Optimal Portfolio Selection: the Role of Illiquidity and Investment Horizon

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Abstract

Modern Portfolio Theory is a single-period model developed for the efficient securities market, in which asset prices are implicitly assumed to follow a random walk. It is widely agreed that real estate does not fit into the efficient market paradigm; however, mixed-asset portfolio analysis continues to rely on Modern Portfolio Theory. This paper proposes an alternative portfolio theory that extends the classical MPT to accommodate multi-period utility maximization as well as the unique characteristics of real estate such as liquidity risk, horizon-dependence of real estate returns and high transaction cost. Using real world data, we find that the optimal allocation to real estate in the mixed-asset portfolio is much lower than previously suggested by the literature, and is quite in line with the reality of institutional portfolios.

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1. Introduction

The appropriate role of private real estate in mixed-asset portfolios is a long standing puzzle in the real estate literature. Despite the general consensus among academics and practitioners that private real estate investment can bring additional diversification benefits to the traditional security-only portfolios, wide disagreement remains with regard to the optimal allocation to real estate. For example, while numerous academic studies since the 1980s have repeatedly suggested that real estate should constitute 15% to 40% or more of a diversified portfolio,1 leading institutional investors typically have only about 3% to 5% of their total assets in real estate.2 Significant effort has been devoted to resolving the discrepancy, but to date there appears to be no agreeable consensus among academics. The current paper attempts to resolve this old issue with a new approach.

The elegance of Modern Portfolio Theory is based on some critical assumptions. For example, the theory premises on a centralized and well-functioning asset market in which trading

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1 See, for example, Hartzell, Hekman and Miles (1986), Fogler (1984), Firstenberg, Ross and Zisler (1988), and Hudson-Wilson, Fabozzi and Gordon (2005), among others.

2 See Goetzmann and Dhar (2005) and an earlier survey by Pension & Investments (2002), which reported that the top 200 defined-benefit pension funds had only about 3% of their total assets in private real estate equities.
is continuous and frictionless, and the complex characteristics of the assets can be conveniently captured with two parameters – the means and variances of their expected returns. Real estate, however, are infrequently traded in decentralized private markets that are highly inefficient, and their returns are neither normal nor stable over time. (Young and Graff (1995), Graff, Harrington and Young (1997), Young, Lee and Devaney (2006), and Young (2008)) Secondly, non-variance factors – illiquidity, immobility, non-divisibility, etc. are unique to real estate and the traditional mean and variance are inadequate in capturing the performance of real estate. (Lusht (1988)) Lastly, while financial assets can be inexpensively traded with ease, the high transaction cost of real estate prevents investors from trading the asset frequently, which implies that real estate has to be held for a much longer period to be a viable investment. Most researchers acknowledge that these features of real estate do not conform with the finance paradigm, but in the absence of an alternative portfolio theory, the issues are often dismissed or downplayed as minor deviations. As a result, mixed-asset portfolio analysis continues to rely on Modern Portfolio Theory.

Previous effort in resolving the real estate allocation puzzle has largely been focused on attempting to “fine-tune” the application of Modern Portfolio Theory (MPT), by either imposing additional constraints that limit the maximum weights on real estate, and/or, in case the NCREIF data are used, inflating the real estate risk by some de-smoothing procedure. To a large extent, though, these ad hoc solutions seem to have confounded the puzzle further rather than solving it.

The current study takes a different approach – instead of tweaking the way the theory is applied, we modify the theory itself. Our objective is to develop a formal framework that extends the classical MPT to accommodate the unique characteristics of real estate. Specifically, our analysis explicitly incorporates three most distinct characteristics of real estate: (1) real estate
returns are horizon-dependent; (2) real estate bears liquidity risk; and (3) real estate involves high transaction costs. Although these characteristics have been extensively studied in the literature in various contexts, the accumulated knowledge has yet to be synthesized to yield a formal model for mixed-asset portfolio analysis. Building upon a series of recent research in this area, this paper attempts to fill the void. We first develop an alternative model for the mixed-asset portfolio and then apply the model by using real world data. Without resorting to any de-smoothing procedure or imposing any *ad hoc* constraints, we find that the optimal allocations to real estate in the mixed-asset portfolio are substantially lower than previous studies suggest and are more consistent with the observed practice of institutional investors.

2. **Investment horizon, illiquidity, and transaction cost of real estate**

   2.1 Horizon-dependence of real estate performance

   Modern Portfolio Theory is essentially a single-period model, which assumes that assets in the portfolio are to be held for “one period,” and the optimal portfolio is the one that maximizes the investor’s objective over such a single-period horizon. The validity of the theory to the multi-period investment reality depends on a critical assumption – all asset returns are independent and identically-distributed (i.i.d.) over time. Early studies by Merton (1969), Samuelson (1969), and Fama (1970) have shown that, under the i.i.d. condition, investor’s utility maximization over multiple periods are indistinguishable from that over a single period, that is, investment horizon is irrelevant. This result has a powerful implication because it effectively implies that portfolio optimization only needs to be based on the assets’ single-period performance, regardless of investor’s expected investment horizon. The importance of the i.i.d.
condition, therefore, should not be underestimated. It is the critical link that bridges the gap between the single-period theory and the multi-period investment reality.

Such a link does not exist for real estate. In fact, numerous studies have repeatedly documented that real estate returns exhibit strong auto-correlation and serial persistence, that is, they are not i.i.d. from period to period. This knowledge is often perceived as being perhaps too basic and encompassing to be practically useful, and it has typically not prevented researchers from treating real estate just like other financial assets in portfolio analysis. Some would argue that the assumption can be adopted for convenience because, after all, the stock returns are probably not perfectly i.i.d. either. But given the critical importance of i.i.d. to the traditional MPT application, two questions naturally arise: First, can real estate returns be considered “reasonably close” to the i.i.d. condition? Second, if not, what is the alternative distribution for real estate returns? Recent literature has addressed these issues.

Most previous studies find that real estate returns are not i.i.d., simply based on the evidence of serial correlation. The limitation of these studies is that they cannot tell how far real estate returns are away from the i.i.d. condition. Cheng, Lin and Liu (2011a) attempt to shed light on the “distance” by conducting a direct test of the i.i.d. hypothesis on a wide variety of asset classes including common stocks, private real estate, and REITs. Using the widely regarded BDS test by Brock, Dechert and Scheinkman (1987), they find that, while financial assets are reasonably close to the i.i.d. assumption, the null hypothesis of the i.i.d. condition is strongly rejected by real estate assets.

