frac{\ln(r/R(0)) m}{\sqrt{s_R^2 T}} \frac{1}{2} s_R^2 T.
\]

We observe that the values of MI under forbearance \( DL_2 \) are greater than those of a portfolio made up of Merton’s put \( DL_1 \), and the excess premiums are the forbearance costs. The above closed-form solution can in fact be considered a general form of MI pricing under capital forbearance and one with a number of special cases. For example, it yields the case of Bardhan et al. (2006) when we set \( R(0) = \tau = 0, \rho = q = 1 \), and \( l = 0 \).

**Capital Forbearance to Mortgage Insurance**

The immediate impact of granting capital forbearance is best demonstrated by the following derivative property:

\[
\frac{\partial DL_2}{\partial \rho} = \frac{\partial DL^C}{\partial \rho} + \frac{\partial DL^L}{\partial \rho} = DL_2 \frac{n(b_1)}{\rho \sqrt{s_R^2 T}} \left[ (R(0) - 1) - \rho N(d_{k2}) - N(d_{k1}) \right] < 0, \quad (16)
\]

where
\[
\frac{\partial DL^C}{\partial \rho} = DL_2 \frac{n(b_1)}{\rho \sqrt{\sigma_R^2 T}} (R(0) - 1) \geq 0 \text{ or } < 0;
\]

\[
\frac{\partial DL^\tau}{\partial \rho} = DL_2 \frac{n(b_1)}{\rho \sqrt{\sigma_R^2 T}} (-\rho N(d_{k_2}) - N(d_{k_1})) < 0;
\]

\[
d_{k_1} = \frac{-\ln \rho + \frac{\sigma_R^2 \tau}{2}}{\sqrt{\sigma_R^2 \tau}}, d_{k_2} = d_{k_1} - \sqrt{\sigma_R^2 \tau};
\]

Here, \( n(.) \) denotes the probability density function of a standard normal variable. The sign of \( \frac{\partial DL^C}{\partial \rho} \) is negative, indicating that a higher value of \( \rho \) makes less undercapitalized insurers to be forborne and this reduces the cost of MI. We also note that the forbearance’s impact on the MI includes a positive (or negative) capital-component effect \( \frac{\partial DL^C}{\partial \rho} \) and a negative time-component effect \( \frac{\partial DL^\tau}{\partial \rho} \). In other words, when the asset value is larger (or less) than the liability value of the mortgage insurer, the positive (or negative) capital-component effect reflects that less (or more) insolvencies will be resolved at \( T \), whereas the negative time-component effect shows that more insolvencies will be resolved at \( T + \tau \). The negative aggregate effect indicates that the time component dominates the capital component. This shows that the forbearance policy can save the cost of a MI temporarily by delaying closures, but the future resolution cost dominates the cost savings and raises the cost of the MI.

**Capital Requirements**

The impact from adjusting capital requirements is demonstrated by the following derivative property:
\[
\frac{\partial DL_2}{\partial q} = \frac{1}{q\sqrt{\sigma_R^2 T}} \left[ R(0)n(a_2)\left(1 + N(d_{k3})\right) + qn(a_2)N(d_{k4}) \right] > 0, \tag{17}
\]

where
\[
d_{k3} = -\ln q + \frac{\sigma_R^2 \tau}{2}, \quad d_{k4} = d_{k3} - \sqrt{\frac{\sigma_R^2 \tau}{2}}.
\]

The positive sign of \(\frac{\partial DL_2}{\partial q}\) indicates that the higher level of required capital increases the cost of MI. This property makes it intuitive that a higher capital requirement may let more undercapitalized, but solvent, insurers to extend their coverage to time \(T + \tau\). The extension of coverage will decrease the wealth of a mortgage insurer.

