A Theory of Capital Structure, Price Impact, and Long-Run Stock Returns under Heterogeneous Beliefs

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Abstract

We analyze a firm's financing decision in an environment of heterogeneous beliefs and short sales constraints. We study a setting in which the insiders of a firm, owning a certain fraction of its equity, choose between equity, debt, or convertible debt to raise additional financing to implement a positive net present value project. The insiders’ objective is to maximize their long-run wealth conditional on their own beliefs about their firm’s future prospects. Market participants, each of whom have limited wealth, have heterogeneous beliefs about the firm’s long-run value. We analyze two different economic settings: one in which there are no market imperfections other than heterogeneous beliefs, and another in which there are also significant costs of issuing securities and of financial distress. We show that, in the absence of these two costs, the average belief of outsiders (“optimism”) and the dispersion in outsider beliefs are the crucial determinants of the firm’s security choice. When outsider beliefs are highly optimistic relative to firm insiders and the dispersion in outsider beliefs is high, the firm issues equity alone; when outsider beliefs are less optimistic (and less dispersed), the firm issues a combination of equity and debt. Neither straight debt alone nor convertible debt alone is optimal in this setting. Once the two costs are significant, we show that the firm always issues equity when outsider beliefs are optimistic and highly dispersed. If outsider beliefs are less optimistic, it issues a combination of equity and debt if issue costs are small; if issue costs are large, it either issues risk-free straight debt (if the investment amount required is small) or convertible debt (if this amount is large). Our model generates a pecking order of external financing under heterogeneous beliefs different from that of asymmetric information models. Further, it generates several unique testable predictions for the price impact of equity, debt, and convertible debt issues and for the firm’s long-run stock returns following the issuance of these securities.

JEL classification: G12, G14, G32

Keywords: Heterogeneous beliefs, Capital structure, Price impact, Long-run stock returns

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1 Introduction

Starting with Miller (1977), a number of authors have theoretically examined the stock price implications of heterogeneous beliefs and short sale constraints on stock valuations. Miller (1977) argues that when investors have heterogeneous beliefs about the future prospects of a firm, its stock price will reflect the valuation that optimists attach to it, because the pessimists will simply sit out the market (if they are constrained from short-selling). A number of subsequent authors have developed theoretical models that derive some of the most interesting cross-sectional implications of Miller’s logic. In an important paper, Morris (1996) shows that the greater the divergence in the valuations of the optimists and the pessimists, the higher the current price of a stock in equilibrium, and hence lower the subsequent returns. In another important paper, Duffie, Gârleanu, and Pedersen (2002) show that, even when short-selling is allowed (but requires searching for security lenders and bargaining over the lending fee), the price of a security will be elevated and can be expected to decline subsequently in an environment of heterogeneous beliefs among investors if lendable securities are difficult to locate. Another important implication of heterogeneous beliefs among investors is that it can lead to a significant amount of trading among investors: see, e.g., Harris and Raviv (1993), who use differences in opinion among investors to explain empirical regularities about the relationship between stock price and volume. However, while the implications of heterogeneous beliefs among investors for capital markets have been examined at some length (see, e.g., Lintner (1969) for one of the earliest contributions), the corporate finance implications of such beliefs have not been adequately studied (with some notable exceptions that we will discuss later: see, e.g., Allen and Gale (1999)). The objective of this paper is to fill this gap in the literature by developing a theory of capital structure, the price impact on equity of issuing various securities, and the long-run stock returns following the issuance of these securities under heterogeneous beliefs.

Several interesting questions arise in the above context. For example, does heterogeneity in beliefs between firm insiders and outsiders, and among outsiders, about the future prospects of a firm affect its security choice when raising external financing? Does increased investor optimism about a firm’s future prospects result in its being more likely to issue equity over debt, or a combination of the two? Under what situations is it optimal to issue convertible debt? Can heterogeneity in beliefs explain the price-impact (i.e., the abnormal return to equity upon a new security issue on the date the security is actually
issued) of a firm’s equity, debt, or convertible debt issue that traditional asymmetric information models cannot explain? In particular, what explains the fact that, while the long-run stock returns of both equity and debt issuers have been empirically shown to be negative, the long-run stock returns of equity issuers are significantly more negative than those of debt issuers? Finally, how does heterogeneity in beliefs affect the long-run stock returns to issuers of equity, debt, and convertible debt?

We answer these and other related questions in a heterogeneous beliefs framework. We analyze a firm’s financing decision in an environment of heterogeneous beliefs and short sales constraints. The setting we study is one in which insiders of a firm, owning a certain fraction of equity in the firm, choose between equity, debt, or convertible debt to raise external financing to implement a positive net present value project. Market participants, each of whom have limited wealth to invest in the firm, have heterogeneous beliefs about the long-run value of the firm, which differs from that of firm insiders. We can think of the average outsider belief as the level of “optimism” among outsiders, and the spread among outsider beliefs as the “dispersion” in their beliefs. The objective of firm insiders is to choose the security (or a combination of securities) to issue such that they maximize the long-run wealth of the firm’s current shareholders, conditional on their own beliefs.

The paper consists of four parts. In the first part of the paper (Section 3), we characterize the solution to the above described problem of the firm in a setting where there are no market imperfections (i.e., no security issue costs or costs of financial distress) other than heterogeneity in outside investors’ beliefs and differences in beliefs between firm insiders and outsiders. We refer to this as our basic model. We first compare the case where the firm chooses between equity alone, debt alone, and convertible debt alone. We show that, in the above setting, insiders of the firm will issue equity if and only if they expect the beliefs of the marginal outside investor to whom they will sell equity to be above their own beliefs about their firm’s prospects. This allows firm insiders to take advantage of outside investors’ optimism and sell overvalued equity to them. On the other hand, if the marginal outside investor’s belief is below insiders’ own beliefs, they will choose to issue debt instead, taking advantage of the fact that the valuation of debt is relatively insensitive to outsider beliefs. If, in this situation, the firm were

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1As in the existing literature on heterogeneous beliefs (see, e.g., Miller (1977) or Morris (1996)) we assume short-sale constraints throughout, so that the effects of differences in beliefs among investors are not arbitraged away. The above standard assumption is made only for analytical tractability: our results go through qualitatively as long as short selling is costly (see, e.g., Duffie, Gärleanu, and Pedersen (2002)).
to issue equity instead, the equity would be significantly undervalued relative to firm insiders’ beliefs. We show that issuing convertible debt is never optimal in this setting, since it will be dominated by either equity alone (if the marginal outsider belief is above insider beliefs) or debt alone (if the marginal outsider belief is below insider beliefs).

We then allow the firm to issue a combination of equity and debt as well as the other three securities in our basic model. We show here that if the marginal outside investor (in the case of pure equity financing) is optimistic enough that his belief is above a certain threshold belief, the firm chooses to issue equity alone. If, however, the marginal investor’s belief is below that threshold, the firm issues a combination of equity and debt, selling equity to the more optimistic outside investors and debt to the less optimistic ones. Further, the above implies that, the more optimistic or the more dispersed outsider beliefs are about the firm (or both), the more likely the firm is to issue equity alone rather than a combination of equity and debt (or to use a larger fraction of equity to raise the required investment amount when a combination of equity and debt is optimal). Finally, the greater the amount of external financing required by the firm, the lower the marginal investor’s belief in the case of pure equity financing, and therefore, the more likely the firm is to use at least some debt to raise this financing (and larger the proportion of debt issued when a combination of equity and debt is optimal).

In the second part of the paper (Section 4), we present our full-fledged model, incorporating costs of issuing each security (e.g., investment banking fees) and costs of financial distress into our basic model. We adopt the simplest possible assumptions for these two costs: a fixed cost of issuing each security, and a fixed cost incurred by the firm in the event of default on debt. Thus, issuing a combination of securities will incur larger total issue costs than issuing only one security. In this full-fledged model, we first compare situations under which the firm chooses between issuing equity alone, debt alone, and convertible debt alone. We show that, as in the basic model, issuing equity is optimal when the marginal outside investor’s belief is above that of firm insiders. However, if the marginal investor’s belief is below that of firm insiders, the firm issues either straight debt or convertible debt depending on the amount of external financing required. If this amount is small enough that, if the firm issues straight debt, there is no probability of default, then risk-free straight debt is the optimal choice of the firm. The intuition here is that, compared to equity or convertible debt, risk-free debt is less sensitive to outsider beliefs and
does not suffer any undervaluation. If, however, the investment amount required is large enough that any straight debt issued incurs a positive probability of default, then the firm prefers to issue convertible debt. This is because, while both risky straight debt and convertible debt will be undervalued to the same extent in this situation, issuing convertible debt with an appropriately chosen conversion ratio (i.e., where the equity and debt components of the convertible debt are chosen optimally) allows the firm to minimize costs of financial distress.

We then study the case in our full-fledged model where the firm may issue a combination of equity and straight debt as well as equity alone, straight debt alone, or convertible debt alone. Like a similar result in our basic model, we first show that, if the marginal outside investor is optimistic enough that his belief is significantly above firm insiders’ beliefs, the firm will raise the required amount by issuing equity alone. If, however, the marginal outsider’s belief is below firm insiders’ beliefs, then the firm will find it optimal to issue a combination of equity and straight debt (risky or risk-free) if the issue costs involved are small. In this case, the amount of equity versus debt issued to raise the required amount of financing depends on whether the marginal investor’s belief is above or below a certain threshold belief. If the marginal investor’s belief is above this threshold belief, the firm issues a combination of equity and risk-free debt; if the marginal investor’s belief is below this threshold belief, the firm issues a combination of a smaller amount of equity and a large amount of (risky) debt. The above threshold belief will depend on the firm’s cost of financial distress as well. Finally, if the issue costs are large enough that issuing a combination of securities is significantly costly, the firm prefers to issue convertible debt instead of a combination of equity and straight debt. The advantage to a firm of issuing a combination of equity and straight debt over convertible debt is that the former allows the firm to price-discriminate, selling equity to the more optimistic investors and straight debt to the more pessimistic ones; clearly, the firm has to sell convertible debt at a uniform price to the same group of investors. The disadvantage of selling a combination of equity and straight debt is the additional issue cost incurred by issuing two different securities, so that issuing convertible debt reduces the firm’s aggregate issue cost.

Our model induces a pecking order of external financing under heterogeneous beliefs that differs from this pecking order under asymmetric information models (see, e.g., Myers and Majluf (1984)), even in the absence of any other market imperfections. First, while issuing equity is the last choice
in an asymmetric information setting, it is the first choice in a setting where outside investors are optimistic enough that the marginal outside investor’s belief is above that of firm insiders. Second, under asymmetric information, if the firm can raise the required amount of external financing by issuing risk-free debt, this will be the most preferred security to issue; in contrast, even when the marginal outside investor is pessimistic relative to firm insiders, under heterogeneous beliefs, the firm prefers to issue a combination of equity and debt rather than risk-free debt alone to raise the required external financing. Third, under asymmetric information, if the firm cannot raise the entire amount of financing required by issuing risk-free debt, it will choose to issue risky debt (or other securities that are less information-sensitive than equity) to raise this amount; in contrast, under heterogeneous beliefs, the firm will raise the required amount by issuing a combination of equity and risky debt under these circumstances, even when the marginal outside investor is more pessimistic than firm insiders. We discuss the pecking order of external financing under heterogeneous beliefs in more detail in Section 5.

In the third part of the paper (Section 6), we study the price impact of equity, debt, and convertible debt issues, and study how the dispersion in investor beliefs affects the price impact of an equity issue. By price impact, we mean the abnormal return to the firm’s equity from the price prevailing before the external security issue to the price prevailing after the issue date (not the announcement date). Since the market is already aware that a security issue has been announced, one would expect a price impact of zero in the absence of heterogeneity in investor beliefs. We demonstrate that, in the presence of heterogeneous beliefs among outside investors, the price impact of an equity issue will be negative, while that of debt and convertible debt issues will be zero. The intuition for the fall in share price on the day of a new equity issue is that the marginal investor holding the firm’s equity after the equity issue turns out to be less optimistic compared to the beliefs of the marginal investor holding the firm’s equity prior to the equity issue, since, to sell additional equity to outsiders, the firm has to go down the belief ladder (i.e., it has to sell the new equity to outside investors who are less optimistic than those currently holding the firm’s equity). Further, we show that the price impact of an equity issue will be more negative if the dispersion in outsider beliefs is greater.

In the fourth (and final) part of the paper (Section 7), we characterize the long-run stock returns of

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2 In other words, asymmetric information models will not be able to generate a significant price impact for an equity issue, since there is no new information-flow from firm insiders to outsiders on the day of an equity issue.
firms following equity, debt, and convertible debt issues. First, our analysis implies that the long-run stock returns after an equity issue will be negative. Second, it implies that the long-run stock returns after a (straight or convertible) debt issue will also be negative, but algebraically greater (i.e., less negative) on average than those following an equity issue. Finally, our analysis predicts that the long-run stock returns following an equity issue will be more negative if the dispersion in outsiders' beliefs is greater. The intuition behind the long-run negative stock returns following an equity issue is that, as additional information about the firm’s operating performance becomes available to outside investors over time, the dispersion in outside investors’ beliefs about the firm’s prospects becomes smaller (as outside investors update on this information, their beliefs become more homogeneous); further, the larger the initial dispersion, the larger the reduction in the dispersion in outsiders' beliefs with the arrival of new information. This reduction in dispersion means that the belief of the marginal investor holding the firm’s equity will be lower after the arrival of new information compared to the marginal investor’s belief at the time of the equity issue, thus leading to a reduction, on average, in the price of the firm’s equity in the long run. Since the dispersion in outsiders investors' beliefs when a firm (optimally) chooses to issue equity will be greater than in situations where it (optimally) chooses to issue straight debt or convertible debt (ceteris paribus), the long-run stock return following an equity issue will be more negative than that following a straight debt or a convertible debt issue. It is worth noting that the above results on the relative magnitudes of the long-run stock returns following equity versus that following straight or convertible debt issues are unique to our model: for example, they cannot be generated by asymmetric information models.