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3 The literature on the subject is too large to be reviewed here fully. A few examples include Case and Shiller (1989), Young and Graff (1995), Englund, Gordon and Quigley (1999) and Gao, Lin and Na (2009), among others.
The non-i.i.d. nature of real estate returns implies that real estate performance is horizon-dependent. In other words, the performance of multi-period real estate investment cannot be properly measured by single-period performance metrics. This is even true for some non-real estate portfolios as well. For example, in a study of hedge fund performances, Clifford, Krail and Liew (2001) find that simple estimate of volatility using monthly returns underestimates the actual fund volatility, and simple monthly beta and correlation greatly underestimate the fund’s equity market exposure. They attribute the cause of such biased monthly performance measures to the fact that hedge funds typically hold illiquid exchange-traded securities or over-the-counter securities that do not trade on a monthly basis, but are held for much longer period of time.

How does investment horizon affect the return and variance of multi-period real estate investment? Lin and Liu (2008) formally model the real estate transaction process and propose an alternative assumption that the variance of real estate return increases with the square of the holding time (as opposed to linearly increasing with holding time under the i.i.d. assumption). They further show that residential real estate data is more consistent with this alternative assumption than with the i.i.d. assumption. In a subsequent study, Cheng, Lin and Liu (2010a) extend the examination to the commercial real estate market and find that the risk structure in the commercial market is even closer to the alternative assumption than the residential market. This result reaffirms the finding that the real estate market rejects the i.i.d. assumption in favor of the alternative assumption, which implies that the variance of real estate returns increases faster than that of stocks as holding-period increases.

In a more recent study, Cheng, Lin and Liu (2011b) conduct a bootstrap simulation to compare the impact of holding-period on the performances of real estate versus stock, and they provide the following findings:
Figure 1 indicates that the pattern of the risk-to-return ratio for the NCREIF Index is quite different from that of the S&P500. The ratio of the S&P500 essentially moves horizontally with modest fluctuation, suggesting it is basically independent of the investment horizon. But the ratio for the NCREIF Index is highly horizon-dependent and consistently increases with the holding period. At short horizons, the NCREIF ratio is consistent with the “exceptionally low volatility” or significant “risk premium” that is so widely documented in the literature. However, as the holding period becomes more realistic and longer, the difference between NCREIF and the S&P500 quickly narrows, and eventually disappears as the holding period approaches 9 years (36 quarters). These results suggest that real estate risk increases as holding period becomes
longer. This is in contrast to the findings of Rehring (2011), ManKinnon and Zaman (2009) and Pagliari (2011), which suggest the variance of real estate returns declines or “decays” in the long run. The cause of the difference may be in their approaches to the question. Whereas the above three studies all rely on return-generating models based on certain assumptions on autocorrelation, mean-reversion, and so on, Cheng, Lin and Liu observe the historical data as-is, and directly compute multi-period return series from quarterly real estate indices without using any return-generating models.\textsuperscript{4} The findings by Cheng, Lin and Liu (2011b) are in fact consistent with Collett, Lizieri and Ward (2003) who directly observe from the U.K. commercial data that “even allowing for appraisal smoothing, private real estate looks less attractive once more realistic and longer time horizons are considered”. (p. 207)

The finding in Figure 1 suggests that real estate becomes less appealing in the long run. On the other hand, real estate has to be held for a long period of time because of its illiquidity and high transaction cost. This leads to an important question: how long is enough to amortize the transaction cost and illiquidity, but not too long to bear more price volatility? In other words, is there an optimal holding period for real estate? This question is important because, if real estate performance is horizon-dependent, one cannot determine the appropriate mean and variance – the inputs to the mean-variance optimization – without first determining the appropriate holding time. Consequently, the optimal real estate allocation in a mixed-asset portfolio is affected by the investment horizon as well. Cheng, Lin, and Liu (2010b) develop a closed-form formula for the optimal holding period of real estate. Their results show that the optimal holding period is longer when transaction costs are high and/or the expected time-on-

\textsuperscript{4} For detail descriptions of their computation process, see Cheng, Lin, and Liu (2011b). An alternative non-bootstrap method is discussed in Cheng, Lin, and Liu (2010a).
market is long. Using NCREIF data, they show that the optimal holding periods for various types of properties to be in the range of 4 – 6 years, which is broadly consistent with the findings by Gau and Wang (1994) and Collett, Lizieri and Ward (2003).

The horizon-dependent performance and the existence of an optimal holding-period for real estate have significant implications for the mixed-asset portfolio optimization. In a mean-variance framework, the optimal allocation to real estate depends on the estimates of its mean and variance, which in turn depend on the optimal holding period of real estate. This suggests that holding period should be part of the optimization process for the mixed-asset portfolio. That is, a mixed-asset portfolio must be simultaneously optimized with regard to both the holding period of real estate as well as the asset allocations.

2.2 Illiquidity and ex ante risk of real estate

Proper estimate of the real estate risk, however, requires more than incorporating the impact of holding period, because real estate is subject to another important risk – liquidity risk. Real estate cannot be easily bought and sold at any time an investor desires. The potential loss of welfare due to such an inability to trade out of a position when needed is a significant source of risk and must be properly accounted for by rational investors. Real estate investors have been dissatisfied with the way the performance is currently measured. A fairly recent survey by Goetzmann and Dhar (2005) reports that majority of the surveyed institutional portfolio managers consider illiquidity as their number one risk factor for real estate investment, and the lack of appropriate performance metric that quantifies such risk in formal analysis is a major challenge to their portfolio decision-makings. This concern is justified for two reasons:
First, liquidity risk is real and substantial in *ex ante*. Portfolio decisions are forward-looking in nature. Proper asset performances, therefore, should be measured with *ex ante* rather than *ex post* metrics. This is particularly important for real estate because, at the time of the acquisition decision, the future sale at the end of the holding period for real estate is very different from that for financial assets. While a stock investor only faces the uncertain sales price, real estate investor faces an additional uncertainty—the uncertain and lengthy time-on-market (TOM). According to the data from the National Association of Realtors, the average TOM in the U.S. residential market during the period from 1989 to 2006 is about 6 months, and it is substantially longer in the commercial property market. While liquidity risk may be mitigated by longer holding period, most properties are not held for very long in practice. As suggested by Gau and Wang (1994) and Collett, Lizieri and Ward (2003), the typical holding period is about 5 – 9 years for commercial properties. Within this range, Cheng, Lin, and Liu (2010a) find that the liquidity risk alone still contribute an additional 6% to 29% to the total *ex ante* risk of quarterly NCREIF index. The notion that liquidity risk is negligible in the long run is not entirely correct.