The MI value is expected to be positively related to time delay and can be shown by the following equation:

\[
\frac{\partial DL_2}{\partial t} = \frac{\sqrt{s_R^2}}{4\sqrt{T + t}} R(0)n(d_1(K_3)) \left\{ N\left(\frac{a_2 - md_1(K_3)}{\sqrt[1 - m^2]}\right) - N\left(\frac{b_2 - md_1(K_3)}{\sqrt[1 - m^2]}\right) \right\} + N\left(\frac{a_1 - md_2(K_3)}{\sqrt[1 - m^2]}\right) - N\left(\frac{b_1 - md_2(K_3)}{\sqrt[1 - m^2]}\right) > 0.
\]

This result indicates that extending the time delay to resolve the undercapitalized mortgage insurer raises the mortgage insurer’s cost, and the longer the time delay is, the higher the cost for the mortgage insurer.

**Cross Effect of Forbearance and Time of Delay**

The two key policy parameters of capital forbearance are the forbearance threshold parameter \((\rho)\) and the time of delay parameter \((\tau)\). We find that the forbearance parameter is negatively related to the delay parameter based on the following partial

The negative relation interprets that a lower capital forbearance threshold allows more undercapitalized mortgage insurers to extend their operations and increases the positive impact of delay on the mortgage insurer’s cost, \( \left( \frac{\partial DL_2}{\partial \tau} \right) \). It also shows that a longer time of delay permits the undercapitalized mortgage insurer to operate over a longer period of time and therefore lengthens the negative impact of the capital forbearance threshold on the mortgage insurer’s cost, \( \left( \frac{\partial DL_2}{\partial \rho} \right) \).

4. **Numerical analysis**

This section estimates the values of MI for alternative scenarios using the least-squares approach provided by Longstaff and Schwartz (2001). This method is simple, but powerful, for valuing American-style options. To be consistent with the fact that payments in practice must be made each month, the simulations are run on a monthly basis with 50,000 paths and then the MI premiums are monthly premiums. Table 1 presents the base set of parameters.

[Table 1 is here]

Deviations from the base values provide insights into how changes in the
characteristics of the asset-liability structure of the mortgage insurer, the frequency and severity of catastrophe events in housing price, the interest rate process, and capital regulatory policy all affect the MI premiums.

We first consider the case where the mortgage insurer may default, i.e., with counterparty default risk, and then report the default-free and default-risky MI premium rates for alternative sets of catastrophic intensity of housing price and the mean and variances of jump size in housing price. The MI premium rates are calculated by the price of the MI premium as a percentage of the initial loan value. The differences between the default-free and default-risky MI premium rates are the default risk premiums, which are shown in Table 2. As expected, default-free MI premium rates are more valuable to the mortgage insurer and have higher values than their corresponding default-risky MI premium rates. The higher the mortgage insurer’s initial asset-liability structure is, the lower the default risk and the higher the MI premium rates will be.

[Table 2 is here]

We observe that the default risk premiums increase with the catastrophic intensity and the mean and variance of jump size in housing price. For instance, in the case where the mortgage insurer’s initial asset-liability structure \((A/L)\) is 1.3, the mean of jump size \((\theta)\) is -0.01 and the variance of jump size \((\delta^2)\) is 1, the default risk premium increases from 0.088 basis points to 0.136 basis points when the catastrophic intensity \((\lambda)\) increases from 0.5 to 1 and rises to 0.360 basis points when the catastrophic intensity increases to 2. The default risk premium increases with catastrophic intensity and the mean of jump size and volatility of housing price, and the premium can be substantial in
the presence of jump risk in housing price and should not be neglected in the valuation of MI. For example, in the case where \((\lambda, \theta, \delta^2) = (2, 0.01, 0.2)\), the default risk premium goes from 0.405 basis points to 0.551 basis points when \((A/L)\) decreases from 1.5 to 1.1, or an increment of 0.146 basis points. The increment of the default risk premium drops down to about 7 basis points for the case of \((\lambda, \theta, \delta^2) = (0.5, 0.01, 0.05)\).