It is important to note that, while outside investors and firm insiders have heterogeneous prior beliefs, all agents in our model are fully rational. As Morris (1995) has argued in an important paper, differences in beliefs are quite consistent with rationality. Thus, in our setting, rational agents with heterogeneous priors “agree to disagree” about the future cash flows of the firm. In other words, our model develops a theory of capital structure, price impact, and long-run stock returns in a fully rational setting with heterogeneous beliefs and short-sale constraints.

The implications of our model have motivated a recent empirical study by Chemmanur, Nandy, and

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Yan (2008). They test some of the above implications of our model using measures of investor optimism developed by Baker and Wurgler (2006), and the two standard proxies for heterogeneity in investor beliefs used in the literature, namely, the dispersion in analyst forecasts and abnormal share turnover. Their results can be summarized as follows. First, they find that the probability of a firm issuing equity rather than debt is increasing in both the level of optimism of outside investors and the dispersion in outsider beliefs. Second, they find that, consistent with our model predictions, the price impact on a firm’s equity is negative for an equity issue and zero for a debt issue (they find an average price impact of -2.8% around equity issues and zero percent around debt issues). These results are robust to controlling for the fact that the choice of security to issue (debt versus equity) is itself determined by the average level of outsider beliefs (optimism) and the dispersion in these beliefs. Finally, they find that, while the long-run stock returns to both debt and equity issuers are negative, the stock returns to equity issuers are significantly more negative than those to debt issuers. Further, they find that, the more optimistic outside investors are at the time of an equity issue and more dispersed their beliefs, the more negative the long-run (one and two year) stock returns are to the firm after equity issuance.

The empirical results of Chemmanur, Nandy, and Yan (2008) indicate that outside investor optimism and the dispersion in outside investor beliefs are indeed important determinants of the external financing choices made by a firm. Further, it is difficult to justify the existence of phenomena such as the negative price impact of an equity issue using other models (with rational investors) relying on imperfections such as asymmetric information: since no new information arrives in the market on the day of an equity issue beyond that released on the day of the announcement of the issue, the price impact should be zero in the absence of heterogeneous beliefs. Their results on the difference in long-run stock returns to equity and debt issuers are also difficult to explain in the absence of heterogeneous beliefs among outside investors (for example, it is difficult to generate long-run negative stock returns to firms issuing equity in a model driven by asymmetric information with rational investors).

The rest of the paper is organized as follows. In Section 2, we discuss the related literature. Section 3 outlines our basic model where heterogeneity in beliefs is the only market imperfection. Section

4See also Spiess and Affleck-Graves (1999), who document the long-run stock underperformance of debt and convertible debt issuers. While they do not compare the long-run stock returns of debt and equity issuers, one can draw the conclusion that the long-run stock returns following straight debt and convertible debt issues are less negative than those following equity issues by comparing their results to those of other studies documenting the long-run underperformance of equity issuers (see, e.g., Spiess and Affleck-Graves (1995), and Loughran and Ritter (1997)).

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4 presents our full-fledged model by incorporating issue costs and costs of financial distress. Section 5 summarizes the implications of our model with regard to the pecking order of security issuance of a firm under heterogeneity in beliefs among investors and between firm insiders and outsiders, and contrasts it to that arising from asymmetric information models. Section 6 analyzes the price impact following equity, debt, and convertible debt issues, and Section 7 analyzes the long-run stock returns of firms following the issuance of these securities. Section 8 describes some of the testable implications of our model, and Section 9 concludes. All proofs as well as lengthy parameter restrictions in various propositions are confined to the Appendix.

2 Related Literature

Our paper is related to several strands in the finance and economics literature. The first is the emerging literature on firm and investor behavior under heterogeneous prior beliefs. As discussed earlier, several authors have examined the asset pricing and trading implications of heterogeneous beliefs (see, e.g., Harrison and Kreps (1978), Morris (1996), Duffie, Gárleanu, and Pedersen (2002), and Chen, Hong, and Stein (2002) for contributions to this literature, and Scheinkman and Xiong (2004) for a review). Several authors have argued that prior beliefs should be viewed as primitives in the economic environment (Kreps (1990)) and that it may be appropriate for economists to allow for differences in prior beliefs to understand economic phenomena (Morris (1995), Allen and Morris (1998)).

Allen and Gale (1999) examine how heterogeneous priors among investors affect the source of financing (banks versus equity) of new projects. In contrast to their paper, our primary focus is on how heterogeneity in beliefs among investors affects the firm’s choice of security to issue. Dittmar and Thakor (2007) study a firm’s choice between issuing debt and equity when insiders and outsiders disagree about the firm’s choice of project to invest in. They assume that while equity holders disagree with insiders about project choice only based on their beliefs, debt holders may disagree with them for additional reasons (such as having a different objective function). The choice between equity and debt in their setting trades off the additional autonomy provided by equity holders to the manager in choosing projects against the tax benefits of debt: their model predicts that equity will be issued when

\footnote{See also Abel and Mailath (1994) who demonstrate that in certain special settings with heterogeneous beliefs, even projects that all investors believe have negative expected value if undertaken may be financed by these investors.}
there is less disagreement between insiders and outsiders and debt will be issued when this disagreement is more. Apart from the fact that there is no heterogeneity among outside investors’ beliefs in their setting (unlike our paper where such heterogeneity is crucial), the trade-off driving the debt versus equity choice in their model is quite different from ours. Further, their prediction regarding the conditions under which a firm will issue equity rather than debt is exactly opposite to that emerging from our analysis (in the sense that, while their model predicts that firms are more likely to issue debt when there is more disagreement between firm insiders and outsiders, our model predicts that firms are more likely to issue equity when there is greater heterogeneity in beliefs among outside investors, and the average outsider is more optimistic about the firm’s future prospects compared to firm insiders). Harris and Raviv (1993) use differences of opinion to explain empirical regularities about the relationship between stock price and volume. Finally, Garmaise (2001) analyzes the optimal design of securities by a cash-constrained firm facing investors with diverse beliefs: however, his focus is on comparing optimal designs when investors have rational beliefs (in the sense of Kurz (1994)) versus rational expectations.

Our model is also related to the empirical literature on heterogeneous beliefs and outside investor optimism. Apart from Chemmanur, Nandy, and Yan (2007), who directly test the predictions of our model and find consistent evidence, our model provides a fully rational explanation of some of the empirical literature on the relation between shareholder optimism and equity issues. For example, our model can explain the empirical results of Baker and Wurgler (2002) that the tendency of firms to issue equity when investors are most enthusiastic about firms’ earnings prospects (“market timing”) has a large, persistent effect on firms’ capital structures. Further, our model can explain these results without resorting to behavioral explanations, purely relying on a model where all agents are rational but where there is heterogeneity in prior beliefs both between firms insiders and outsiders and among outside investors. Finally, our paper is broadly related to the large theoretical and empirical literature on corporate capital structure driven by considerations other than heterogeneity in beliefs (see Harris...
3 The Basic Model

There are three dates in the model: time 0, 1, and 2. At time 0, insiders of a firm own a fraction $\alpha$ of the firm’s equity. The remaining $1-\alpha$ is held by a group of outside shareholders. The total number of shares in the firm is normalized to 1, so that $\alpha$ can be thought of as either the fraction of equity or the number of shares held by insiders. At time 1, the firm needs to raise an amount of $I$ from outside investors to fund the firm’s project.\(^8\) At time 2, the cash flows from the firm’s project are realized and become common knowledge to all market participants, which can be either $X^H$ or $X^L$, where $X^H > X^L > 0$.\(^9\)

There is a continuum of investors in the market, with an aggregate wealth of $W > 0$. Each investor has the same amount of wealth. Market participants have heterogeneous beliefs about the future (time 2) cash flows of the firm. Firm insiders believe that with probability $\theta_f$, the cash flow will be $X^H$, and with probability $1-\theta_f$, the cash flow will be $X^L$. We assume that $\theta_f X^H + (1-\theta_f) X^L > I$ so that firm insiders believe that the project has positive net present value. Potential (new) outside investors’ beliefs about the value of the firm are uniformly distributed over the interval $[\theta^m - d, \theta^m + d]$. We can think $\theta^m$ as the “average” or “mean” belief of outsiders, and $d$ as the dispersion in outsiders’ beliefs (we will sometimes refer to $\theta^m$ as the level of “optimism” among potential outside investors). We use $\theta$ to index an agent whose belief is $\theta$. Agent $\theta$ believes that with probability $\theta$ the firm’s time-2 cash flow will be $X^H$, and with probability $1-\theta$, the cash flow will be $X^L$.\(^{10}\) Clearly, existing investors who

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\(^8\)When outsiders’ valuation of the new project is greater than that of firm insiders, it may be beneficial for the latter to sell equity that raises an amount larger than $I$ to take advantage of the optimistic beliefs of outsiders with respect to the firm’s new project. In this case, the amount raised by the firm may exceed $I$, and will be that amount that maximizes the firm insiders’ surplus conditional on their own beliefs. The optimal amount raised will then depend on the following trade-off: as the firm sells more shares, insiders are able to capture value from a larger number of outsiders by selling them a larger number of shares at an overvalued price, but the price per share falls, since the belief of the marginal outside investor, which determines the price at which these shares are sold, will be less optimistic. However, given that the focus of this paper is not on the determination of the optimal amount of equity raised by the firm, but on the optimal choice of security to issue to raise a given investment amount, we assume here that the firm raises only the minimum amount required, $I$, to fund the firm’s project due to considerations of corporate control or other reasons we do not model here. Modeling the optimal amount of external financing raised complicates our model considerably without changing the qualitative nature of our results.

\(^9\)Note that the cash flows $X^H$ and $X^L$ are realized conditional on the project being financed and implemented.

\(^{10}\)Further, there are enough outsiders who believe that the project has positive net present value so that, for all securities among the menu of securities available to the firm, the marginal outside investor providing funding for implementing the project believes it to have net present value large enough that the firm insiders’ participation constraint is satisfied (i.e., they are better off implementing the new project by selling that security to outsiders than not implementing it).
already hold the firm’s stock at time 0 will be the most optimistic outside investors, and their beliefs are greater than \((\theta^m + d)\). We assume that the existing outside shareholders holding the outstanding stock in the firm have already exhausted their wealth so that they cannot buy any additional securities newly issued by the firm at time 1. The beliefs of insiders and outsiders about the cash flows of the firm are illustrated in Figure 1.

The menu of securities available to the firm consists of common equity, straight debt, and convertible debt. In the basic model (Section 2), we assume that the firm does not incur any frictional cost of issuing securities (i.e., no issue or underwriting costs). Further, in the basic model we also assume that the firm does not incur any deadweight cost of financial distress even if it is in default on its promised payment on debt issued (either straight debt or convertible debt). We will introduce both the above costs in our full-fledged model (starting with Section 3). Throughout the paper, we assume that all investors are subject to a short-sale constraint: i.e., no short selling in the firm’s security is allowed in the economy. We also assume that the amount of total wealth available to all investors is relatively large compared to the amount of money the firm wants to raise, so that \(W > 2I\).

The objective of firm insiders is to choose the optimal security to issue such that they maximize the expected time-2 payoff of current shareholders, based on firm insiders’ belief, \(\theta^f\).\(^{11}\) There is a risk-free asset in the economy, the net return on which is normalized to 0. All agents are risk-neutral. Thus,

\(^{11}\)Since firm insiders hold a fraction \(\alpha\) of the firm’s shares, maximizing the value of current shareholders is equivalent to maximizing the value of shares held by firm insiders.
At time 0, insiders of a firm own a fraction \( \alpha \) of the firm's equity. The remaining \( 1 - \alpha \) is held by a group of outside shareholders.

The total number of shares outstanding in the firm is normalized to 1.

The required amount of investment \( I \) for the project is raised from outside investors by issuing either equity, or straight debt, or convertible debt, or a combination of these securities.

Figure 2: Sequence of Events

Firm insiders choose the optimal security, \( S \), to maximize the following objective function

\[
\max_S E_1[CF_2^{equity}|S, \theta^f]
\]  

where \( E_1[CF_2^{equity}|S, \theta^f] \) is the time-1 expected value (according to firm insiders' belief) of the time-2 cash flows to the current equity holders of the firm, conditional on issuing security \( S \), where \( S \) can be either equity, straight debt, or convertible debt. The sequence of events in the basic model is given in Figure 2.

3.1 The Case where the Firm Issues Equity alone

We first analyze the case where the firm is constrained to issue only equity to outside investors in order to raise the required amount of investment \( I \) at time 1.\(^{12}\) The issuing firm maximizes its expected

\(^{12}\)We assume that, in the case where the firm raises its external financing through an equity issue, current shareholders do not participate in the issue, either as buyers or sellers. As discussed earlier, a wealth constraint will prevent current shareholders from buying any additional equity in the firm. We also assume that current shareholders are affiliated with firm insiders, and thus prevented from selling into the equity issue (e.g., through lock-up provisions). However, it should be noted that, even if there is a limited amount of selling into the equity issue by current shareholders, the qualitative nature of our results do not change, as long as such selling by current shareholders does not constitute a significant fraction
payoff based on insiders’ own belief about the firm’s future cash flow at time 2. The following lemma characterizes the main properties of this equity issue.

**Lemma 1.** When the issuing firm chooses to issue common stock alone to raise the amount of investment $I$, it has to issue a total of

$$E_1 = \frac{I}{\hat{\theta}X^H + (1 - \hat{\theta})X^L - I}$$

shares of new stock to outside investors at the price $P_{E_1}^{\text{Equity}} = \hat{\theta}X^H + (1 - \hat{\theta})X^L - I$, where the marginal investor in the firm’s equity has the belief $\hat{\theta} = \theta^m + d - \frac{2d}{W}$ about the firm’s cash flow at time 2. The equity price $P_{E_1}^{\text{Equity}}$ is decreasing in the amount of investment $I$.