Second, liquidity risk is a systematic risk. Generally speaking, the expected TOM is a function of market conditions, and is not under the full control of the seller. In hot markets all properties are sold rather quickly, while in cold markets the average TOM will be substantially longer. Individual sellers may be able to influence their individual selling time with listing strategies subject to their financial constraints, but they cannot control the average TOM under a given market condition. A quick sale typically results in significant price discount from the property’s fair value. While the (deep) discount may reflect the degree of financial distress of the seller, it does not properly reflect the trading strategy of normal sellers who would take the time
necessary under given market conditions and search until a desirable offer arrives. In other words, liquidity risk is priced by the market. Conventional risk measures which ignore liquidity risk fail to account for this component of systematic risk.

As part of systematic risk, illiquidity cannot be diversified away in a portfolio. Although Bond, Huang, Lin, and Vandell (2007) suggest that the risk related to marketing time uncertainty can be diversified away by constructing a portfolio of ten or more properties, a subsequent study by Lin, Liu and Vandell (2009) clarifies the issue by showing that this is true only under the i.i.d. assumption. Otherwise they show that the risk due to uncertain marketing time does not in general approach zero in the limit, in fact could increase or decrease depending upon the illiquidity characteristics of the individual assets and the correlation among individual property returns and marketing periods. They conclude that “even large institutional real estate portfolio managers must consider the illiquidity present in their portfolios and cannot assume that its effect will be diversified away”. (p. 191)

2.3 Transaction cost and holding period

Real estate involves high transaction costs. Using U.K. data, Collett, Lizieri and Ward (2003) report “the round-trip lump-sum costs” were approximately 7-8% of the asset value. It should be noted that the effect of high transaction cost is more than just lowering the net sales price or returns, it also prevents frequent trading and affects investment horizon, which in turn affects the risk of real estate investment. Modern Portfolio Theory does not consider this effect of transaction cost. But a number of studies have documented that, in the presence of transaction cost, single-period utility maximization is not the same as multi-period utility maximization. Mayshar (1979) incorporates fixed transaction cost into a single-period mean-variance model and finds that even a small transaction cost results in investors holding fewer assets (as opposed
to the market portfolio) for longer periods, which implies that the Sharpe-Lintner CAPM is no longer valid. Constantinides (1986) finds that proportional transaction cost substantially reduces demand for assets and increases their holding periods. He also finds that a single-period model cannot simply be extended to multiple periods, because the appropriate holding periods are asset-specific. Amihud and Mendelson (1986) develop a theoretical model that predicts that higher bid–ask spreads (proxies for transaction costs) are correlated with longer holding periods. Their finding is supported by the empirical evidence in Atkins and Dyl (1997), who find significant correlations between holding period and the bid-ask spreads. Collectively, these studies imply that the classical single-period MPT is not appropriate for multi-period real estate investment because of high transaction cost.

In summary, the complex nature of real estate, as well as the complexity of its performance measurement, suggests that mixed-asset portfolio theory will be more complex than the classical MPT. The horizon-dependent performance, liquidity risk, and high transaction cost of real estate implies that optimal diversification of mixed-asset portfolio should be based on multi-period utility maximization. In the next sections, we present an alternative model that extends the classical MPT into a multi-period model, and demonstrate its application using real world data. Our results indicate that real estate allocation turns out to be much lower than that MPT suggests, thus reconciling the gap between academics and practitioners with regard to the decades-old real estate allocation puzzle.

3. A Model for the Mixed-asset Portfolio

A basic premise of Modern Portfolio Theory is that investors are rational and risk averse. They like returns but dislike risk. The optimal portfolio decision, therefore, is to seek the
efficient portfolio that (a) has the highest expected return for a given level of risk, or (b) has the minimum risk for a desired level of expected return. Since these are equivalent optimization problems, one way to solve the problem is to maximize the expected return for a given level of risk. This is commonly known as the mean-variance analysis. A general representation of the mean-variance analysis can be described as follows.

Consider a simple world where there are $N$ classes of assets available for investment. Suppose that an investor’s investment horizon is $T$ periods. At any given level of risk $\Sigma^2$, the investor’s objective is to choose an optimal weight $w_i$ on asset $i$ ($i = 1, 2, 3, \ldots, N$) to satisfy the following:

$$\max_{(w_1, w_2, \ldots, w_N, \sum w_i = 1)} \left\{ E \left[ \sum_{i=1}^{N} w_i \tilde{R}_{i,T} \right] \right\}$$

$$\text{St.}\quad \text{Var} \left( \sum_{i=1}^{N} w_i \tilde{R}_{i,T} \right) = \Sigma^2$$

where $\tilde{R}_{i,T}$ is the total return on individual asset $i$ over $T$ periods and $E \left[ \sum_{i=1}^{N} w_i \tilde{R}_{i,T} \right]$ is the expected holding-period (total) return of the portfolio. Mathematically, (1) is equivalent to solving the following Lagrangian function,

$$L(w_1, w_2, \ldots, w_N, \sum_{i=1}^{N} w_i = 1, \lambda) = E \left[ \sum_{i=1}^{N} w_i \tilde{R}_{i,T} \right] - \lambda (\text{Var} \left( \sum_{i=1}^{N} w_i \tilde{R}_{i,T} \right) - \Sigma^2)$$

For a given level of risk, there exists a unique $\lambda$ in Lagrangian function (2), which $\lambda$ essentially captures the degree of the investor’s risk aversion. Mathematically, Lagrangian

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5 Modern Portfolio Theory was developed by Markowitz (1952), Sharpe (1963), and Brennan (1975), and among others.
function (2) is equivalent to maximizing the portfolio’s risk-adjusted return for a given risk aversion parameter $\lambda$.