Table 3 reports the premium rate of a MI contract while fixing \((\lambda, \theta, \delta^2) = (1, 0.01, 1)\). We set \((\phi_{L,H}) = -2, -1, 0\) to measure different scenarios of the housing sensitivity of liabilities and set the interest rate elasticity pair to \((\phi_{A,r}, \phi_{L,r}) = (-2, -1), (\phi_{A,r}, \phi_{L,r}) = (-1L/A, -1)\) and \((\phi_{A,r}, \phi_{L,r}) = (0, 0)\), respectively, so as to measure different scenarios of interest rate elasticity gaps. We find that the housing price sensitivity of liabilities is an increasing function of the MI premium rate. Moreover, in the case of \((\phi_{A,r}, \phi_{L,r}) = (0, 0)\), both assets and liabilities are interest rate insensitive, whereas the scenario of \((\phi_{A,r}, \phi_{L,r}) = (-1L/A, -1)\) has a zero size-adjusted interest rate elasticity gap, which can be regarded as an active interest rate risk management practice that strives to eliminate the interest rate risk facing the mortgage insurer by adjusting the asset-liability mix. The case of \((\phi_{A,r}, \phi_{L,r}) = (-2, -1)\) presents a positive duration gap on the asset-liability structure. The difference in the premium rates for \((\phi_{A,r}, \phi_{L,r}) = (-2, -1)\) and that for \((\phi_{A,r}, \phi_{L,r}) = (-1L/A, -1)\) reflects the interest rate risk. For instance, when \((A/L)\) is 1.5, the duration gap on the asset-liability structure equals to 2.

In the case of \((A/L) = 1.5\) and \(\phi_{L,H} = 0\), the MI premium decreases 0.16\% (1.789\% - 1.629\%) when the interest rate elasticity gaps of the mortgage insurer becomes
$(\phi_{A,r}, \phi_{L,r}) = (-2, -1)$, rather than $(\phi_{A,r}, \phi_{L,r}) = (0, 0)$. This implies that, comparing with zero duration gap on the asset-liability structure, positive duration gap on the asset-liability structure causes the asset value of the mortgage insurer decline and thus the MI premium rate decreases. Furthermore, the lower the mortgage insurer’s initial asset-liability structure is, the higher effect of positive duration gap on the asset-liability structure on the MI premium rates will be. For example, under the situation of $\phi_{L,H} = 0$, the premium rate of MI increases from 0.16% to 0.27% when the asset-liability structure $(A/L)$ decreases from 1.5 to 1.1. Hence, the interest rate risk effect is quite substantial and is more obvious for a lower asset-liability structure mortgage insurer. It is also not surprising to see no effect caused by the interest rate risk premium when both assets and liabilities are interest rate insensitive, i.e., for the case of $(\phi_{A,r}, \phi_{L,r}) = (0, 0)$. Note that the MI premium rates for the zero-gap and insensitivity scenarios are very similar, indicating that active interest risk management can achieve the same effect as sticking exclusively to interest rate insensitive assets and liabilities, regardless of the housing price sensitivity of liabilities. This means that a mortgage insurer that employs interest rate management can effectively have a much lower interest rate risk and can increase the values of its MI contracts.

[Table 3 is here]

Table 4 reports the premium rate of MI under alternative levels of the forbearance threshold $(\rho)$, capital requirement $(q)$, and time to delay $(\tau)$. As forbearance extends the insurance coverage to the undercapitalized mortgage insurer, the premium rate of MI increases with a lower forbearance threshold. For example, under the situations of
$(\lambda, \delta^2, \tau) = (2, 0.2, 6)$ and $A/L = 1.3$, the premium rate of MI increases from 323.7 basis points to 359.0 basis points when the forbearance threshold $(\rho)$ decreases from 100% to 95%, and rises to 39.73 basis points when the forbearance threshold $(\rho)$ decreases to 90%.