Under heterogeneous beliefs and short-sale constraints, the firm will offer equity only to the most optimistic investors in the market. The (uniform) price at which the firm sells shares to outsiders depends on the belief of the marginal outside investor in the firm’s equity, denoted by $\hat{\theta}$. This marginal investor is determined by starting with the most optimistic outside investor willing to invest in the firm (whose belief is given by $(\theta^m + d)$) and working down the ladder of outside investors’ beliefs until the entire amount $I$ required for investment in the firm is raised by selling equity. This means that the price of the firm’s equity depends on two factors. The first factor is the average belief of investors in the market: the higher this average belief, the more optimistic the marginal investor’s beliefs. The second factor that affects the price is the dispersion in outside investors’ beliefs: holding the average belief constant, a higher dispersion in outside investors’ beliefs means that the marginal investor’s beliefs are more optimistic. Finally, the higher the amount of money the firm needs to raise from outsiders, the lower down the belief-ladder the firm needs to go, and therefore the less optimistic the marginal investor holding the firm’s equity subsequent to the equity issue will be. Since the less optimistic the marginal investor holding the firm’s equity, the lower the price of the firm’s equity will be, this implies that a larger investment amount results in a lower equity issue price.

### 3.2 The Case where the Firm Issues Straight Debt alone

We now assume that the firm issues straight debt alone to raise the required investment amount $I$. We normalize the face value of each unit of straight debt is 1 (in other words, the price of debt we derive of the equity issue. Introducing such selling only introduces additional complexity into our model without generating commensurate insights.
here is the price per dollar of face value). The price of the firm’s straight debt is determined by the belief of the marginal investor in the firm’s debt. The following lemma characterizes the main properties of this debt issue.

Lemma 2. When the issuing firm chooses to issue straight debt alone to raise the required amount of investment $I$:

i) If $I > \frac{X}{L}$, the firm issues risky straight debt. The price of each unit of debt is given by:

$$PD_1 = \frac{\hat{\theta}I}{I - (1 - \hat{\theta})X^L}. \quad (3)$$

The firm needs to issue a total of

$$F = \frac{I - (1 - \hat{\theta})X^L}{\hat{\theta}} \quad (4)$$

units of straight debt to raise the amount $I$, where the marginal investor in the firm’s debt has the belief $\hat{\theta} = \theta^m + d - \frac{2dI}{W}$ about the firm’s cash flow at time 2.

ii) If $I \leq \frac{X}{L}$, the firm issues risk-free straight debt. The price $PD_1$ of each unit of debt is 1, and the firm needs to issue a total of $F = I$ units of straight debt to raise the required amount of financing $I$.

When the firm issues straight debt alone in order to raise the required amount of new financing $I$, it raises these funds from the same group of investors as in the above case where it issues equity alone. In other words, similar to an equity issue, the firm starts with the outside investor who is the most optimistic about the firm’s future cash flows and works down the ladder of outsiders’ beliefs until the entire amount $I$ is raised by selling straight debt. Therefore, Lemma 2 shows that the marginal investor in the firm’s debt is the same as the marginal investor in its equity if the firm were to issue equity alone instead of debt alone (as in Lemma 1). We therefore denote the belief of this marginal debt investor also by $\hat{\theta}$, which is equal to $\theta^m + d - \frac{2dI}{W}$.\footnote{One should note that, unlike an equity issue, the straight debt issue has no impact on the price of the firm’s existing equity since the firm’s marginal equity investor is the same as before the straight debt issue.}

The price at which each unit of straight debt is sold by the firm, denoted by $PD_1$, is the price at which the marginal investor breaks even, given his belief $\hat{\theta}$. The firm issues $F$ units of straight debt such that it is able to raise the entire investment amount $I$. One should note that in the case of risk-free debt, the security price is independent of the marginal investor’s belief $\hat{\theta}$. However, in the case of risky debt, when the required amount of investment $I$ is large, the debt price is also sensitive to the marginal outside investor’s belief $\hat{\theta}$, though this sensitivity is much smaller than in the case of the price of equity.
3.3 The Case where the Firm Issues Convertible Debt alone

We now analyze the case where the firm issues convertible debt alone to raise the required amount of investment \( I \). The terms of the convertible debt security are as follows: each unit has a face value of 1 and is sold at a price \( p \) at time 1; each unit of convertible debt can be converted into \( x \) shares of equity at time 2 if the investor chooses to exercise this option. We assume that there are restrictions on the conversion ratio \( x \) so that convertible debt will be a truly hybrid security between equity and straight debt (we specify these in lemma 3).

We normalize the number of shares of equity outstanding in the firm before it issues the convertible debt to 1. To raise the amount \( I \), the firm has to issue a total of \( I/p \) units of convertible debt. If investors decide to convert into equity at time 2, then the value of each unit of convertible debt from conversion is \( \frac{x}{1+xI/p}V \), where \( V \) is the firm’s market value at time 2, which is equal to either \( X^H \) or \( X^L \). Investors will convert to common stock only if the payoff from conversion is greater than the face value of the convertible debt, 1, that is, if

\[
\frac{x}{1+xI/p}V > 1, \quad (5)
\]

or equivalently

\[
\frac{V}{1+xI/p} > \frac{1}{x}. \quad (6)
\]

The quantity on the RHS of the inequality, \( \frac{1}{x} \), is the conversion price of the convertible debt, whereas the LHS of the inequality corresponds to the firm value per share after the conversion. The following lemma characterizes the optimal conversion ratio \( x \) and the price \( p \) of the convertible debt, if the firm issues convertible debt alone in order to raise the required amount of investment financing \( I \).

**Lemma 3.** Let \( x < \frac{\hat{\theta}X^H+(1-\hat{\theta})X^L}{X^L(\hat{\theta}X^H+(1-\hat{\theta})X^L-I)} \). Further, let \( x > \frac{1}{X^H-I} \) if \( I \leq X^L \), and \( x > \frac{\hat{\theta}}{\hat{\theta}X^H+(1-\hat{\theta})X^L-I} \) otherwise.\(^{14}\) If the firm decides to issue convertible debt alone to raise the required investment amount of \( I \), then:

\(^{14}\)These parametric restrictions ensure that the convertible debt is truly a hybrid of equity and straight debt. If the conversion ratio \( x \) is too high, new investors holding convertible debt will find it optimal to convert into equity at time 2 regardless of the value of the firm’s cash flow. Thus, there will be practically no difference between convertible debt and equity. Similarly, if the conversion ratio \( x \) is too low, there will be practically no difference between convertible debt and straight debt. Thus, convertible debt will be a truly hybrid security between equity and straight debt, only if the conversion ratio \( x \) is between a lower bound and an upper bound. Existing shareholders can also impose an upper bound on the conversion ratio simply due to their concerns about maintaining control of the firm. Please see footnote 22 for a numerical example on convertibles.
When outsiders are optimistic about the firm on average and their beliefs are more dispersed so that the marginal investor’s belief \( \hat{\theta} \) satisfies \( \hat{\theta} = \theta^m + d - \frac{2dI}{W} \geq \theta^f \), it is optimal for the firm to set the conversion ratio at \( x = \bar{x} \) given in (A.27). In this case, the firm needs to issue

\[
F = \frac{I}{p}
\]  

(7)

units of convertible debt, where the convertible debt price \( p = \bar{p} \) is given by (A.29).

When outsiders are pessimistic about the firm on average and their beliefs are less dispersed so that the marginal investor’s belief \( \hat{\theta} \) satisfies \( \hat{\theta} = \theta^m + d - \frac{2dI}{W} < \theta^f \), it is optimal for the firm to set the conversion ratio at \( x = x \) given by (A.23). In this case, the firm needs to issue

\[
F = \frac{I}{p}
\]  

(8)

units of convertible debt, where the convertible debt price \( p = \bar{p} \) is given by (A.26).

When the firm issues convertible debt alone in order to raise the required amount of new financing \( I \), it raises these funds from the same group of investors as in the above cases where it issues equity alone or straight debt alone. Thus, the marginal investor in the firm’s convertible debt is determined by starting with the outside investor who is most optimistic about the firm’s future cash flows and working down the ladder of outsider beliefs until the entire amount \( I \) required for investment in the firm is raised by selling convertible debt. Therefore, the belief of the marginal outside investor in the firm’s convertible debt is identical to the belief of the marginal investor in the above cases where the firm issues equity or straight debt alone. We therefore denote the belief of the marginal investor in the firm’s convertible debt also by \( \hat{\theta} \) which is equal to \( \theta^m + d - \frac{2dI}{W} \). Given the price \( p \), the conversion ratio \( x \), and the expected cash flows offered by each unit of the convertible debt, the marginal investor breaks even in return for his investment in the firm.

The above lemma shows that the difference in beliefs between firm insiders and outside investors plays a critical role in the pricing and the design of the convertible debt security. When outsiders are sufficiently more optimistic about the firm’s future cash flows on average (i.e., the outsiders’ average belief \( \theta^m \) is higher) and their beliefs are more dispersed, the marginal outside investor with belief \( \hat{\theta} \) will also be more optimistic about the firm’s future cash flows than firm insiders (i.e., \( \hat{\theta} \geq \theta^f \)). In this case, we show that it is optimal for firm insiders to set the conversion ratio \( x \) at the highest possible value \( \bar{x} \), and thereby maximize the equity component of the convertible debt. This makes sense since this
equity component will be overvalued by the marginal outside investor relative to firm insiders’ belief, and therefore, firm insiders will seek to benefit from capturing the outsiders’ optimism on behalf of the existing shareholders by maximizing the equity component of convertible debt. The price of the convertible debt in this case is given by equation (7) in this scenario.\textsuperscript{15} On the other hand, when outsiders are less optimistic about the firm’s future cash flows on average and their beliefs are less dispersed, the marginal outside investor will also be less optimistic about the firm’s future cash flows than firm insiders. In this case, it is optimal for firm insiders to set the conversion ratio at the lowest possible value $x$ in order to minimize the equity component of the convertible debt, since this component will now be undervalued relative to firm insiders’ belief. The price of the convertible debt in this case is then given by equation (8).\textsuperscript{16}

3.4 The Choice between Equity alone, Straight Debt alone, and Convertible Debt alone

In this subsection, we assume that the firm has the choice of issuing either equity alone, debt alone, or convertible debt alone in order to finance the project. Thus, we assume that the firm cannot issue a combination of different securities, and the entire investment amount $I$ needs to be raised by issuing only one type of security. The following proposition characterizes the conditions under which the firm chooses to issue each security.

**Proposition 1. (The Choice between Equity alone, Straight Debt alone, and Convertible Debt alone)** Let $\hat{\theta}_X H + (1 - \hat{\theta}) X_L > I$ so that the firm’s project has positive NPV based on the marginal outside investor’s belief $\hat{\theta} = \theta^m + d - \frac{2dI}{W}$. If the firm can issue only one type of security in order to raise the required amount of $I$ for the project from outside investors, then:

(i) The firm will choose to issue equity alone if outsiders are optimistic about the firm on average, and their beliefs are very dispersed so that the marginal outside investor is more optimistic than firm insiders, i.e., if $\hat{\theta} = \theta^m + d - \frac{2dI}{W} > \theta^f$;

(ii) The firm will choose to issue straight debt alone if outsiders are pessimistic about the firm on average, and their beliefs are not so dispersed so that the marginal outside investor is less optimistic than firm insiders, i.e., if $\hat{\theta} = \theta^m + d - \frac{2dI}{W} \leq \theta^f$;

\textsuperscript{15}However, we will later show in Proposition 1 that if the firm is unconstrained with regard to its choice of security, so that it can choose among equity, straight debt, and convertible debt, it will always choose to issue equity rather than convertible debt under this scenario, since equity will be even more overvalued than convertible debt in this situation.

\textsuperscript{16}One should again note that, unlike an equity issue, the convertible debt issue has no impact on the price of the firm’s existing equity since the firm’s marginal equity investor remains the same before and after the convertible debt issue.
(iii) The firm will never choose to issue convertible debt since convertible debt will be dominated by either equity alone or straight debt alone, depending on outsiders’ beliefs.

As we discussed earlier when we considered the issuance of each individual security to fund the new project under lemmas 1, 2, and 3, the marginal outside investor in the firm’s securities is determined by starting with the most optimistic investor willing to invest in the firm and working down the ladder of outsider beliefs until the entire investment amount of \( I \) is raised. Therefore, in each case, we showed that the marginal outside investor has the same belief \( \hat{\theta} \) about the firm’s future cash flow at time 2: i.e., \( \hat{\theta} = \theta^m + d - \frac{2dI}{W} \), regardless of the particular security the firm chooses to issue at time 1 in order to externally raise the total amount of \( I \). However, since each security has its own unique payoff structure depending on the state of the world at time 2, the expected payoffs of insiders and existing shareholders will be different across all three different securities despite the fact that the marginal outside investor’s belief does not depend on the type of security issued by the firm. The above proposition shows that the sensitivity of the value of a particular security to the beliefs of outside investors about the firm’s future cash flows is a critical factor affecting the choice of security to raise external finance for the firm.