For mathematical simplicity, we assume that there are only three assets available to investors: An illiquid real estate asset, a liquid financial asset and a risk-free asset. By directly applying Modern Portfolio Theory, we can obtain the optimal asset allocations by solving the following problem:

$$
\max_{w_{RE}, w_L} \left\{ w_{RE} u_{RE} + w_L u_L + (1 - w_{RE} - w_L) r_f - \right. \\
\lambda \left[ w_{RE}^2 \sigma_{RE}^2 + 2 w_{RE} w_L \sigma_{RE} \sigma_L \rho + w_L^2 \sigma_L^2 \right] \right\}
$$

(3)

where

$u_{RE}$ and $\sigma_{RE}$ are the period return and risk of real estate asset;

$u_L$ and $\sigma_L$ are the period return and risk of the financial asset;

$\rho$ is the correlation between the real estate and financial asset;

$r_f$ is the period return for the risk-free asset;

$w_{RE}$ and $w_L$ are the weights on the real estate and financial assets, respectively;

$\lambda$ is investor’s risk-averse parameter.

Therefore, we can calculate the optimal weight on the real estate as follows,

$$
w_{RE}^* = \frac{u_{RE} - r_f}{\sigma_{RE}} - \rho \left( \frac{u_L - r_f}{\sigma_L} \right)
$$

$$
= \frac{2 \lambda (1 - \rho^2) \sigma_{RE}}{\sigma_{RE}^2 + 2 \sigma_{RE} \sigma_L \rho + \sigma_L^2}
$$

(4)

By directly applying Modern Portfolio Theory, a number of studies such as Hartzell, Hekman and Miles (1986), Fogler (1984), Firstenberg, Ross and Zisler (1988), and Hudson-
Wilson, Fabozzi and Gordon (2005), among others, have suggested that any diversified portfolio should contain 15% to 40% in real estate.

We next extend the classical MPT to accommodate multi-period utility maximization as well as the unique features of real estate. Figure 2 illustrates the chain of events occurring in the sale of a portfolio. Suppose that the investor acquires a portfolio at time 0 and wishes to sell the entire portfolio after holding it for $T$ periods.\(^6\) The financial asset can be sold immediately at time $T$, but the real estate will have to wait to be sold at $T + \tilde{t}$, where time-on-market ($\tilde{t}$) and the eventual sales price are both uncertain and not fully within the control of the investor.\(^7\) Note that the investor’s actual holding period for the real estate is $T + \tilde{t}$. Since $\tilde{t}$ is random, the decision to choose the optimal holding period for the investor is essentially to choose the optimal $T^*$.  

**Figure 2. The Sequence of Events in the Sale of the Portfolio**

(1) Place the portfolio in the market

\[ 0 \quad \longrightarrow \quad T \quad \longrightarrow \quad \tilde{t} \quad \text{(random)} \quad \longrightarrow \]

\[ \text{Time on market} \]

(2) Sale of the financial asset

(3) Sale of the real estate asset with lump-sum cost: $C$

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\(^6\) For mathematical simplicity, we assume that there is no partial sale.

\(^7\) Immediate sale of real estate (as in the case of imminent foreclosure) would characterize the behavior of an extremely distressed seller. While the likely (deep) price discount reflects the degree of distress of the seller, it does not properly reflect the illiquidity of the real asset under normal situations. Put differently, since normal sellers do not attempt immediate sale of their real estate, the price discount that would be necessary for such immediate sale is not an appropriate measure of the asset’s illiquidity. Lippman and McCall (1986) define such illiquidity as the expected time until an asset is successfully exchanged for cash under optimal policies.
Now, we incorporate the three features of real estate: (1) the horizon-dependent performance; (2) liquidity risk; and (3) transaction cost.

First, for the financial asset in the portfolio, the periodic risk-adjusted return can be readily obtained as

$$r_L^{adj}(w_L) = (w_L u - \lambda w_L^2 \sigma_L^2)$$ \hspace{1cm} (5)

But for the illiquid real estate asset, the computation of the periodic risk-adjusted return is much more complex than the financial asset with trivial time on market. As discussed earlier, there are two sources of risk associated with real estate transaction, as well as a high transaction cost \((C)\).

Figure 2 indicates that, by the time the property is sold, the eventual holding-period of the investment is \((T + t)\), which is uncertain in *ex ante*. The total risk-adjusted return for the real estate can then be expressed as,

$$r_{RE}^{adj}(w_{RE}) = E^{ex-ante}\{w_{RE} \tilde{R}_{T+t,RE}\} - \lambda \times Var^{ex-ante}\{w_{RE} \tilde{R}_{T+t,RE}\} - w_{RE} C$$ \hspace{1cm} (6)

Given the fact that the time-on-market \(\tilde{t}\) and \(\tilde{R}_{T+t,RE}\) can be regarded as two stochastic variables, by applying the conditional variance formula for any two stochastic variables, the *ex ante* variance in Equation (6), \(Var^{ex-ante}\{w_{RE} \tilde{R}_{T+t,RE}\}\), can be computed as,

$$Var^{ex-ante}\{w_{RE} \tilde{R}_{T+t,RE}\} = Var\{E\{w_{RE} \tilde{R}_{T+t,RE}\mid \tilde{t}\}\} + E\{Var\{w_{RE} \tilde{R}_{T+t,RE}\mid \tilde{t}\}\}$$ \hspace{1cm} (7)

Without specifying a particular distribution of time-on-market \(\tilde{t}\), we denote its mean and variance as \(t_{TOM}\) and \(\sigma_{TOM}^2\), respectively. Both \(t_{TOM}\) and \(\sigma_{TOM}^2\) essentially capture the degree of
real estate illiquidity. In terms of *ex post* return \( \tilde{R}_{T_t, RE} \), recent studies by Lin and Liu (2008) and Cheng, Lin and Liu (2010a) find that the total return upon sale follows the distribution with mean \( (T + \tilde{t})u_{RE} \) and standard deviation \( (T + \tilde{t})\sigma_{RE} \). We can thus simplify Equation (7) as

\[
Var^{ex-ante} (w_{RE} \tilde{R}_{T_t, RE}) = w_{RE}^2 ((T + t_{TOM})^2 \sigma_{RE}^2 + (\sigma_{RE}^2 + u_{RE}^2)\sigma_{TOM}^2)
\]

Equation (8) integrates two source of risks in real estate investment, price risk and liquidity risk \( (t_{TOM}, \sigma_{TOM}^2) \), into one unified risk metric.