We see here that the sensitivities of the forbearance threshold effect increase with their initial asset-liability structure. For instance, the mortgage premium rate for a borrower with an initial asset-liability structure at 1.1 increases by 52.6 basis points when the forbearance threshold reduces from 100% to 90%, and the changes are more substantial (86.5 basis points) for mortgage insurers with an asset-liability structure at 1.5. At the same time it shows that the premium rate of MI can be saved by delaying a resolution for undercapitalized mortgage insurers, which ultimately increases the cost of MI.

The time to delay effect could be more substantial as the initial asset-liability structure increases. For instance, the mortgage premium rate with an initial asset-liability structure at 1.1 increases by 85 basis points when the time to delay rises from 4 to 6 months, whereas the change goes up to 103.4 basis points for mortgage insurers with an asset-liability structure at 1.5. Moreover, an increased capital requirement will result in higher MI premium rates. In brief, the counterparty default effect, forbearance threshold effect, time to delay effect, and capital requirement effect are all critical factors in the premium of MI.

[Table 4 is here]
5. Conclusions

We have developed a model in this paper for measuring the MI premium rate by taking into account the default risk of the mortgage insurer, interest rate risks, catastrophic intensity and severity of the housing price, and capital regulatory policy. We treat forbearance as an option to delay the resolution of undercapitalized mortgage insurers in the current regulatory framework and then derive a closed-form solution with some specific conditions in order to examine and measure its impacts and to discuss its possible implications.

The results indicate that the default risk premium can be substantial in the presence of catastrophic risk in the housing price and should not be neglected in valuating the MI policy. Our results have interesting implications for forbearance policy. We show that the forbearance policy can save the cost of a mortgage insurer temporarily by delaying closures, but the future resolution cost dominates the cost savings and raises the cost of the mortgage insurer. Furthermore, time to delay effects are positively related by the value of MI, a lower forbearance threshold increases the impact of delay on the MI value, and a longer time of delay amplifies the impact of forbearance threshold on MI value. The forbearance threshold and time to delay effects could be more significant when the initial asset-liability ratio increases. This implies that it would ultimately raise a loss for a mortgage insurer, instead of saving a cost for the mortgage insurer, when a capital forbearance policy is considered. In sum, the counterparty default effect, forbearance threshold effect, time to delay effect, and capital requirement effect are all critical factors for valuating MI.
References


### Table 1
Parameter definitions and base values

<table>
<thead>
<tr>
<th>Asset parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A(0)$</td>
<td>Mortgage insurer’s asset</td>
</tr>
<tr>
<td>$\phi_{A,r}$</td>
<td>Interest rate elasticity of asset -1, 0</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Volatility of credit risk 5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Liability parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L(0)$</td>
<td>Mortgage insurer’s liability 100</td>
</tr>
<tr>
<td>$\phi_{L,r}$</td>
<td>Interest rate elasticity of liability -1, 0</td>
</tr>
<tr>
<td>$\phi_{L,H}$</td>
<td>Housing price elasticity of liability -1, 0</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>Volatility of credit risk 5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interest rate parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r(0)$</td>
<td>Initial instantaneous interest rate 5%</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Magnitude of mean-reverting force 0.2</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Long-run mean of interest rate 5%</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Volatility of interest rate 10%</td>
</tr>
<tr>
<td>$\lambda_r$</td>
<td>Market price of interest rate risk -0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Housing price parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H(0)$</td>
<td>Borrower’s housing price 200,000</td>
</tr>
<tr>
<td>$s_H$</td>
<td>Volatility of housing price risk 5%</td>
</tr>
<tr>
<td>$f_{H,r}$</td>
<td>Interest rate elasticity of housing price -1, 0</td>
</tr>
<tr>
<td>$q$</td>
<td>Mean of jump size -1% and 1%</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>Variance of jump size 5%, 10%, and 20%</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Jump intensity 0.5, 1, and 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_R$</td>
<td>Insurance coverage level 25%</td>
</tr>
<tr>
<td>$y$</td>
<td>Installments monthly 500</td>
</tr>
<tr>
<td>$c = r + \text{spread}$</td>
<td>Contract rate Spread=2%</td>
</tr>
<tr>
<td>$T$</td>
<td>Loan maturity 360 months</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Forbearance parameter 100%, 95%, and 90%</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Time of delay 2, 4, and 6 months</td>
</tr>
<tr>
<td>$q$</td>
<td>Capital standard 1.04, 1.05, and 1.06</td>
</tr>
</tbody>
</table>
Table 2
Mortgage insurance premium rates with insurer’s default risk but no capital forbearance