In the case where outside investors are more optimistic about the firm’s future cash flows on average, i.e., the average outsider belief \( \theta^m \) is relatively high, and their beliefs are more dispersed, i.e., the dispersion \( d \) in beliefs across outside investors is relatively large, the belief of the most optimistic new investor in the firm’s security (given by \( (\theta^m + d) \)) is likely to be significantly higher than that of firm insiders, i.e., \( \theta^f \). Then, starting with this most optimistic investor willing to invest in the firm and working down the ladder of outsider beliefs until the entire investment amount of \( I \) is raised, the belief of the marginal outside investor, \( \hat{\theta} \), should also be more likely to be above that of firm insiders. In this situation, all these securities (equity, straight debt, or convertible debt) will be overvalued relative to firm insiders’ belief. However, since equity is the most sensitive security to outsider beliefs, it will also be the most overvalued security based on insiders’ beliefs if the marginal outside investor is more optimistic than firm insiders.\(^\text{17}\) Therefore, in this scenario, we show that the firm chooses to issue equity alone instead of the other two securities in order to best capture outside investors’ optimism. Figure

\(^{17}\)Note that if we rank each security based on its value sensitivity to outsiders’ beliefs about the firm’s future cash flows, equity is the most sensitive security, since its payoffs are perfectly positively correlated with the state of the world. Straight debt is the least sensitive security to investor beliefs, since it promises the repayment of a fixed face value \( F \) unless the firm defaults in the future. Convertible debt which is a hybrid of straight debt and equity ranks in between the two with respect to its price sensitivity to outsider beliefs.
Figure 3: Beliefs of insiders and outsiders in the scenario where the firm chooses to issue equity alone.

On the other hand, when outside investors are more pessimistic about the firm’s future cash flows on average, and their beliefs are less dispersed, the belief of the most optimistic outside investor will not be as optimistic as in the scenario discussed in the previous paragraph. In this case, if the marginal investor’s belief, $\hat{\theta}$, is below that of firm insiders, and the firm chooses to sell equity, its equity will be substantially undervalued relative to the insiders’ belief. Therefore, the firm will choose to issue straight debt since this security is less sensitive to outsider beliefs than either equity or convertible debt, and therefore the least undervalued. Figure 4 illustrates the scenario (the beliefs of insiders and outsiders) under which the firm chooses to issue straight debt.

The above proposition shows that, in the absence of issue costs and costs of financial distress, issuing convertible debt is never optimal for the firm in either of the above two scenarios. When the marginal outside investor is more optimistic than firm insiders, i.e., $\hat{\theta} \geq \theta^f$, the equity component of convertible debt will be overvalued. In other words, the embedded option to convert into equity and thereby share the upside of the firm will be overvalued by the marginal outside investor who is more optimistic than firm insiders. However, in this case, firm insiders would be even better off by issuing common equity instead of issuing convertible debt with an overvalued equity component. One should note that there is
always a positive probability that convertible debt holders will find it optimal not to convert into equity and instead prefer to claim the fixed face value of the convertible debt promised by the firm. Hence, the payoff of the convertible debt is less than perfectly positively correlated with the future cash flows of the firm at time 2, whereas the equity payoff is perfectly positively correlated. Thus, the valuation of equity is even more sensitive to outside investors’ beliefs than that of convertible debt. Therefore, if the marginal outside investor is more optimistic than firm insiders, equity will be even more overvalued than convertible debt, and insiders can capture outside investors’ optimism better by issuing equity rather than convertible debt.

On the other hand, when the marginal outside investor is more pessimistic than firm insiders, i.e., \( \hat{\theta} < \theta^f \), the equity component of convertible debt will be undervalued. In this case, while firm insiders are better off issuing convertible debt rather than equity (since the undervaluation of equity is more severe than that of convertible debt), they are even better off by issuing straight debt rather than convertible debt. Since straight debt always promises the repayment of a fixed face value no matter how good the state of the world is, its undervaluation based on insiders’ belief will be less severe than that of convertible debt which gives the investor the right to participate in the upside potential of the firm by converting to equity. In other words, if the marginal outside investor is more pessimistic about the firm’s upside potential than firm insiders, this conversion option will be undervalued based on insiders’
belief, and therefore, it will be optimal for the firm to issue straight debt which will be less undervalued.

In summary, convertible debt is a dominated security (in the absence of issue costs and costs of financial distress).

3.5 The Case of External Financing by Issuing a Combination of Securities

We now consider the possibility that the firm can issue a combination of debt and equity to raise the necessary financing for its project.

Proposition 2. (The Choice between Equity alone, Straight Debt alone, and a Combination of Straight Debt and Equity) Let $\theta^f < \theta^m + d$.

(i) The firm will choose to issue equity alone if outsiders are very optimistic about the firm on average, and their beliefs are very dispersed so that the marginal outside investor’s belief $\hat{\theta}$ is above the upper threshold belief $\theta_1$, i.e., $\hat{\theta} \geq \theta_1$.

(ii) The firm will choose to issue a combination of risk-free straight debt and equity if outsiders are moderately optimistic about the firm on average, and their beliefs are moderately dispersed so that the marginal outside investor’s belief $\hat{\theta}$ is between the lower threshold belief $\theta_2$ and the upper threshold belief $\theta_1$, i.e., $\theta_2 \leq \hat{\theta} < \theta_1$.

(iii) The firm will choose to issue a combination of risky straight debt and equity if outsiders are pessimistic about the firm on average, and their beliefs are not very dispersed so that the marginal outside investor’s belief $\hat{\theta}$ is below the lower threshold belief $\theta_2$, i.e., $\hat{\theta} < \theta_2$.

(iv) It is never optimal for the firm to issue debt alone.

When the average outside investor is very optimistic about the firm’s future cash flows and outsiders’ beliefs are very dispersed, the marginal outside investor will be willing to pay a relatively high price for the firm’s equity with respect to the insiders’ beliefs. In this case, the above proposition shows that it will be optimal for the firm to issue equity alone in order to capture the high degree of optimism of the marginal outside investor. Issuing equity alone clearly dominates issuing debt alone in this scenario, since the value of equity is much more sensitive to the beliefs of outside investors than the value of debt, and therefore, if outside investors are much more optimistic than firm insiders, equity will be much more overvalued than straight debt based on insiders’ beliefs. Issuing equity alone in this case also dominates issuing a combination of debt and equity because of the following trade-off the firm faces when issuing a combination of debt and equity. While raising part of the total funding $I$ through debt issuance will increase the equity price (since less money is raised through equity issuance), the debt price will not be
as sensitive to the optimism in outsiders’ beliefs as the equity price. When the marginal outside investor has a very optimistic view of the firm even in the case where the entire amount of funding is raised by issuing equity, issue equity alone better captures the optimism of outside investors than issuing a combination of equity and debt. Thus, firm insiders will choose to maximize the overvaluation benefit they capture due to the large difference in equity valuation based on the beliefs of insiders and those of the marginal outside investor.

When the average outside investor is not so optimistic about the firm’s future, and outsiders’ beliefs are not so dispersed, issuing equity alone to raise the entire funding will hurt the firm’s existing shareholders (and insiders), if the marginal outside investor has a lower valuation of the firm than the insiders do. Similarly, if the marginal outside investor’s valuation of the firm is only slightly higher than the insiders’ valuation of the firm (assuming that the firm issues equity alone), the firm will be better off by raising part of the total funding \( I \) through debt, thereby increasing the equity price paid by the marginal equity investor. In such cases, the above proposition shows that it is optimal for the firm to issue a combination of debt and equity to raise the required funding \( I \) for the firm’s project. Starting with the most optimistic outside investor with belief \( \theta_m + d \) and going down the ladder of outsider beliefs, the firm can raise some money \( (I - I_D) \) by issuing equity to the most optimistic investors and the rest \( I_D \) by issuing debt to the less optimistic investors until the entire amount of \( I \) is raised. In this way, as long as the most optimistic outside investor is more optimistic than firm insiders, i.e., \( \theta_I < (\theta_m + d) \), the firm can still capture and benefit from the optimism of the most optimistic outsiders by issuing some equity. On the other hand, by issuing some debt simultaneously, the firm will not be hurt by the beliefs of the less optimistic and downright pessimistic outside investors.

The above proposition shows that, to raise a given level of required investment funding \( I \), the firm will prefer to issue equity alone if the marginal outside investor’s belief \( \theta \) exceeds the threshold value of \( \theta_1 \), which may be above the insiders’ belief \( \theta_I \). This condition will be satisfied when the average outside investor is very optimistic about the firm’s future cash flows (\( \theta_m \) is high relative to \( \theta_I \)) and outsiders’ beliefs are very dispersed (the dispersion in outsiders’ belief \( d \) is large). As the average optimism of outsiders \( \theta_m \) and/or the dispersion in their beliefs \( d \) decrease, the marginal outside investor becomes less optimistic. Hence, the cost of issuing undervalued equity increases, and the firm chooses to issue
some amount of debt ($I_D$) in combination with selling equity in order to reduce this undervaluation cost. As long as the marginal outsider investor is moderately optimistic (i.e., $\theta_2 \leq \theta < \theta_1$), the size of the debt issue will be small, and the firm will choose to issue a combination of risk-free debt ($I_D \leq X_L$) and equity. However, if the marginal outsider is sufficiently pessimistic (i.e., $\theta < \theta_2$), the firm will increase the size of its debt issue, and choose to issue a combination of risky debt ($I_D > X_L$) and equity in order to strike the optimal balance between the firm’s objective to reduce the cost of issuing undervalued equity to pessimistic outsiders by selling them some debt and its objective of capturing the optimism of the most optimistic outsiders by selling them some equity.

If it is feasible for the firm to issue a combination of debt and equity, issuing debt alone is never optimal since this fails to capture the optimism of those investors with very optimistic beliefs about the firm. If there exist some very optimistic outside investors who value the firm higher than the insiders, the firm can benefit from the optimism of these outsiders by issuing some equity to them. Thus, even if the average outside investor is not so optimistic about the firm’s future prospects, issuing a combination of equity and debt dominates issuing debt alone as long as there exists some heterogeneity in outsiders’ beliefs and the most optimistic outside investor is more optimistic than firm insiders.\(^{18}\)

**Proposition 3. (The Choice between Convertible Debt alone and a Combination of Straight Debt and Equity)** Convertible debt is always dominated by a combination of straight debt and equity.

The above proposition shows that (in the absence of issue costs) convertible debt is dominated by a combination of straight debt and equity. When the firm issues a combination of straight debt and equity, it can sell equity to the most optimistic outside investors at a relatively high price and sell debt to the less optimistic outsiders. Starting with the most optimistic outside investor with belief ($\theta^m + d$) and going down the ladder of outsider beliefs, the firm can raise some money ($I - I_D$) by issuing equity to the most optimistic investors and the rest ($I_D$) by issuing debt to the less optimistic investors until the entire amount of $I$ is raised. In other words, there will be two different marginal investors, with the marginal equity investor having a more optimistic belief $\tilde{\theta}$ about the firm than the marginal debt investor.

\(^{18}\)However, this particular result is true only under the assumption that there are no issue costs. When issue costs are significant (as we assume in later sections), we will show that it can be optimal for the firm to issue straight debt alone as well as equity alone under certain conditions.
investor with belief $\tilde{\theta}$. The relationship between these two beliefs is given by the following equation:

$$
\tilde{\theta} = \theta^m + d - \frac{2d(I - ID)}{W} = \hat{\theta} + \frac{2dID}{W}.
$$

(9)

In contrast, when the firm issues convertible debt, the equity component and the debt component of the convertible security have to be sold to the same group of investors at a uniform price. Thus, the marginal outside investor who is pricing the equity component of convertible debt is the same marginal investor who is pricing the debt component of it, so that both components are priced by the marginal investor with belief $\tilde{\theta}$. However, if the firm instead issues a combination of equity and debt, it can sell equity to the more optimistic outside investor with belief $\tilde{\theta}$ who is willing to pay a higher price than the marginal convertible debt investor with belief $\tilde{\theta}$. Thus, when the firm issues convertible debt, it is unable to achieve the optimal price differentiation between its debt and equity components. Therefore, in the absence of issuance costs, convertible debt is always dominated by a combination of debt and equity.

4 The Full-Fledged Model with Costs of Financial Distress and Issue Costs: The Choice between Equity, Straight Debt, and Convertible Debt

straight debt, and convertible debt.

4.1 The Choice between Equity alone, Straight Debt alone, and Convertible Debt alone

First, we analyze the case of prohibitively expensive issue costs where the firm can issue only one type of security to finance its investment. We assume that, if the face value $F$ of a debt security (straight or convertible) is strictly greater than the firm’s cash flow $X_L$ in the bad state, the firm faces a financial distress cost $C^B > 0$ when the low cash flow $X_L$ is realized, since the firm has to default on a fraction of its promised payment $F$ in this scenario. Otherwise, if $F \leq X_L$, the firm’s cost of financial distress is 0. The following proposition characterizes the conditions under which the firm issues equity alone, debt alone, or convertible debt alone.\(^{21}\)

**Proposition 4. (The Choice between Equity alone, Straight Debt alone, and Convertible Debt alone)**

(i) If outside investors are optimistic about the firm’s future cash flows on average, and their beliefs are widely dispersed so that the marginal outside investor is more optimistic than firm insiders, i.e., if $\theta = \theta^m + d - \frac{2dI}{W} \geq \theta^I$, it is optimal for the firm to issue equity alone.

(ii) If outside investors are pessimistic about the firm’s future cash flows on average, and their beliefs are not so dispersed so that the marginal outside investor is less optimistic than firm insiders, i.e., if $\theta = \theta^m + d - \frac{2dI}{W} \leq \theta^I$, the firm’s optimal security choice is as follows:

a) If the required investment amount $I$ is small so that $I \leq X_L$, it is optimal for the firm to issue risk-free straight debt.

b) If the required investment amount $I$ is large so that $I > X_L$, it is optimal for the firm to issue convertible debt with total face value $F = X_L$.

The intuition behind the above proposition is as follows. When the average outside investor is much more optimistic about the firm’s future cash flows than firm insiders, and outsiders’ beliefs are very dispersed, the marginal outside investor will be more optimistic than firm insiders. In this situation, it is optimal for the firm to issue equity alone. In this case, equity dominates both straight debt and convertible debt from the point of view of firm insiders since it best allows the firm to take advantage of

\(^{21}\)Throughout the paper, we assume that, if the firm issues convertible debt, it is optimally designed from the firm’s point of view, and that its design satisfies the parametric restrictions specified in Lemma 3. In other words, we require the convertible debt to be truly a hybrid of equity and debt, which will be the case only if the restrictions specified in Lemma 3 are satisfied (see also the discussion in footnote 14).
the optimism among outsiders and thus sell a security that is most overvalued relative to firm insiders’ valuation conditional on their own belief. Furthermore, issuing equity alone minimizes the firm’s costs of financial distress (i.e., they will be zero).