For the *ex ante* expected return - the first term of Equation (6), first calculate the expectation of the *ex post* expected return conditional on time-on-market \( (\tilde{t}) \), and then use the Law of Iterated Expectations:

\[
E^{ex-ante} [w_{RE} \tilde{R}_{T_t, RE}] = E[E[w_{RE} \tilde{R}_{T_t, RE} | \tilde{t}]]
\]

\[
= E[w_{RE} (T + \tilde{t})u_{RE}]
\]

\[
= w_{RE} (T + t_{TOM})u_{RE}
\]

Substituting Equations (8) and (9) into Equation (6) yields

\[
r_{RE}^{Adj} (w_{RE}) = w_{RE} (T + t_{TOM})u_{RE} - \lambda w_{RE}^2 [(T + t_{TOM})^2 \sigma_{RE}^2 + \sigma_{TOM}^2 (u_{RE}^2 + \sigma_{RE}^2)] - w_{RE} C
\]

Therefore, the *periodic* risk-adjusted return for the illiquid real estate is

\[
r_{RE}^{Adj} (w_{RE}) = w_{RE} u_{RE} - \lambda w_{RE}^2 \left( (T + t_{TOM})\sigma_{RE}^2 + \frac{\sigma_{TOM}^2 (u_{RE}^2 + \sigma_{RE}^2)}{T + t_{TOM}} \right) - w_{RE} \frac{C}{T + t_{TOM}}
\]
For mathematical simplicity, we assume that the correlation between the real estate and financial assets is zero, i.e., \( \rho = 0 \). Although this assumption may seem strong, it is not entirely unreasonable as it has been well documented in the empirical studies that real estate historically has low correlation with financial assets. For example, using annual U.S. data from 1947 to 1982, Ibbotson and Siegel (1984) found real estate's correlation with S&P stocks to be -0.06, Worzala and Vandell (1993) estimate the correlation between NCREIF quarterly index from 1980 to 1991 and stocks of the same period to be about -0.0971. Eichholtz and Hartzell (1996) document correlations between real estate and stock indexes to be -0.08 for U.S., -0.10 for Canada, and -0.09 for U.K. Quan and Titman (1999) examined the correlation for 17 countries and, unlike earlier studies, they find a generally positive correlation pattern in most countries, but for the U.S. such positive correlation is insignificant.

Using the quarterly NCREIF and S&P500 Indices during the period of 1978Q1 to 2008Q3, we compute the correlations over different investment horizons and the results are displayed in Table 1.

### Table 1. Long-run Correlation Coefficients between NCREIF and S&P500

<table>
<thead>
<tr>
<th>Holding Period (years)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
<tbody>
<tr>
<td>NCREIF Overall vs. S&amp;P500</td>
<td>0.116</td>
<td>0.114</td>
<td>0.074</td>
<td>0.028</td>
</tr>
<tr>
<td>NCREIF Apartment vs. S&amp;P500</td>
<td>0.094</td>
<td>0.078</td>
<td>0.069</td>
<td>0.074</td>
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<tr>
<td>NCREIF Industrial vs. S&amp;P500</td>
<td>0.168</td>
<td>0.118</td>
<td>0.107</td>
<td>0.104</td>
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<tr>
<td>NCREIF Office vs. S&amp;P500</td>
<td>0.149</td>
<td>0.165</td>
<td>0.142</td>
<td>0.115</td>
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<tr>
<td>NCREIF Retail vs. S&amp;P500</td>
<td>-0.008</td>
<td>-0.079</td>
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</tbody>
</table>

From Table 1, we can conclude that the magnitudes of the correlations are actually quite consistent with previous studies in that all coefficients are rather small. There is somewhat a
trend that the correlations seem to slightly decline as holding period increases. This result, along with the findings by previous studies, reaffirms that real estate has low correlations with stocks and provides some reasonable justifications to the assumption of zero correlation between real estate and stocks in this study.  

Under the assumption of $\rho = 0$, the portfolio’s risk-adjusted return is the sum of the risk-adjusted return of each asset in the portfolio—the financial asset, the illiquid real estate and the risk-free asset. We can thus express the periodic risk-adjusted return for the portfolio as,

$$
R_{Port}^{adj} = R_L^{adj}(w_L) + R_{RE}^{adj}(w_{RE}) + (1 - w_L - w_{RE})r_f
$$

The optimization problem for the investor is to choose the optimal weights ($w_L^*$ and $w_{RE}^*$) and the optimal holding period $T^*$ to maximize $R_{Port}^{adj}$. By substituting Equations (5) and (11) into Equation (12), we can thus formulate the optimal asset allocations among the three assets at time zero as the following optimization problem.

$$
\max_{w_{RE}, w_L, T} \left\{ \left( u_L w_L - \lambda w_L^2 \sigma_L^2 \right) + (1 - w_{RE} - w_L)r_f + w_{RE} \left[ u_{RE} - \frac{C}{T + t_{TOM}} \right] - \lambda w_{RE}^2 \left( T + t_{TOM} \right) \sigma_{RE}^2 + \frac{\sigma_{TOM}^2}{T + t_{TOM}} \left( u_{RE}^2 + \sigma_{RE}^2 \right) \right\}
$$

The important difference between this model and the classical MPT is that the optimal policy in this model involves not only the optimal weights ($w_L^*$ and $w_{RE}^*$) but also the investor’s optimal holding period $T^*$, whereas the classical MPT is a single-period model. In addition, the alternative model treats the real estate differently from the financial asset in three aspects:

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8 It should be noted that the model developed in this study can be readily extended to the case of non-zero correlation.
(1) the model distinguishes the price behavior of real estate from that of financial assets by recognizing the fact that real estate returns are not i.i.d.; (2) the real estate performance requires a different measure from that of financial assets: the ex ante measure, which integrates price risk with real estate liquidity risk, is the appropriate real estate performance measure; (3) the model explicitly incorporates the high transaction cost of real estate.

Now suppose that real estate can be traded instantly with trivial time-on-market risk, i.e., \( t_{\text{TOM}} \approx 0 \) and \( \sigma_{\text{TOM}}^2 \approx 0 \), and further suppose that the transaction cost is negligible when selling real estate (\( C \approx 0 \)). Equation (13) can then be rewritten as

\[
\max_{w_{RE}, w_L, T} \left\{ w_{RE} u_{RE} + w_L u_L + (1 - w_{RE} - w_L) r_f - \lambda \left[ w_{RE}^2 \sigma_{RE}^2 + w_L^2 \sigma_L^2 \right] \right\}
\]

(14)

This is still different from the classical MPT as in Equation (3) with \( \rho = 0 \) because of the holding period \( T \) in Equation (14). However, we can readily show that the classical MPT holds only when real estate returns are i.i.d.