<table>
<thead>
<tr>
<th>$(\lambda, \theta, \delta^2)$</th>
<th>Default-free MI rate</th>
<th>Default-risky MI rate</th>
<th>Default risk premiums</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A/L$</td>
<td>$A/L$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>1.3</td>
<td>1.5</td>
</tr>
<tr>
<td>(0.5, -0.01, 0.05)</td>
<td>0.811%</td>
<td>0.686%</td>
<td>0.723%</td>
</tr>
<tr>
<td>(0.5, -0.01, 0.1)</td>
<td>0.864%</td>
<td>0.698%</td>
<td>0.755%</td>
</tr>
<tr>
<td>(0.5, -0.01, 0.2)</td>
<td>1.567%</td>
<td>1.197%</td>
<td>1.273%</td>
</tr>
<tr>
<td>(1, -0.01, 0.05)</td>
<td>0.915%</td>
<td>0.718%</td>
<td>0.779%</td>
</tr>
<tr>
<td>(1, -0.01, 0.1)</td>
<td>1.086%</td>
<td>0.688%</td>
<td>0.765%</td>
</tr>
<tr>
<td>(1, -0.01, 0.2)</td>
<td>1.912%</td>
<td>1.480%</td>
<td>1.587%</td>
</tr>
<tr>
<td>(2, -0.01, 0.05)</td>
<td>1.193%</td>
<td>0.752%</td>
<td>0.833%</td>
</tr>
<tr>
<td>(2, -0.01, 0.1)</td>
<td>1.647%</td>
<td>1.180%</td>
<td>1.251%</td>
</tr>
<tr>
<td>(2, -0.01, 0.2)</td>
<td>2.980%</td>
<td>2.519%</td>
<td>2.562%</td>
</tr>
<tr>
<td>(0.5, 0.01, 0.05)</td>
<td>0.871%</td>
<td>0.706%</td>
<td>0.777%</td>
</tr>
<tr>
<td>(0.5, 0.01, 0.1)</td>
<td>0.934%</td>
<td>0.738%</td>
<td>0.797%</td>
</tr>
<tr>
<td>(0.5, 0.01, 0.2)</td>
<td>1.637%</td>
<td>1.258%</td>
<td>1.293%</td>
</tr>
<tr>
<td>(1, 0.01, 0.05)</td>
<td>0.975%</td>
<td>0.758%</td>
<td>0.801%</td>
</tr>
<tr>
<td>(1, 0.01, 0.1)</td>
<td>1.146%</td>
<td>0.738%</td>
<td>0.785%</td>
</tr>
<tr>
<td>(1, -0.01, 0.2)</td>
<td>1.982%</td>
<td>1.540%</td>
<td>1.611%</td>
</tr>
<tr>
<td>(2, 0.01, 0.05)</td>
<td>1.273%</td>
<td>0.812%</td>
<td>0.893%</td>
</tr>
<tr>
<td>(2, 0.01, 0.1)</td>
<td>1.737%</td>
<td>1.240%</td>
<td>1.311%</td>
</tr>
<tr>
<td>(2, 0.01, 0.2)</td>
<td>3.030%</td>
<td>2.479%</td>
<td>2.582%</td>
</tr>
</tbody>
</table>