On the other hand, when the average outside investor is not so optimistic or downright pessimistic about the firm’s future prospects and the dispersion in outsider beliefs is low (so that the marginal outside investor is less optimistic than firm insiders), equity is no longer the optimal security to issue. This is because, in this case, equity (or any other security with an equity component) will be undervalued relative to the belief of firm insiders. The firm will then issue either risk-free straight debt or convertible debt depending on the size of the required investment $I$.

If the size of the investment is small, i.e., $I \leq X^L$, so that the firm can issue risk-free straight debt, the choice between straight debt and convertible debt depends on the following trade-off. On the one hand, the convertible debt has an equity component embedded in it, which will be undervalued relative to firm insiders’ beliefs in this situation (unlike risk-free straight debt, whose valuation is totally insensitive to outsider beliefs): we will call this the “undervaluation effect.” On the other hand, the option to convert to equity embedded in the convertible debt is also valuable, since it reduces the face value of the debt to be offered to outsiders in return for a given amount of financing, thereby reducing the probability of the firm going into financial distress (and consequently reducing the expected financial distress cost incurred by the firm): we will call this the “embedded option effect.” In particular, the firm can always set the total face value of the convertible debt such that it avoids bankruptcy with probability 1. However, if the investment amount is small enough so that the firm can raise this amount by issuing risk-free straight debt, then it will also have no cost of financial distress. But if the firm issues risk-free convertible debt, this security with its embedded equity options will be undervalued with respect to the risk-free straight debt. Therefore, if the size of the required investment amount is small, the undervaluation effect will dominate the embedded option effect, and the firm will prefer to issue risk-free straight debt rather than convertible debt.

However, if the size of the required investment is large, i.e., $I > X^L$, the firm cannot issue risk-free straight debt to raise this entire amount $I$. In this case, the embedded option effect of the convertible debt will favor issuing convertible debt to issuing risky straight debt. As we showed in Lemma 2,
risky straight debt is sensitive to the marginal investor’s belief, and it is more undervalued than risk-free straight debt. In addition, the firm will also face financial distress cost $C^B$ if the cash flow at time 2 is $X^L$. However, we also show in the above proposition that, while the firm can minimize the financial distress cost by reducing the face value of debt to be issued using embedded equity options (i.e., convertible debt), risky straight debt will have the same undervaluation cost as convertible debt if the face value $F$ of the convertible debt is greater than or equal to the cash flow $X^L$ in the bad state. Even though the embedded equity component of the convertible debt will be more undervalued than risky straight debt by the marginal investor, the straight debt component of the convertible debt will be less undervalued than risky straight debt, as the face value of convertible debt issued will be less than the face value of risky straight debt issued to raise the required investment amount. Since these two effects cancel each other out, risky straight debt has the same undervaluation cost as convertible debt for any face value $F \geq X^L$. We also know from Lemma 3 that the firm has no incentive to reduce the face value of the convertible debt below $X^L$, since this increases the undervaluation of convertible debt relative to risky straight debt. Thus, since the firm will optimally set the face value $F$ of convertible debt to $X^L$ and avoid costs of financial distress, the embedded option effect will dominate the undervaluation effect, and the firm will prefer to issue convertible debt with face value $F = X^L$ rather than risky straight debt when the required investment amount is large.\footnote{Consider the following illustration of the optimal design of the convertible debt issued in this case. Let $X^H = 100$, $X^L = 10$, $I = 50$, $W = 1500$, $\theta^m = 0.5$, and $d = 0.2$. Then, the marginal investor’s belief is equal to $\hat{\theta} = 0.6867$. Suppose also that the insider belief is equal to $\theta^I = 0.8$. Each unit of convertible debt has a face value of 1. If costs of financial distress are nonzero, Proposition 4 implies that it is optimal for the firm to issue 10 units of risk-free convertible debt with a total face value of $F = 10$. The pricing of convertible debt and investors’ break-even condition determine the conversion ratio $x$ and the price $p$ per unit of convertible debt. The conversion price per unit of convertible debt is $1/x$. $I = 50$ and $F = 10$ imply that $p = 5$, since $p = 1/F$ (note that $F$ also represents the number of units of convertible debt issued, since each unit has face value of $\$1$). The pricing equation of convertible debt implies that $x = 0.21498$, and $1/x = 4.6515$. Since there is one share of equity outstanding initially, if convertible debt holders convert there will be $(1 + xF)$ shares of equity outstanding. Given that $1 + xF = 3.14985$, if convertible debt holders convert, there will be 3.14985 shares of equity outstanding. Prior to conversion, we assume that the following condition holds: $X^L < F + 1/x < X^H$. This condition implies that the highest possible total face value of convertible debt is equal to the total face value of straight risky debt, which in this example is equal to 68.2524 > $I = 50$. The corresponding conversion price is equal to 31.7476. Above this conversion price, convertible debt is equivalent to straight risky debt. This restriction also implies that the lowest possible value of $F$ is 6.96, and the lowest possible value of the conversion price is 3.03621, since, below these values, convertible debt is equivalent to equity. Convertible debt holders will convert to equity if and only if the cash flow per share of equity after conversion is greater than the conversion price ($1/x$), which is 4.6515 here. If the low cash flow ($X^L = 10$) is realized, the cash flow per share of equity is $10/3.14985 = 3.17476$. Since 3.17476 is less than the conversion price 4.6515, convertible debt holders do not convert. If the high cash flow ($X^H = 100$) is realized, the cash flow per share of equity is $100/3.14985 = 31.7476$, and convertible debt holders convert. In our example, if $F$ is less than $X^L = 10$ (i.e., the conversion price is less than 4.6515), the undervaluation cost of issuing convertible debt will be higher than that of issuing risky straight debt. If $10 \leq F < 68.2524$, the undervaluation cost of issuing convertible debt will be the same as that of issuing risky straight debt. However, if the firm sets $F$ strictly above 10 (between 10 and 68.2524), it will also incur a positive financial distress cost.
4.2 The Choice between Equity alone, Straight Debt alone, Convertible Debt alone, and a Combination of Straight Debt and Equity

In this subsection, we relax the assumption made in the previous subsection that the issue cost $C^I$ is always prohibitively large enough that the firm finds it optimal to issue only one type of security: in other words, we will analyze situations where the firm may find it optimal to issue a combination of securities to raise the required amount of financing. We start with the simplest case where the menu of securities available to the firm consists of equity, straight debt, or a combination of equity and straight debt. The following proposition characterizes the firm’s choice of external financing between equity alone, straight debt alone, and a combination of straight debt and equity when the firm faces issue costs and costs of financial distress.

Proposition 5. (The Choice between Equity alone, Straight Debt alone, and a Combination of Straight Debt and Equity) Let $\theta^I < \theta^m + d$.

(i) The firm will choose to issue equity alone if outsiders are very optimistic about the firm on average, and their beliefs are very dispersed so that the marginal outside investor’s belief $\hat{\theta}$ is above an upper threshold belief $\theta_{1,b}$, i.e., $\hat{\theta} > \theta_{1,b}$. The threshold belief $\theta_{1,b}$ is less than the threshold $\theta_1$ given in Proposition 2, and decreasing in the issue cost $C^I$.

(ii) If the marginal outside investor’s belief $\hat{\theta}$ is between a lower threshold belief $\theta_{2,b}$ and the upper threshold belief $\theta_{1,b}$ (i.e., $\theta_{2,b} < \hat{\theta} < \theta_{1,b}$), the firm will choose to issue either a combination of risk-free straight debt and equity if the issue cost is low ($C^I \leq C^I_1$), or straight debt alone if the issue cost is high ($C^I > C^I_1$).

(iii) If the marginal outside investor’s belief $\hat{\theta}$ is below the lower threshold belief $\theta_{2,b}$ (i.e., $\hat{\theta} < \theta_{2,b}$), the firm will choose to issue either a combination of risky straight debt and equity (with the amount of equity issued smaller than in (ii) above, with the face value of debt issued larger than in (ii)), if the issue cost is low ($C^I \leq C^I_2$), or straight debt alone if the issue cost is high ($C^I > C^I_2$). The threshold belief $\theta_{2,b}$ is less than the threshold $\theta_2$ given in Proposition 2, and decreasing in the cost of financial distress $C^B$.

Similar to the intuition behind Proposition 2, when the average outside investor is very optimistic about the firm’s future cash flows and the dispersion in outsider beliefs is high, the marginal equity investor will be much more optimistic about the firm than firm insiders. Hence, the equity price determined by the marginal outside investor will be much higher than the insiders’ equity valuation. In this case, the above proposition shows that the firm will issue equity alone in order to fully capture

Thus, it is optimal for the firm to set the face value of the convertible, $F = 10$. 

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the greater optimism of outside investors provided that the marginal outside investor’s belief $\hat{\theta}$ exceeds a certain threshold value $\theta_{1,b}$. Conversely, when the average outside investor is less optimistic and the dispersion in outsider beliefs is lower, the marginal outside investor will value the firm only slightly higher or lower than firm insiders. Then, the firm will choose between straight debt alone and a combination of equity and debt based on the following trade-off. On the one hand, issuing a combination of debt and equity allows the firm to raise some money by selling equity to the most optimistic investors and the rest by selling debt to less optimistic outsiders. In this way, the firm can capture the optimism of the very optimistic investors in the market by selling equity to them at a relatively high price. By issuing debt to raise the remaining amount, the firm will not be hurt by the views of these less optimistic outside investors, since the pricing of straight debt is much less sensitive to the beliefs of outside investors than the pricing of equity. On the other hand, issuing a combination of debt and equity also means that the firm has to pay issue costs on two tranches of securities instead of just one. Therefore, this proposition shows that the marginal outsider’s threshold belief $\theta_{1,b}$, above which the firm prefers to issue equity alone in our full-fledged model, is less than the threshold $\theta_1$ given in Proposition 2 (in our basic model), and decreasing in the issue cost $C^I$.

When the firm prefers to issue a combination of debt and equity rather than equity alone (i.e., $\hat{\theta} < \theta_{1,b}$), the marginal outside investor’s belief $\hat{\theta}$ and the cost of financial distress $C^B$ determine the proportion of debt versus equity that it chooses to issue. For a given level of required investment funding $I$, if the marginal outside investor is moderately optimistic (i.e., $\theta_{2,b} \leq \hat{\theta} < \theta_{1,b}$), the firm will choose to issue a combination of risk-free straight debt and equity. One should recall from Proposition 2 that as the average optimism of outsiders and (or) the dispersion in their beliefs $d$ decrease, the marginal outside investor becomes less optimistic, and therefore, the cost of issuing undervalued equity increases. As a result of this, the firm chooses to issue a larger amount of debt ($I_D$) in combination with equity ($I - I_D$). On the other hand, if the amount of debt that needs to be issued to meet the firm’s investment requirement is sufficiently large ($I_D > X_L$), and therefore, the debt is risky, the firm will also face the risk of incurring a cost of financial distress $C^B$. In this case, if the cost of financial distress $C^B$ is substantial and the marginal outside investor is not very pessimistic, the firm will tolerate issuing undervalued equity to a greater extent (compared to the situation in our basic model) and will choose to issue a
combination of risk-free debt and equity by keeping the amount of debt issued \((I_D)\) less than or equal to the low state cash flow \(X_L\). However, if the marginal outside investor is rather more pessimistic about the firm’s future cash flow prospects (i.e., \(\hat{\theta} < \theta_{2,b}\)) and the cost of financial distress \(C^B\) is not too large, the firm will choose to issue a combination of (a larger amount of) risky straight debt and equity, since the cost of issuing undervalued equity as a result of the marginal investor’s pessimism will be greater than the expected cost of financial distress. This threshold level of marginal outsider’s belief \(\theta_{2,b}\), below which the firm prefers to issue a combination of risky debt and equity, is decreasing in the cost of financial distress \(C^B\), and it is less than the similar threshold belief \(\theta_2\) in our basic model.

The above proposition also shows that, when the marginal outside investor is not very optimistic, the firm will choose to issue straight debt alone rather than a combination of debt and equity, when the issue cost \(C^I\) is large. On the other hand, the firm will choose to issue a combination of debt and equity when the issue costs are relatively low compared to the price differentiation benefits of the debt-equity combination.

We now include convertible debt in the menu of securities available to the firm as well. Given that we have already analyzed the firm’s choice between equity alone, debt alone, and convertible debt alone in the previous subsection, we will now confine ourselves to analyzing the optimality of the firm issuing convertible debt versus a combination of debt and equity.

**Proposition 6. (The Choice between Convertible Debt alone and a Combination of Straight Debt and Equity)** Suppose that outsiders are not very optimistic about the firm on average, and their beliefs are not very dispersed so that the marginal outside investor’s belief \(\hat{\theta}\) is below the upper threshold belief \(\theta_{1,b}\): i.e., \(\hat{\theta} \geq \theta_{1,b}\) in Proposition 5. Then:

(i) If the marginal outside investor’s belief \(\hat{\theta}\) is between the lower threshold belief \(\theta_{2,b}\) and the upper threshold belief \(\theta_{1,b}\) (i.e., \(\theta_{2,b} \leq \hat{\theta} < \theta_{1,b}\)), the firm will choose to issue either a combination of risk-free straight debt and equity if the issue cost is low \((C^I \leq C^I_{3})\), or convertible debt alone if the issue cost is high \((C^I > C^I_{3})\).