We next solve the optimization problem in Equation (13). First, we take the partial derivative of Equation (13) with respect to \( T \), to obtain,

\[
- \lambda w_{RE}^2 \sigma_{RE}^2 - \frac{\sigma_{TOM}^2 (u_{RE}^2 + \sigma_{RE}^2)}{(T + t_{TOM})^2} + \frac{w_{RE} C}{(T + t_{TOM})^2} = 0
\]

(15)

Similarly, the partial derivative of Equation (13) with respect to \( w_{RE} \) results in

\[
u_{RE} - r_f - \frac{C}{T + t_{TOM}} - 2 \lambda w_{RE} \left[ (T + t_{TOM}) \sigma_{RE}^2 + \frac{\sigma_{TOM}^2}{T + t_{TOM}} (u_{RE}^2 + \sigma_{RE}^2) \right] = 0
\]

(16)
Equations (15) and (16) have two questions and two unknown variables: \( w_{RE} \) and \( T \). We can solve the optimal real estate allocation \( (w_{RE}^*) \) and the optimal holding period \( (T^*) \) simultaneously. The existence of the optimal holding period suggests that the optimal mixed-asset allocation is related to the investor’s holding period. In contrast, the classical MPT only solves a single-period optimization problem and implies that the optimal asset allocation is independent of holding-period. To obtain a closed-form solution to Equations (15) and (16) is not easy, but we can examine some important properties of the optimal holding-period \( (T^*) \).

From Equation (15), we have

\[
\frac{\lambda w_{RE} \sigma_{TOM}^2 (u_{RE}^2 + \sigma_{RE}^2) + C}{(T + t_{TOM})^2} = \lambda w_{RE} \sigma_{RE}^2 .
\]  

(17)

Hence,

\[
T^* + t_{TOM} = \sqrt{\frac{\lambda w_{RE} \sigma_{TOM}^2 (u_{RE}^2 + \sigma_{RE}^2) + C}{\lambda w_{RE} \sigma_{RE}^2}} .
\]  

(18)

Therefore,

\[
\frac{\partial T^*}{\partial C} = \frac{1}{2 \lambda w_{RE} \sigma_{RE}^2 \sqrt{\frac{\lambda w_{RE} \sigma_{TOM}^2 (u_{RE}^2 + \sigma_{RE}^2) + C}{\lambda w_{RE} \sigma_{RE}^2}}} > 0
\]  

(19)

\[
\frac{\partial T^*}{\partial \sigma_{TOM}^2} = \frac{1}{2 \sqrt{\frac{\lambda w_{RE} \sigma_{TOM}^2 (u_{RE}^2 + \sigma_{RE}^2) + C}{\lambda w_{RE} \sigma_{RE}^2}}} \left( \frac{(u_{RE}^2 + \sigma_{RE}^2)}{\sigma_{RE}^2} \right) > 0
\]  

(20)
\[
\frac{\partial T^*}{\partial \sigma^2_{RE}} = \frac{1}{2 \sqrt{\frac{\lambda W_{RE} \sigma^2_{TOM} (u_{RE}^2 + \sigma^2_{RE}) + C}{\lambda W_{RE} \sigma^2_{RE}}} \left( - \frac{\sigma^2_{TOM} u_{RE}^2}{\sigma^4_{RE}} + \frac{C}{\lambda W_{RE} \sigma^4_{RE}} \right)} < 0 \quad (21)
\]

These results can be summarized in Theorem 1.

**Theorem 1.** Real estate transaction cost, liquidity risk, and price risk play a crucial role in determining the optimal holding period. In particular, we have

\[
\frac{\partial T^*}{\partial C} > 0; \quad \frac{\partial T^*}{\partial \sigma^2_{TOM}} > 0; \quad \frac{\partial T^*}{\partial \sigma^2_{RE}} < 0
\]

Other things being equal, a less liquid market (a larger \( \sigma^2_{TOM} \)) with a higher transaction cost (higher \( C \)) implies a longer optimal holding period. However, a higher price volatility (\( \sigma^2_{RE} \)) implies a shorter optimal holding period.

The important insight revealed by Theorem 1 is that the optimal holding period is determined by both systematic and non-systematic factors—market condition (\( \sigma^2_{TOM} \) and \( C \)) and property-specific risk (\( \sigma^2_{RE} \)). This is consistent with the findings by a recent study of Cheng, Lin and Liu (2010b), which concludes that the holding period of real estate is affected by both market conditions and property-specific performance. Theorem 1 is also consistent with the findings by Mayshar (1979) and Constantinides (1986), both of which find that even a small transaction cost results in a longer holding period with fewer assets (as opposed to the market portfolio).

With the optimal holding-period (\( T^* \)), we can readily solve the optimal weight of real estate (\( w_{RE}^* \)) by using Equation (16), which can be summarized in Theorem 2.
Theorem 2. The optimal weight on the real estate \( w_{RE}^* \) is given by,

\[
    w_{RE}^* = \frac{u_{RE} - r_f - \frac{C}{T^* + t_{TOM}}}{2\lambda[(T^* + t_{TOM})\sigma_{RE}^2 + \frac{\sigma_{TOM}^2}{T^* + t_{TOM}}(u_{RE}^2 + \sigma_{RE}^2)]}
\]  

(22)

In addition, \( w_{RE}^* \) is negatively related to real estate liquidity risk and its transaction cost, i.e.,

(a). \( \frac{\partial w_{RE}^*}{\partial C} < 0 \); (b). \( \frac{\partial w_{RE}^*}{\partial \sigma_{TOM}} < 0 \).

Two conclusions can be drawn from Theorem 2. First, Modern Portfolio Theory, which implicitly assumes that all assets have zero time-on-market risk, is not directly applicable to mixed-asset portfolios that include real estate. Second, by taking the liquidity risk, high transaction cost, and non-i.i.d. nature of real estate returns into consideration, we demonstrate that optimal real estate allocation is affected by liquidity risk, investor time horizon and real estate transaction cost. In particular, the optimal weight on the real estate asset is always less than that suggested by the classical MPT. In other words, the optimal real state allocation suggested by pervious literature is exaggerated.

5. A Numerical Example

In this section, we use an example to study how we can numerically solve both the optimal real estate allocation and the optimal holding period.