This table presents mortgage insurance premium rates and default risk premiums with counterparty default risk but no capital forbearance. This table also presents the default risk premium for an alternative interest rate and housing sensitivity structure. The mortgage insurance premium rates are calculated by the prices of the mortgage insurance premium as a percentage of the initial loan value and reported for alternative sets of catastrophe intensities $(\lambda)$ and catastrophe loss mean and volatilities $(\theta, \delta^2)$. Here, $A/L$ represents the initial asset-liability structure of the mortgage insurers. All estimates are computed using 50,000 sample paths. Other parameter values are specified in Table 1.
### Table 3
Mortgage insurance premium rates with insurer’s default risk for alternative interest-rate and housing sensitivity structures

<table>
<thead>
<tr>
<th>$\phi_{L,H}$</th>
<th>$(\phi_{A,r}, \phi_{L,r})$ = (0, 0)</th>
<th>1.1</th>
<th>1.3</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>1.498%</td>
<td>1.564%</td>
<td>1.789%</td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td>0.709%</td>
<td>1.061%</td>
<td>1.341%</td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td>0.228%</td>
<td>0.616%</td>
<td>0.918%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\phi_{L,H}$</th>
<th>$(\phi_{A,r}, \phi_{L,r})$ = (−1 × $L / A$, −1)</th>
<th>1.1</th>
<th>1.3</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>1.505%</td>
<td>1.514%</td>
<td>1.768%</td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td>0.694%</td>
<td>1.011%</td>
<td>1.311%</td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td>0.208%</td>
<td>0.566%</td>
<td>0.885%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\phi_{L,H}$</th>
<th>$(\phi_{A,r}, \phi_{L,r})$ = (−2, −1)</th>
<th>1.1</th>
<th>1.3</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>1.225%</td>
<td>1.304%</td>
<td>1.629%</td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td>0.514%</td>
<td>0.891%</td>
<td>1.231%</td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td>0.112%</td>
<td>0.486%</td>
<td>0.834%</td>
</tr>
</tbody>
</table>

This table presents the mortgage insurance premium rates with counterparty default risk for an alternative interest rate and housing sensitivity structure. The mortgage insurance premium rates are calculated by the prices of the mortgage insurance premium as a percentage of the initial loan value and reported for alternative sets of housing sensitivity ($\phi_{L,H}$) of the mortgage insurer’s liabilities, interest rate sensitivities ($\phi_{A,r}$) of the mortgage insurer’s assets, and liabilities. Here, $A / L$ represents the initial asset-liability structure of the mortgage insurers. All estimates are computed using 50,000 sample paths. Other parameter values are specified in Table 1.
Table 4
Mortgage insurance premium rates with insurer’s default risk and capital forbearance