(ii) If the marginal outside investor’s belief \(\hat{\theta}\) is below the lower threshold belief \(\theta_{2,b}\) (i.e., \(\hat{\theta} < \theta_{2,b}\)), the firm will choose to issue either a combination of risky straight debt and equity (with the amount of equity issued smaller than in (i) above, with the face value of debt issued larger than in (i)), if the issue cost is low \((C^I \leq C^I_{4})\), or convertible debt alone if the issue cost is high \((C^I > C^I_{4})\). The threshold belief \(\theta_{2,b}\) is decreasing in the firm’s cost of financial distress \(C^B\).

Issuing a combination of straight debt and equity means that the firm can sell its package of securities at higher prices because the firm can sell equity to the most optimistic outside investors at a relatively
high price and sell debt to the less optimistic outsiders. In contrast, when the firm issues convertible debt, the equity component and the debt component of the security have to be sold to the same group of investors, and therefore the firm is unable to capture the optimism among the more optimistic outside investors, since the price of the entire package of hybrid securities (i.e., convertible debt) will be determined by the belief of a single marginal investor (rather than by the beliefs of two different marginal investors, the beliefs of one for equity and one for debt). The benefits of issuing a combination of debt and equity over issuing convertible debt increases with the dispersion in outsider beliefs, since more heterogeneous outsider beliefs imply the existence of a greater mass of very optimistic outside investors to whom the firm can sell equity at a relatively high price compared to firm insiders’ beliefs. On the other hand, issuing convertible debt rather than a combination of debt and equity can save the firm issue costs. The benefit of convertible debt in terms of saving issue costs increases with the magnitude of the issue cost $C^I$. When the issue cost $C^I$ is high, the cost saving benefit of convertible debt outweighs the valuation benefit of the debt-equity combination, and it is optimal for the firm to issue convertible debt. Conversely, when the the issue cost is low, the valuation benefit of the debt-equity combination outweighs the issue–cost saving benefit of convertible debt, and it is optimal for the firm to issue a combination of debt and equity.

Finally, if the firm chooses to issue a combination of equity and debt, the fraction of equity versus debt issued in this combination depends on the belief of the marginal outside investor: if the marginal outsider’s belief is as specified in (i) (i.e., $\theta_{2,b} \leq \hat{\theta} < \theta_{1,b}$), then the firm issues a significant amount of equity, so that the debt issued is risk-free; in contrast, if the marginal outsider’s belief is lower, as specified in (ii) (i.e., $\hat{\theta} < \theta_{2,b}$), then the firm issues a small amount of equity and a larger amount of debt compared to the situation in (i), so that the debt issued is risky. As discussed under Proposition 5, the threshold value of marginal outsider belief $\theta_{2,b}$, below which the firm issues a combination of risky debt and equity (rather than risk-free debt and a larger amount of equity), will be lower as the firm’s cost of financial distress $C^B$ is larger.23

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23Since the threshold beliefs involved are the same across propositions 5 and 6, a combination of these propositions can be viewed as comparing all four financing choices available to the firm, namely, equity alone, debt alone, a combination of equity and debt, and convertible debt. We chose to split up these comparisons across two propositions mainly for clarity of exposition.
5 The Pecking Order of External Financing under Heterogeneous Beliefs

In this section, we discuss the pecking order implications of our basic model (Section 3) and our full-fledged model (Section 4) for the security issuance of a firm under heterogeneous beliefs.

5.1 The Pecking Order of External Financing under Heterogeneous Beliefs in the Absence of Other Market Imperfections

Our model implies a “pecking order” of external financing under heterogeneous beliefs. If the marginal outside investor is much more optimistic than firm insiders, the firm issues the most belief-sensitive security (equity only) to capture outsiders’ optimism, rather than straight debt or convertible debt, which are less belief-sensitive securities, and therefore less overvalued than equity. If outside investors are moderately more optimistic or less optimistic than firm insiders on average, and the dispersion in their beliefs is not too large, the firm will issue a combination of equity and debt to cater to outside investor clienteles with different beliefs about the firm’s future prospects. In particular, if the marginal outsider’s belief is above a certain threshold belief, then the firm issues a combination of risk-free straight debt and equity. If, however, the marginal outsider’s belief is below this threshold value, the firm issues a small amount of equity in combination with risky straight debt to raise the required investment amount. Risky straight debt is more undervalued than risk-free debt, but at the same time, it is less undervalued than equity in the latter scenario.

Further, in a setting where the only market imperfection is heterogeneity in beliefs, convertible debt is dominated by either straight debt or equity (or a combination of these securities) depending on the belief of the marginal investor relative to that of firm insiders. This is because convertible debt is moderately belief-sensitive, and unlike a combination of straight debt and equity, it cannot be issued to separate investor clienteles with different beliefs.

Finally, it is important to note that the pecking order of external financing is quite different in a setting where heterogeneity in beliefs is the sole market imperfection compared to one where asymmetric information is the sole market imperfection. First, while issuing equity is the last choice in an asymmetric information setting (e.g., Myers and Majluf, 1984), it is the first choice in a setting where outside
investors are optimistic enough so that the marginal outside investor’s belief is above that of firm insiders. Second, under asymmetric information, if the firm can raise the required amount of external financing by issuing risk-free debt, this will be the most preferred security to issue; in contrast, even when the marginal outside investor is pessimistic relative to firm insiders, under heterogeneous beliefs, the firm prefers to issue a combination of equity and debt rather than risk-free debt alone to raise the required external financing. Third, under asymmetric information, if the firm cannot raise the entire amount of financing required by issuing risk-free debt, it will choose to issue risky debt (or other securities that are less information-sensitive than equity) to raise this amount; in contrast, under heterogeneous beliefs, the firm will raise the required amount by issuing a combination of equity and risky debt under these circumstances, even when the marginal outside investor is more pessimistic than firm insiders.

5.2 The Pecking Order of External Financing under Heterogeneous Beliefs in a Setting with Costs of Financial Distress and Issue Costs

Using our full-fledged model with costs of financial distress and issue costs, we also showed that the belief of the marginal investor relative to that of firm insiders determines a “pecking order” of external financing under heterogeneous beliefs even when these imperfections are present. Consider first the case where the issue cost and the cost of financial distress are moderate (not too large). In this case, when the average outsider is very optimistic, and the dispersion in outsider beliefs is relatively large (so that the marginal outside investor is more optimistic than firm insiders), the firm always chooses to issue equity alone to raise the required amount of external financing. In other words, if the marginal outside investor is more optimistic than firm insiders, the firm can capture the benefit of this optimism by issuing the most belief-sensitive security, i.e., equity, rather than less belief-sensitive securities like convertible debt or straight debt. However, when the average outside investor is less optimistic, and the dispersion in outsider beliefs is smaller (so that the marginal investor is less optimistic than firm insiders), the firm issues a combination of equity and straight debt. In this case, the amount of equity versus debt issued to raise the required amount of financing depends on whether the marginal investor’s belief is above or below a certain threshold belief. If the marginal investor’s belief is above the threshold
belief, the firm issues a combination of equity and risk-free debt; if the marginal investor’s belief is below this threshold belief, the firm issues a combination of a smaller amount of equity and a large amount of risky debt. The above threshold belief will depend on the firm’s cost of financial distress.

Consider now the case where the firm’s issue cost is significantly larger (so that issuing a combination of securities is prohibitively expensive) and its cost of financial distress is significant. In this case, the firm continues to choose to issue equity alone to meet its financing requirements if the marginal outside investor is more optimistic than firm insiders. If, however, the marginal outside investor is less optimistic than firm insiders, the firm chooses to issue convertible debt or risk-free debt depending upon the investment amount it needs to raise. If this investment amount is small enough, the firm issues risk-free straight debt; if, however, this investment amount is larger, the firm issues convertible debt to raise the required amount of external financing.

6 The Price Impact of Security Issues

In this section, we investigate the price impact of equity, straight debt, and convertible debt issues on the current stock price of the firm at the time of a security issue. The price impact of a security issue is measured as the abnormal return to the firm’s equity from the price prevailing before a security issue (not the announcement date) to the price prevailing after the issue. Since the market already is aware that a security issue has been announced, one would expect a price impact of zero in the absence of heterogeneity in investor beliefs. While, in practice, some time will elapse between the announcement date and the issue date of a security, for simplicity, we model the issue date and the announcement date together.

Proposition 7. (Price Impact of Security Issues) (i) If the firm issues equity at time 0, there is a negative impact on the stock price on the issue date:

\[ \Delta P_{E}^{\text{Equity}} = -\left(1 - \frac{I}{\bar{\theta}X^{H}} + \frac{(1 - \bar{\theta})X^{L}}{1 - \bar{\theta}}\right) \frac{2dI}{W} (X^{H} - X^{L}) < 0. \] (10)

Note that, empirically, the price impact of a security issue is quite different from an announcement effect in the abnormal return measured on the day of the announcement of the security issue (before the issue becomes effective), while the price impact is the abnormal return measured on the day the security issue actually comes into effect. If we separate the two dates, there should be no announcement effect in our setting since investors do not update their beliefs based on others’ actions (insiders actions do not convey any information to outsiders in our setting since there is no information asymmetry in our model).
(ii) If the firm issues straight debt or convertible debt, there is no impact on the stock price on the issue date:
\[ \Delta P E^{\text{Debt}} = \Delta P E^{\text{Convertible}} = 0. \]  

(11)

(iii) The greater the dispersion in outsiders’ beliefs, \(d\), the greater the price impact of an equity issue
\[ \frac{\partial |\Delta P E^{\text{Equity}}|}{\partial d} > 0. \]  

(12)

When the firm issues equity, it must sell the equity to investors who are less optimistic about the firm’s value than current shareholders (since the current shareholders have limited wealth). Hence, the valuation of the new marginal equity investor (i.e., the stock price just after the equity issue) will be lower than the valuation of the marginal equity investor before the equity issue. This results in a fall in the firm’s share price, yielding a negative price impact. On the other hand, if straight debt or convertible debt is issued, the equity price will remain at the same level as before the security issue. This is because the marginal investor who holds equity in the firm remains the same in these cases, so that the valuation of the marginal equity holder is unaffected, resulting in a zero price impact of the straight debt issue or convertible debt issue on the firm’s equity.\(^{26,27}\)

This proposition also shows that the negative price impact of an equity issue will be larger in absolute value as the dispersion in outsiders’ beliefs increases. If outsiders’ beliefs are more dispersed (\(d\) is greater), the distance the firm has to go down the ladder of outside investors’ beliefs (to raise the entire investment amount of \(I\)) increases, yielding a more negative price impact of an equity issue.\(^{28}\)

\(^{26}\)However, if the firm has outstanding debt, the debt issue will have a negative impact on the price of the firm’s debt, through a mechanism similar to that generating a negative price impact of an equity issue on the firm’s outstanding equity.\(^{27}\)

\(^{27}\)Note that the mechanism generating a differential price impact of an equity issue versus a debt issue on a firm’s outstanding equity in our setting of heterogeneous beliefs among outside investors is completely different from that generating differences in announcement effect of equity and debt issues on the firm’s equity in asymmetric information models such as Myers and Majluf (1984).\(^{28}\)

\(^{28}\)Chemmanur, Nandy, and Yan (2008) show that there is indeed such a negative price impact on the date of an equity issue, and this negative impact is increasing in the dispersion in outsider beliefs. One may wonder why such a price impact is not arbitrated away by traders who short-sell the firm’s equity on the announcement date of the equity issue (and buy it back after the share price has fallen on the issue date), thus moving forward the price drop to the announcement date. Note that such a trade is not a riskless arbitrage, since there will be market movements in the weeks between the announcement date and the issue date. As Mitchell, Pulvino, and Stafford (2002) show in the analogous context of risky arbitrage around negative stub values in equity carve-outs, traders attempting such arbitrage often earn a rate of return lower than the risk-free interest rate.
At time 0, insiders of a firm own a fraction $\alpha$ of the firm's equity. The remaining $1 - \alpha$ is held by a group of outside shareholders.

The total number of shares outstanding in the firm is normalized to 1.

Additional (noisy) information about the firm’s prospects arrives.

The required amount of investment $I$ for the project is raised from outside investors by issuing either equity, or straight debt, or convertible debt, or a combination of these securities.

The period over which long-term stock returns are measured.

All cash flows are realized.

Figure 5: Sequence of Events in the Long-run Stock Returns Model

7 Long-Run Stock Returns following Security Issues

In this section, we will analyze the long-run stock returns of firms following equity, straight debt, and convertible debt issues that have been studied in the empirical literature. Here we will make the additional assumption that, in the long run, the dispersion in outside investors’ beliefs goes down significantly due to the arrival of additional noisy public information about the firm’s prospects subsequent to its security issue. We denote the final date of our model as time 3, and introduce another date (time 2) between the issue date (time 1) and the final date (time 3), when noisy new information about the firm’s prospects becomes available to outsiders. The sequence of events in this modified model is given in Figure 5.