From Equation (22), we have
and from Equation (18), we have

\[
T^* + t_{TOM} = \sqrt{\frac{\lambda w_{RE}^* \sigma_{TOM}^2 (u_{RE}^2 + \sigma_{RE}^2) + C}{\lambda w_{RE}^* \sigma_{RE}^2}}. \tag{18'}
\]

Substituting Equation (18') into Equation (22'), we can obtain an equation of only one unknown variable (\( w_{RE}^* \)). To obtain a closed form solution for \( w_{RE}^* \) is difficult. However, we can find a numerical solution if we have all the model parameters: (1) real estate illiquidity, i.e., \( t_{TOM} \) and \( \sigma_{TOM}^2 \); (2) the periodic return and risk of real estate asset, \( u_{RE} \) and \( \sigma_{RE} \); and (3) the transaction cost (\( C \)), the risk-free rate (\( r_f \)), and the risk aversion parameter (\( \lambda \)). Now we discuss these parameters in turn.

1. **Real estate illiquidity.** Although not required for model development, the distribution of TOM needs to be reasonably assumed in order to estimate \( t_{TOM} \) and \( \sigma_{TOM}^2 \). Following Cheng, Lin, and Liu (2010a), we assume the distribution of TOM to be negative exponential for demonstration purposes. The negative exponential distribution has a simple mathematical property: its variance is equal to the square of its mean. Therefore, the variance of the time-on-market (TOM) is equal to the square of the expected TOM, i.e., \( \sigma_{TOM}^2 = t_{TOM}^2 \). In addition, based on information from the National Association of Realtors, the average TOM for the US residential market during the period from 1982 to 2010 was about 7.5 months. Considering the
fact that average marketing periods in commercial markets are often longer than those in residential markets, we thus choose $t_{tOM}$ ranging from 8 to 16 months.

2. *The periodic return* ($u_{RE}$) *and risk* ($\sigma_{RE}$) *of the real estate*. Using the annualized NCREIF property index during 1978 – 2008, we obtain an average annual return of 9.1% and a standard deviation of 8.3%, and use them as estimate for $u_{RE}$ and $\sigma_{RE}$. For demonstration purpose, we make no attempt to correct any smoothing bias in the NCREIF data. Although smoothing is a known issue with the NCREIF index, recent studies have found that the effect of smoothing is rather modest.\(^9\)

3. *The real estate transaction cost*. According to Collett, Lizieri and Ward (2003), the “round-trip lump-sum costs” of real estate can be approximately 7 to 8 percent of the value of an asset. In order to see how transaction cost affects the optimal holding period and real estate allocation, we choose $C$ ranging from 4% to 9%. In addition, we choose $r_f = 4.5\%$ based on the estimation of risk-free rate for the corresponding period.

4. *The risk aversion parameter* ($\lambda$). The presence of a risk-aversion parameter in the model suggests that optimal asset allocation is investor specific. However, although the market is full of investors with various degrees of risk-aversion, the competitive force of the marketplace ensures that only the highest bidder gets the deal, and these highest bidders are likely to be investors who have a particular degree of risk-aversion such that the property’s expected risk-adjusted return is the highest to them. The degree of risk-aversion of those highest bidding investors, therefore, can be implied from the observed market data. As mentioned before, studies in the past using the classical MPT have reported real estate allocations to be between 15% and

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\(^9\) See, for example, Cheng, Lin, and Liu (2011b).
If we take the middle point of this range, 27% for instance, it will imply a risk-aversion parameter \( \lambda = 12 \), according to Equation (4) with \( \rho = 0 \).

With the model parameters at hand, we can obtain numerical solutions to \( w_{RE}^* \) and \( T^* \). The results are reported in Tables 2 and 3.

### Table 2. The Optimal Holding Period of Real Estate

<table>
<thead>
<tr>
<th>Transaction Cost</th>
<th>Expected TOM</th>
<th>8 months</th>
<th>10 months</th>
<th>12 months</th>
<th>14 months</th>
<th>16 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td></td>
<td>3.0</td>
<td>3.2</td>
<td>3.4</td>
<td>3.6</td>
<td>3.8</td>
</tr>
<tr>
<td>5%</td>
<td></td>
<td>3.5</td>
<td>3.7</td>
<td>3.9</td>
<td>4.1</td>
<td>4.3</td>
</tr>
<tr>
<td>6%</td>
<td></td>
<td>4.1</td>
<td>4.3</td>
<td>4.5</td>
<td>4.6</td>
<td>4.8</td>
</tr>
<tr>
<td>7%</td>
<td></td>
<td>4.7</td>
<td>4.9</td>
<td>5.0</td>
<td>5.2</td>
<td>5.4</td>
</tr>
<tr>
<td>8%</td>
<td></td>
<td>5.3</td>
<td>5.5</td>
<td>5.6</td>
<td>5.8</td>
<td>5.9</td>
</tr>
<tr>
<td>9%</td>
<td></td>
<td>6.0</td>
<td>6.1</td>
<td>6.2</td>
<td>6.4</td>
<td>6.5</td>
</tr>
<tr>
<td>10%</td>
<td></td>
<td>6.6</td>
<td>6.7</td>
<td>6.8</td>
<td>6.9</td>
<td>7.1</td>
</tr>
</tbody>
</table>

Note: This table displays the optimal holding period \( (T^* + t_{TOM}) \) under various assumptions of transaction cost and TOM. All numbers are in years unless noted. Tables 2 is quite consistent with the finding by Cheng, Lin and Liu. (2010b)

### Table 3. The Optimal Weights for Real Estate

<table>
<thead>
<tr>
<th>Transaction Cost</th>
<th>Expected TOM</th>
<th>8 months</th>
<th>10 months</th>
<th>12 months</th>
<th>14 months</th>
<th>16 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td></td>
<td>5.87%</td>
<td>5.45%</td>
<td>5.05%</td>
<td>4.69%</td>
<td>4.37%</td>
</tr>
<tr>
<td>5%</td>
<td></td>
<td>4.97%</td>
<td>4.69%</td>
<td>4.42%</td>
<td>4.16%</td>
<td>3.92%</td>
</tr>
<tr>
<td>6%</td>
<td></td>
<td>4.29%</td>
<td>4.10%</td>
<td>3.91%</td>
<td>3.72%</td>
<td>3.54%</td>
</tr>
<tr>
<td>7%</td>
<td></td>
<td>3.76%</td>
<td>3.63%</td>
<td>3.49%</td>
<td>3.35%</td>
<td>3.21%</td>
</tr>
<tr>
<td>8%</td>
<td></td>
<td>3.34%</td>
<td>3.25%</td>
<td>3.15%</td>
<td>3.04%</td>
<td>2.93%</td>
</tr>
<tr>
<td>9%</td>
<td></td>
<td>3.01%</td>
<td>2.94%</td>
<td>2.86%</td>
<td>2.78%</td>
<td>2.69%</td>
</tr>
<tr>
<td>10%</td>
<td></td>
<td>2.73%</td>
<td>2.67%</td>
<td>2.62%</td>
<td>2.55%</td>
<td>2.48%</td>
</tr>
</tbody>
</table>

Note: This table displays the optimal weights of real estate given the corresponding optimal holding-periods in Table 2.