<table>
<thead>
<tr>
<th>(λ, δ², τ)</th>
<th>Capital requirement 1.04</th>
<th>Capital requirement 1.05</th>
<th>Capital requirement 1.06</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100%</td>
<td>95%</td>
<td>90%</td>
</tr>
<tr>
<td>A/L = 1.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.5, 0.05, 2)</td>
<td>0.185%</td>
<td>0.221%</td>
<td>0.299%</td>
</tr>
<tr>
<td>(0.5, 0.1, 4)</td>
<td>0.194%</td>
<td>0.236%</td>
<td>0.310%</td>
</tr>
<tr>
<td>(0.5, 0.2, 6)</td>
<td>0.202%</td>
<td>0.255%</td>
<td>0.324%</td>
</tr>
<tr>
<td>(1, 0.05, 2)</td>
<td>0.281%</td>
<td>0.329%</td>
<td>0.386%</td>
</tr>
<tr>
<td>(1, 0.1, 4)</td>
<td>0.594%</td>
<td>0.686%</td>
<td>0.801%</td>
</tr>
<tr>
<td>(1, 0.2, 6)</td>
<td>0.924%</td>
<td>1.026%</td>
<td>1.147%</td>
</tr>
<tr>
<td>(2, 0.05, 2)</td>
<td>1.491%</td>
<td>1.594%</td>
<td>1.747%</td>
</tr>
<tr>
<td>(2, 0.1, 4)</td>
<td>2.514%</td>
<td>2.693%</td>
<td>2.900%</td>
</tr>
<tr>
<td>(2, 0.2, 6)</td>
<td>3.224%</td>
<td>3.477%</td>
<td>3.750%</td>
</tr>
<tr>
<td>A/L = 1.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.5, 0.05, 2)</td>
<td>0.204%</td>
<td>0.258%</td>
<td>0.319%</td>
</tr>
<tr>
<td>(0.5, 0.1, 4)</td>
<td>0.207%</td>
<td>0.262%</td>
<td>0.327%</td>
</tr>
<tr>
<td>(0.5, 0.2, 6)</td>
<td>0.216%</td>
<td>0.273%</td>
<td>0.342%</td>
</tr>
<tr>
<td>(1, 0.05, 2)</td>
<td>0.304%</td>
<td>0.361%</td>
<td>0.424%</td>
</tr>
<tr>
<td>(1, 0.1, 4)</td>
<td>0.607%</td>
<td>0.699%</td>
<td>0.818%</td>
</tr>
<tr>
<td>(1, 0.2, 6)</td>
<td>0.997%</td>
<td>1.099%</td>
<td>1.240%</td>
</tr>
<tr>
<td>(2, 0.05, 2)</td>
<td>1.506%</td>
<td>1.647%</td>
<td>1.879%</td>
</tr>
<tr>
<td>(2, 0.1, 4)</td>
<td>2.527%</td>
<td>2.750%</td>
<td>2.977%</td>
</tr>
<tr>
<td>(2, 0.2, 6)</td>
<td>3.237%</td>
<td>3.590%</td>
<td>3.973%</td>
</tr>
<tr>
<td>A/L = 1.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.5, 0.05, 2)</td>
<td>0.219%</td>
<td>0.275%</td>
<td>0.345%</td>
</tr>
<tr>
<td>(0.5, 0.1, 4)</td>
<td>0.224%</td>
<td>0.285%</td>
<td>0.364%</td>
</tr>
<tr>
<td>(0.5, 0.2, 6)</td>
<td>0.233%</td>
<td>0.298%</td>
<td>0.372%</td>
</tr>
<tr>
<td>(1, 0.05, 2)</td>
<td>0.321%</td>
<td>0.389%</td>
<td>0.461%</td>
</tr>
<tr>
<td>(1, 0.1, 4)</td>
<td>0.624%</td>
<td>0.721%</td>
<td>0.845%</td>
</tr>
<tr>
<td>(1, 0.2, 6)</td>
<td>1.050%</td>
<td>1.161%</td>
<td>1.301%</td>
</tr>
<tr>
<td>(2, 0.05, 2)</td>
<td>1.523%</td>
<td>1.674%</td>
<td>1.996%</td>
</tr>
<tr>
<td>(2, 0.1, 4)</td>
<td>2.544%</td>
<td>2.788%</td>
<td>3.095%</td>
</tr>
<tr>
<td>(2, 0.2, 6)</td>
<td>3.264%</td>
<td>3.637%</td>
<td>4.129%</td>
</tr>
</tbody>
</table>

This table presents the mortgage insurance premium rates with counterparty default risk and capital forbearance for alternative capital parameters. The mortgage insurance premium rates are calculated by the prices of the mortgage insurance premium as a percentage of the initial loan value and reported for alternative sets of capital requirement (q), capital forbearance (ρ), and time to delay (τ). Here, A/L represents the initial asset-liability structure or capital position of the mortgage insurers. All estimates are computed using 50,000 sample paths. Other parameter values are specified in Table 1.