This noisy new information is hard and credible: an example is the information that can be collected from the firm’s quarterly reports and earnings announcements. Due to the arrival of this new information, we assume that outside investor beliefs about the firm’s cash flows become less heterogeneous in the long run. In particular, we assume that, in the long run (i.e., at time 2), the dispersion in outsider beliefs decreases from $d$ to $d(1 - \delta)$ where $\delta$ is the percentage reduction in the dispersion in outside
investors’ beliefs about the firm’s future cash flow. We also assume that the average belief of outside investors does not shift upward or downward at time 2, and remains fixed at its prior mean value $\theta^m$. In the interest of analytical simplicity, we do not explicitly model the revision in outsider beliefs in response to the arrival of new information. However, in general, the greater the precision of the new information available to outsiders, the greater the reduction $\delta$ in the dispersion in outsider beliefs (see footnote 29 for a discussion of one way of explicitly modeling the revision in outsider beliefs in response to new information). Note that for a firm that issues equity alone at time 1 to raise the investment amount $I$, the marginal equity investor’s belief at time 1 is equal to $\hat{\theta} = \theta^m + d \left( \frac{1}{2I} \right)$. If the dispersion in investor beliefs about the firm goes down by amount of $\delta d$ by time 2, the marginal equity investor’s belief at time 2 will be equal to $\hat{\theta}_2 = \theta^m + d(1-\delta) \left( \frac{1}{2I} \right)$, and the market value of the firm will then be given by

$$V_2 = \hat{\theta}_2 X^H + (1 - \hat{\theta}_2) X^L.$$  \hfill (13)

**Proposition 8. (Long-Run Stock Returns following Equity Issues)** The long-run stock return of firms issuing equity is negative and is given by

$$LR^{Equity} = \frac{PE_2^{Equity} - PE_1^{Equity}}{PE_1^{Equity}} = \frac{-d\delta(X^H - X^L) \left( \frac{1}{2I} \right)}{\hat{\theta}X^H + (1 - \hat{\theta})X^L} < 0.$$  \hfill (14)

The above proposition states that the long-run stock return of firms following equity issues will always be negative. The key to understanding the intuition here is to recall that the firm’s equity at time 1 is priced by the marginal outside investor whose belief $\hat{\theta} = \theta^m + d \left( \frac{1}{2I} \right)$ is determined by going down the ladder of investor beliefs (starting with the most optimistic outsider belief $(\theta^m + d)$) until the entire amount of $I$ is raised. Thus, the higher the dispersion in outsider beliefs $d$ at the time of security

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29 For analytical simplicity, we have chosen not to explicitly model the outside investors’ updating of their prior beliefs in response to the new public information arriving at time 2. However, one scenario that is consistent with what we have assumed here is as follows. Suppose that the true probability of high cash flow realization at time 3 is $T$. Everyone will observe a noisy signal $t$ about $T$ at time 2. Let’s assume that $t$ is equal to $T$ with probability $\delta$, and that $t$ is not equal to $T$ and is completely uninformative with probability $(1 - \delta)$. Then, consider an outside investor with an arbitrary belief $\theta$. After observing $t$, he knows that with probability $\delta$, the probability of high cash flow is $t = T$, and with probability $(1 - \delta)$ the signal is uninformative and he sticks to his prior belief $\theta$, so that his posterior belief should be $\delta t + (1 - \delta) \theta$. Therefore, outsiders’ posterior will range from $\delta t + (1 - \delta)(\theta^m + d)$ to $\delta t + (1 - \delta)(\theta^m + d)$, with a dispersion of $(1 - \delta)d$, down from the initial dispersion $d$ at the time of security issue. Further, the reduction $\delta d$ is proportional to the original dispersion $d$. Note that, in this scenario, the reduction in dispersion is more when the signal is more informative: $\delta d$ is increasing in $\delta$ as well. We can see two extremes: when $\delta = 0$, there will be no reduction in dispersion; when $\delta = 1$, everyone will have the same posterior belief. The posterior mean belief of all investors at time 2 is equal to $\delta t + (1 - \delta)(\theta^m)$, a weighted average of the new signal $t$ and the prior mean belief $\theta^m$. If the signal $t$ is equally likely to be above or below $\theta^m$, the expected value of the posterior mean belief of outsiders is also equal to the prior mean belief $\theta^m$. 37
issue (time 1), the higher the marginal equity investor’s belief $\theta$ at time 1, and therefore, the higher
the firm’s share price at the time of the equity issue. If the dispersion in investor beliefs decreases
over time by an amount of $\delta d$, this implies that the existing marginal equity holder of the firm will
become less optimistic about the firm and, therefore, the stock price will fall in the long-run (by time
2) corresponding to the decline in the dispersion in outsiders’ beliefs. This is the case, since, if there
is a fall in the dispersion $d$ around the mean belief $\theta^m$ and the belief rankings of all investors remains
the same with respect to each other, all investors with a belief above $\theta^m$ at the time of equity issue will
update their beliefs in the long run so that they become less optimistic about the firm. Thus, the
long-run stock returns of firms subsequent to equity issues will always be negative.

**Proposition 9. (Comparative Statics on Long-Run Stock Returns following Equity Issues)**

The long-run stock returns of equity issuers is decreasing in the initial dispersion in outsiders’ beliefs $d$ at the time of equity issue and the percentage reduction $\delta$ in the dispersion in outsiders’ beliefs; i.e.,

$$\frac{\partial LR_{Equity}}{\partial d} < 0 \quad \text{and} \quad \frac{\partial LR_{Equity}}{\partial \delta} < 0.$$  

The greater the extent of hard information about the firm that arrives after an equity issue, the
greater the reduction in the dispersion in outsiders’ beliefs since investors update their beliefs about
the firm using this new public information. Further, the greater the reduction in dispersion in outsider
beliefs, the greater the fall in the marginal outside investor’s optimism, resulting in the long-run stock
returns following equity issues becoming more negative: since this long-run reduction in the dispersion
in outside investor beliefs is given by $d\delta$, the long-run stock returns of equity issues will be decreasing
in both the initial dispersion $d$ and the percentage reduction in dispersion, $\delta$.

**Proposition 10. (Long-Run Stock Returns following Straight Debt and Convertible Debt
Issues)**

(i) The long-run stock return of firms issuing straight debt or convertible debt is negative and is given
by

$$LR^{Debt} = LR^{Convertible} = -\frac{d\delta}{\theta^m + d} < 0.$$  

(ii) The long-run stock return of firms issuing straight debt or convertible debt is decreasing in the
initial dispersion in outsiders’ beliefs $d$ at the time of equity issue and the percentage reduction $\delta$
in the dispersion in outsiders’ beliefs; i.e.,

$$\frac{\partial LR^{Debt}}{\partial d} < 0 \quad \text{and} \quad \frac{\partial LR^{Debt}}{\partial \delta} < 0.$$  

$^{30}$Similarly, all investors with beliefs below $\theta^m$ at the time of security issue will become more optimistic as the spread
around the mean belief level decreases over time. Note that since $W > 2I$, the marginal equity investor’s belief $\theta$ at time
1 is always greater than $\theta^m$.  

38
The intuition behind the above proposition is as follows. Unlike in the case of an equity issue, issuing straight debt or convertible debt does not affect the marginal investor in the firm’s equity. Therefore, the stock price of the firm immediately after a debt (either straight or convertible) issue will reflect only the beliefs of the current marginal investor in the firm’s equity, since the marginal investor in the firm’s equity prior to the debt issue will be the same as the marginal investor in the firm’s equity after the debt issue. Further, the long-run (time-2) stock price of these firms will reflect the belief of the same marginal equity investor who is expected to have an updated belief of \( \theta^m + d(1-\delta) \) at time 2. Therefore, since the marginal equity investor becomes less optimistic in the long run, the long-run stock return following a debt (or a convertible debt) issue will also be negative, and similar to the case for an equity issue, it will also be decreasing in the initial dispersion in outside investors’ beliefs and the percentage reduction in the dispersion in outsiders’ beliefs.

We now compare the average long-run stock returns of issuing firms following equity, straight debt, and convertible debt issues. Given insiders’ beliefs and market conditions (i.e., the mean belief of outsiders and the dispersion in outsider beliefs), any given firm will issue only one kind of security in equilibrium, namely, equity, straight debt, or convertible debt: in other words, it does not make sense to compare the long-run stock returns of a given firm facing identical market conditions issuing equity versus issuing debt, since only one will occur in equilibrium. Therefore, we can only make comparisons across the average long-run stock returns of samples of firms issuing each security. The following proposition compares the average long-run stock return of firms following issues of equity, straight debt, and convertible debt.

**Proposition 11. (Comparison of Average Long-Run Stock Returns across Security Issues)**
Let \( \theta^f > \theta^m \), and let (A.73) hold. Then, the average long-run stock return of firms following a debt issue (straight debt or convertible debt) will be algebraically greater than that following an equity issue; i.e.,

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31In other words, the relevant marginal investor in determining the long-run stock returns following a straight debt or a convertible debt issue is the marginal investor in the firm’s equity at the time of the debt issue (since the marginal investor in the firm’s equity does not change as a result of the debt issue). In contrast, the marginal investor determining firms’ long-run stock returns following an equity issue is the marginal investor holding the firm’s equity immediately after the equity issue (since the marginal investor in the firm’s equity after the equity issue is different from the marginal investor in the equity prior to the equity issue).

32While, in Sections 3 and 4, we show that firms may also choose to issue combinations of equity and debt under certain circumstances, in this section we confine ourselves to comparing the long-run stock returns of firms issuing equity alone, straight debt alone, or convertible debt alone since the empirical literature has focused only on these three securities. However, the general intuition behind our results here go through even if we were to consider firms issuing a combination of equity and debt, provided we classify firms raising a larger fraction of the total amount of external financing by issuing equity as equity issuers, and those raising a larger fraction of the external financing amount by issuing debt as debt issuers.
We showed in Proposition 1 and Proposition 4 that a firm prefers to issue equity rather than debt only if the marginal outside investor is more optimistic than firm insiders. If the marginal outside investor is more pessimistic than firm insiders, the firm issues convertible debt or straight debt depending on the size of required investment \( I \) and the firm’s cost of financial distress. Recall that the marginal outside investor is more optimistic than firm insiders only if the mean outsider belief, and the dispersion in outside investors’ beliefs are sufficiently high. Thus, for a given level of mean outside investor beliefs, we know that the dispersion in outsiders’ beliefs at the time of the security issue will be higher for equity-issuing firms than that for debt-issuing firms. We also showed in Propositions 9 and 10 that the long-run stock return subsequent to a security issue is decreasing in the initial dispersion in outside investors’ beliefs. This means that the average long-run stock return of firms that issue (straight or convertible) debt will be greater (less negative) than the average long-run stock return of those that issue equity.

8 Empirical Implications

We now highlight some of the testable implications of the model and their relationship to the existing empirical literature.

(i) Relationship between investor optimism, dispersion in investor beliefs, and the choice of equity versus debt: Our model predicts that the greater the level of optimism (average belief) among outsiders, and the greater the dispersion in outsider beliefs (or both), the more likely the firm is to choose to issue equity rather than debt. This is because the belief of the marginal investor in the firm’s equity is more likely to be above that of firm insiders if either the level of optimism, or dispersion, or both, among outsiders is higher. Evidence supporting this prediction is provided by Chemmanur, Nandy, and Yan (2008). This prediction also provides a fully rational explanation of the empirical results of Baker and Wurgler (2002), who document that firms tend to issue equity (rather than debt) when outsider optimism is greatest, and this “market timing” of equity issues has a large significant long-run impact on firms’ capital structures.

(ii) Relationship between investment amount and the choice of equity versus debt: Our model predicts

\[ LR^{Debt} = LR^{Convertible} > LR^{Equity}. \]
that, the greater the investment amount to be raised by the firm, the less likely it is to issue equity rather than debt. This is because, since each investor has limited wealth to invest in the firm, the beliefs of the marginal investor is more likely to fall below that of insiders as the amount raised by the firm is greater. Evidence supporting this prediction is provided by Chemmanur, Nandy, and Yan (2008).

(iii) The price impact of an equity issue: The price impact refers to the return to a firm’s equity upon the actual issue (not the announcement) of a security (in our context, debt or equity). The price impact will be given by the change in share price of the firm’s equity (or return) from the price prevailing before the security issue to the price prevailing after the issue. In our setting, the price prevailing before an equity issue will be determined by the beliefs of the marginal investor holding the firm’s equity prior to the issue. Since current shareholders have limited wealth, when new equity is issued it will have to be sold to new investors who are less optimistic about the firm’s long-run prospects, so that the beliefs of the marginal investor after the equity issue will be less optimistic compared to that before the equity issue, resulting in a lower share price after the equity issue: i.e. the price impact of an equity issue will be negative. Further, the above fall in share price upon an equity issue will be greater as the dispersion in outsider beliefs is greater: i.e., greater the dispersion, more negative the price impact. Evidence supporting this prediction is provided by Chemmanur, Nandy, and Yan (2008), who find that the price impact of an equity issue is negative, and is decreasing (more negative) as the dispersion in outsider beliefs is greater.

(iv) The price impact of a debt or convertible debt issue: Our analysis predicts that the price impact of a straight debt or convertible debt issue will be zero (and therefore smaller in magnitude than that of an equity issue). This is because no new equity is issued, so that the beliefs of the marginal equity investor before the firm’s debt issue and that of the marginal equity investor after the debt issue will be the same in this case. In other words, the price of the firm’s equity will be unchanged on the day of the debt issue, so that the price impact (on equity) of a straight or convertible debt issue will be zero. Consistent with this, the empirical analysis of Chemmanur, Nandy, and Yan (2008) shows that the price impact of a straight debt issue is insignificant.

(v) Long-run stock returns following an equity issue: We define long-run stock return as the one year, two year, or three year stock return starting from the closing price on the day of the actual equity issue
(as is standard in the relevant empirical literature). In other words, long-run stock returns exclude the price impact of a security issue. Our model predicts that the long-run stock returns following an equity issue will be negative. Evidence consistent with this has been documented in a number of empirical papers (see, e.g., Loughran and Ritter (1997), or Spiess and Affleck-Graves (1995)). Further, our model predicts that the greater the dispersion in outsider beliefs regarding a firm’s prospects at the time of the equity issue, the more negative its long-run stock return following the equity issue will be. Evidence supporting this prediction is provided by Chemmanur, Nandy, and Yan (2008), who document that the more optimistic outsiders are on average about a firms prospects at the time of an equity issue and the more dispersed their beliefs, the more negative the long-run stock returns of the issuing firm.

(vi) **Long-run stock returns following a debt or convertible debt issue:** Our model predicts that the average long-run stock return of firms following a (straight or convertible) debt issue will be negative, but algebraically greater (less negative) than the average long-run stock return of firms following an equity issue. Evidence supporting this prediction is provided by Chemmanur, Nandy, and Yan (2008), who document that, while the average long-run stock returns to both equity and debt issuers are negative, the average stock return to equity issuers is significantly more negative than that of debt issuers (see also Spiess and Affleck-Graves (1995) and Spiess and Affleck-Graves (1999)).