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10 Numerous academic studies since the 1980s often conclude that real estate should constitute 15% to 40% or more of a diversified portfolio. See, for example, Hartzell, Hekman and Miles (1986), Fogler (1984), Firstenberg, Ross and Zisler (1988), and Hudson-Wilson, Fabozzi and Gordon (2005), among others.
From Table 2, the expected optimal holding period for real estate ranges from three to seven years, depending on market condition (degree of illiquidity) and transaction cost. The optimal holding periods are longer when transaction costs are high and/or the time-on-market is expected to be longer, which is consistent with the findings of previous studies in the finance literature. The numerical results in Table 2 are also in line with the empirical finding by Gau and Wang (1994), and reasonably close to the finding by Collett, Lizieri and Ward (2003).

The main point of interest, of course, is the optimal real estate allocation displayed in Table 3, which ranges from 2.48% to 5.87%. Compared to previous findings based on the classical MPT, these real estate allocations are much more in line with the institutional portfolio reality. Clearly, the optimal allocation to real estate is determined by investment horizon, liquidity risk and transaction cost. Holding other things equal, higher transaction cost and higher liquidity risk (longer TOM) correlate with lower real estate allocations. These results suggest that the alternative model is able to provide a viable solution to the long-standing real estate allocation puzzle.

It may be of interest to compare our approach and results with a few recent studies that have attempted at resolving the real estate allocation puzzle. MacKinnon and Zaman (2009) examine the effect of long investment horizon on optimal real estate allocation using the VAR model developed in Campbell and Viceira (2005). Applying the model to U.S. property data, they find that, although real estate does exhibit long-run mean-reversion, the behavior is weaker than equities, which means real estate is almost as risky as equities in the long-run. But despite of this, they find the optimal allocations to real estate remains rather high (17-31%) and has the tendency to be higher as the investment horizon gets longer. Rehring (2011) adopt the same Campbell-Viceira method to examine U.K. commercial property data. Unlike MacKinnon and
Zaman (2009), he finds that the model-predicted real estate returns to exhibit decreasing variance in the longer horizon, which is consistent with his finding that the optimal real estate allocation increases rather rapidly as holding period prolongs. Therefore, neither study is able to resolve the real estate allocation puzzle. In contrast, using a somewhat different auto-regression model, Pagliari (2011) finds that, although real estate variance “decays” over the long-run, the variance of financial asset decays faster, thus making real estate relatively more risky as holding period gets longer. This finding, accompanied by what he finds to be increased correlation between private and public assets in the long run, causes real estate to be less appealing and thus carry less weight in mixed-asset portfolios. He suggests about 12% allocation in real estate for investors with a four-year investment horizon, which is still much higher than the reported 3-5% institutional reality. None of these three studies addresses real estate liquidity risk. In an effort to incorporate illiquidity into mixed-asset portfolio decision, Anglin and Gao (2011) develops a model to discuss the impact of liquidity and liquidity shock on portfolio. But they did not provide what the optimal real estate weight should be.

The differences between our approach and these afore-mentioned studies are obvious. First, the approaches of MacKinnon and Zaman (2009), Rehring (2011) and Pagliari (2011) essentially remain in the realm of empirically modifying the way MPT is applied to real estate, that is, they attempt to “fine-tune” the way we use MPT, not MPT itself. Our approach, on the other hand, is to modify the theory by extending it to explicitly accommodate the unique real estate features based on multi-period utility maximization. Second, while the studies above focus on exploring better empirical approaches for estimating the input data (long-run mean and variances) to MPT, we focus on developing an alternative model by explicitly incorporate unique features of real estate.
5. Conclusions

Modern Portfolio Theory (MPT) is a single-period asset allocation model. Its validity on multi-period portfolio decisions hinges on a critical assumption – an efficient market where asset returns are independent and identically distributed (i.i.d.) over time. Because real estate does not fit in this paradigm, the classical MPT needs to be extended to accommodate the more complex features of real estate and mixed-asset portfolio analysis. Building upon a series of recent research on real estate illiquidity and performance metrics, this paper synthesizes some of the latest advances in the literature to develop an alternative model that extends the classical MPT for mixed-asset portfolio analysis. Unlike many previous efforts that attempt to empirically solve the real estate allocation puzzle with ad hoc solutions, we provide a formal model that explicitly incorporates the three most unique features of real estate – horizon-dependent performance, liquidity risk and high transaction cost – into a multi-period mean-variance analysis. Using commercial real estate data, the alternative model produces a range of optimal real estate allocations that are quite in line with the reality of institutional portfolios.

It should be acknowledged that, although our model is able to succeed where the conventional approach has failed in resolving the long-standing real estate allocation puzzle, this work should only be viewed as a first step in the search for a more general portfolio theory for mixed-asset portfolio analysis. Many issues still remain. To mention a few: First, we have implicitly assumed that the investors are “normal” sellers who are not under any liquidity shock to force liquidation of real estate. A more general theory should incorporate seller heterogeneity and the possibility of liquidity shock into the model. Second, we do not consider the indivisibility or partial sale of real estate asset. We assume the entire real estate holding will be sold together. In reality, investors facing liquidity shock may need to liquidate only part of their
real estate holdings to satisfy the need for cash. This issue is discussed in Anglin and Gao (2011) to some extent. Third, while the alternative model breaks away from the efficient market paradigm in which asset prices are assumed to follow the random walk, it is still confined within the mean-variance framework, which implicitly assumes that investors have a symmetric aversion to price volatility. The reality, though, is that investor’s risk perception is often asymmetric. To the extent that asset returns are not jointly normally-distributed, the concept of downside-risk is an appealing risk metric (and more complex as well). This, of course, is a much broader issue that pertains to nearly all mainstream finance theories as well. Despite these simplifications, the empirical analysis and theoretical exploration accomplish the two modest objectives of this paper—to extend the MPT for mixed-asset portfolio analysis and to suggest a solution to the decades-old real estate allocation puzzle. While this line of work is certainly at the stage of early exploration, the prospect is promising.
References