(vii) **A new rationale for issuing convertible debt:** Our model suggests a new rationale for issuing convertible debt. When a firm faces significant issue costs (so that it prefers to raise the required external financing by issuing a single security) and the marginal outside investor in an external financing would be less optimistic about the firm’s prospects compared to firm insiders, then the firm would issue convertible debt in equilibrium provided that the costs of financial distress in its industry are non-trivial. In other words, convertible debt is a mechanism for minimizing the expected financial distress costs of a firm, when the amount of external financing is large enough that this financing requirement cannot be met by issuing risk-free debt, in a setting where issuing equity is suboptimal. Consistent with this, there is significant evidence indicating that convertible debt is issued by smaller, riskier, and high-growth firms in industries characterized by higher costs in the event of financial distress: e.g., high-tech firms.
9 Conclusion

We analyzed a firm’s financing decision in an environment of heterogeneous beliefs and short sales constraints. We studied a setting in which the insiders of a firm, owning a certain fraction of its equity, choose between equity, debt, or convertible debt to raise additional financing to implement a positive net present value project. The insiders’ objective is to maximize their long-run wealth conditional on their own beliefs about their firm’s future prospects. Market participants, each of whom have limited wealth, have heterogeneous beliefs about the firm’s long-run value. We analyzed two different economic settings: one in which there are no market imperfections other than heterogeneous beliefs, and another in which there are also significant costs of issuing securities and of financial distress. We showed that, in the absence of these two costs, the average belief of outsiders (“optimism”) and the dispersion in outsider beliefs are the crucial determinants of the firm’s security choice. When outsider beliefs are highly optimistic relative to firm insiders and the dispersion in outsider beliefs is high, the firm issues equity alone; when outsider beliefs are less optimistic (and less dispersed), the firm issues a combination of equity and debt. Neither straight debt alone nor convertible debt alone is optimal in this setting. Once the two costs are significant, we showed that the firm always issues equity when outsider beliefs are optimistic and highly dispersed. If outsider beliefs are less optimistic, it issues a combination of equity and debt if issue costs are small; if issue costs are large, it either issues risk-free straight debt (if the investment amount required is small) or convertible debt (if this amount is large). Our model generates a pecking order of external financing under heterogeneous beliefs different from that of asymmetric information models. Further, it generates several unique testable predictions for the price impact of equity, debt, and convertible debt issues and for the firm’s long-run stock returns following the issuance of these securities.
References


Appendix: Proofs of Propositions

Proof of Lemma 1:

If the equity is traded at a price of $PE_1^{Equity}$ per share at time 1 when the firm issues equity, all investors whose valuation higher than $PE_1^{Equity}$ will be willing to buy. Denote $\theta$ as the belief of the marginal investor, whose valuation of the equity equals the market price $PE_1^{Equity}$. Because potential outside investors have a total wealth of $W$, and they are uniformly distributed over the interval with a length of $2d$, each investor has a wealth of $W/2d$. Because the firm needs to raise an amount $I$ from investors in the interval $[\theta, \theta^m + d]$, we have

$$\int_{\theta}^{\theta^m + d} \frac{W}{2d} d\theta = I. \quad (A.1)$$

The above equation means that the total wealth of those who buy the new issues equals the amount the firm wants to raise, $I$. Solve for $\theta$, we have

$$\theta = \theta^m + d - \frac{2dI}{W}. \quad (A.2)$$

The market price of new shares sold should be determined by the marginal investor’s valuation of the shares, i.e.,

$$PE_1^{Equity} = \frac{[\theta X^H + (1 - \theta) X^L]}{1 + E_1}, \quad (A.3)$$

where the left side is the market price of each share of equity and the right side is the marginal investor’s valuation of each share of equity. Further, the amount raised by the firm is $I$, which means

$$PE_1^{Equity} \times E_1 = I. \quad (A.4)$$

Solving for equations (A.3) and (A.4) leads to

$$PE_1^{Equity} = \theta X^H + (1 - \theta) X^L - I \quad (A.5)$$

and

$$E_1 = \frac{I}{\theta X^H + (1 - \theta) X^L - I} \quad (A.6)$$

The expected payoff to the firm’s current shareholders is

$$EU^{Equity} = \left(1 - \frac{I}{\theta X^H + (1 - \theta) X^L}\right)[\theta^f X^H + (1 - \theta^f) X^L]. \quad (A.7)$$

Q.E.D.

Proof of Lemma 2

Suppose the firm needs to issue $F$ units of straight debt to raise the amount $I$. Because potential outside investors have a total wealth of $W$, and they are uniformly distributed over the interval with length of $2d$, the marginal investor in the firm’s debt is also $\theta = \theta^m + d - \frac{2dI}{W}$, similar to the argument in the proof of lemma 1.

First, assume that $X^H > I > X^L$, i.e., the debt is risky. The payoff to each unit of straight debt is 1 in the good state and $\frac{X^L}{F}$ in the bad state, so the market price of each unit of debt, which is determined by the marginal investor’s valuation of the debt, is given by

$$PD_1 = \theta + (1 - \theta) \frac{X^L}{F}. \quad (A.8)$$

The firm has to sell $I/PD_1$ units of straight debt to raise an amount of $I$. Since we already assume that the firm
issues a total of $F$ units of debt, we must have

$$F = \frac{I}{PD_1} = \frac{I}{\widehat{\theta} + (1 - \widehat{\theta}) \frac{X^L}{F}}$$

or equivalently,

$$F = \frac{I - (1 - \widehat{\theta}) X^L}{\widehat{\theta}} \quad (A.9)$$

The payoff to the equity holders of the firm is 0 in the bad state and $X^H - F$ in the good state, so the expected payoff to current shareholders of the firm is

$$EU_{Debt}^{\text{Debt}} = \theta f \left( X^H - \frac{I - (1 - \widehat{\theta}) X^L}{\widehat{\theta}} \right). \quad (A.10)$$

Now, assume that the firm can issue risk-free straight debt, i.e., $I \leq X^L$. The payoff to each unit of straight debt is 1 in every state, so the market price of each unit of risk-free debt is

$$PD_1 = \widehat{\theta} + (1 - \widehat{\theta}) = 1. \quad (A.11)$$

The firm has to sell $I/PD_1$ units of risk-free straight debt to raise an amount of $I$. Thus, $F = I/PD_1 = I$. The payoff to the equity holders of the firm is $X^L - I$ in the bad state and $X^H - F$ in the good state, so the expected payoff to current shareholders of the firm is

$$EU_{RiskFreeDebt}^{\text{RiskFreeDebt}} = \theta f X^H + (1 - \theta f) X^L - I. \quad (A.12)$$

Q.E.D.

**Proof of Lemma 3:** We impose the condition that not all investors prefer to convert to equity at time 2 if the bad state with low cash flow ($X_L$) is realized. Otherwise, there would be no difference between convertible debt and equity. Thus, we have the following restriction on the conversion ratio $x$ and the price $p$:

$$x \frac{X^L}{1 + x \frac{I}{p}} < 1, \quad (A.13)$$

which translates into the following no-conversion condition:

$$F = \frac{I}{p} > X^L - \frac{1}{x}. \quad (A.14)$$

In addition, we impose the restriction that all convertible debt investors prefer to convert to equity at time 2 if the good state with high cash flow ($X_H$) is realized. Otherwise, there would be no difference between convertible debt and straight debt. Thus, we have the following restriction on the conversion ratio $x$ and the price $p$:

$$F = \frac{I}{p} < X^H - \frac{1}{x}. \quad (A.15)$$

Combining these two conversion conditions we obtain the following restriction:

$$X^L < \frac{I}{p} + \frac{1}{x} < X^H. \quad (A.16)$$

First consider the case where there is no default of the convertible at time 2. Then the following condition

\footnotetext{33}{This condition also guarantees that none of the convertible debt investors has the incentive to convert to equity at time 2 if the cash flow is equal to $X^L$, i.e., it is equivalent to this condition: $x \frac{X^L - (1 - \theta f)}{1 + x} < 1.$}
has to be satisfied:

$$F = \frac{I}{p} \leq X^L. \quad (A.17)$$

Thus, in the case of default-free convertible, the valuation of the marginal investor for the convertible security at time 1 is given by

$$p = \hat{\theta} \frac{p_x}{p + I_x} X^H + (1 - \hat{\theta}). \quad (A.18)$$

If we solve this equation for the conversion ratio $x$, we get

$$x = \frac{p^2 - (1 - \hat{\theta})p}{\hat{\theta} X^H p + (1 - \hat{\theta})I - pI}. \quad (A.19)$$

It is easy to show that $x$ is increasing in $p$. Note also that if the firm wants to issue risk-free convertible debt (i.e., if $F \leq X^L$), the conversion condition (A.15) in the good state will be satisfied only if $p > 1$, or equivalently, if $F < I$. The expected payoff to the current shareholders is

$$EU^{RiskFreeConvertible} = \theta f \frac{p}{p + I_x} X^H + (1 - \theta f) \left( X^L - \frac{I}{p} \right). \quad (A.20)$$

If we plug in the value of $x$ from (A.19), we have

$$EU^{RiskFreeConvertible} = \theta f \frac{p}{p + I_x} X^H + (1 - \theta f) \left( X^L - \frac{I}{p} \right)$$

$$= \theta f X^H + (1 - \theta f) X^L - \frac{\theta f I}{\hat{\theta}} + \frac{(\theta f - \hat{\theta})I}{\hat{\theta} p} \quad (A.21)$$

If we plug in the value of $x$ from (A.19), we have

$$EU^{RiskFreeConvertible} = \theta f \frac{p}{p + I_x} X^H + (1 - \theta f) \left( X^L - \frac{I}{p} \right)$$

$$= \theta f X^H + (1 - \theta f) X^L - \frac{\theta f I}{\hat{\theta}} + \frac{(\theta f - \hat{\theta})I}{\hat{\theta} p} \quad (A.22)$$

If $\hat{\theta} < \theta f$, it is optimal to set $p$ as low as possible, i.e., $x = \hat{x}$. Given the no default condition in (A.17) and the conversion condition (A.15) in the good state of the world, the lowest possible value of $p$, which we denote by $p$, can be determined. If the firm can issue risk-free straight debt (if $I \leq X^L$), the total face value of risk-free convertible is equal to 1, and $p$ must be strictly greater than 1. In other words, the total face value $F = \frac{I}{p}$ of the risk-free convertible must be slightly less than $I$. Otherwise, if $I > X^L$, the total face value $F$ of the risk-free convertible issue must be less than or equal to $X^L$. Therefore, given the no-default condition (A.17), $p$ is equal to $\frac{I}{X^L} > 1$. If we plug in these lower bounds of $p$ in (A.19), we obtain the following optimal conversion ratio $x$, i.e., $\hat{x}$:

$$\hat{x} = \begin{cases} \frac{I - (1 - \hat{\theta})X^L}{X^H + (1 - \hat{\theta})X^L - I} & : I > X^L, \\ \frac{1}{X^H} + \epsilon & : I \leq X^L. \end{cases} \quad (A.23)$$

where $\epsilon$ is an arbitrarily small positive number. Consider the case when $I > X^L$. Note that since the optimal conversion ratio $x = \hat{x}$ from (A.23) implies that $p = \frac{I}{X^L} > 1$ in this case, it follows from (A.22) that

$$EU^{RiskFreeConvertible} = \frac{\theta f}{\hat{\theta}} (\hat{\theta} X^H + (1 - \hat{\theta}) X^L) = EU^{Debt}. \quad (A.24)$$

Note that in this case, the total face value $F$ of the risk-free convertible debt issue is optimally set to $\frac{I}{p} = X^L$, i.e., the firm’s cash flow at the bad state.

Similarly, in the case where $I \leq X^L$, we have $p = \frac{I}{X^L} > 1$ for convertible debt. Therefore, it follows from (A.22) that

$$EU^{RiskFreeConvertible} < \theta f X^H + (1 - \theta f) X^L - I = EU^{RiskFreeDebt}. \quad (A.25)$$
In both cases, the price \( p = \overline{p} \) of the convertible debt as a function of \( x \) is given by

\[
p = \frac{(1 - \theta) - x I + \theta \theta X^H + \sqrt{(1 - \theta - x I + \theta \theta X^H)^2 + 4(1 - \theta) x I}}{2}
\]  

(A.26)

Thus, we showed that risk-free straight debt always dominates risk-free convertible debt, if the marginal investor is more pessimistic than firm insiders. If the required investment \( I \) is greater than \( X^L \), we also showed that risky straight debt generates the same expected payoff to firm insiders as the risk-free convertible debt (\( F = X^L \)), when the marginal investor is more pessimistic than firm insiders.

If \( \hat{\theta} \geq \theta^f \), it follows from (A.22) that it is optimal to set \( p \) as high as possible, that is, \( x = \overline{x} \). Given the no-conversion condition from (A.14) and the value of \( x \) from (A.19), the highest possible value of \( p \) should be less than \( \frac{\theta X^H + (1 - \theta) X^L}{\overline{x}} \). If we plug in this upper bound of \( p \) in (A.19), we obtain the following upper bound on \( \overline{x} \):

\[
\overline{x} = \frac{\theta X^H + (1 - \theta) X^L}{X^L (\theta X^H + (1 - \theta) X^L - I)} - \epsilon,
\]  

(A.27)

where \( \epsilon \) is a small positive number. Note that since we have \( p < \frac{\theta X^H + (1 - \theta) X^L}{\overline{x}} \), it follows from (A.22) that

\[
EU^{RiskFreeConvertible} < \theta f \, X^H + (1 - \theta f) X^L - I \frac{\theta f \, X^H + (1 - \theta f) X^L}{\theta X^H + (1 - \theta) X^L} = EU^{Equity}.
\]  

(A.28)

Thus, if the marginal investor is more optimistic than firm insiders, issuing equity only is preferred to issuing risk-free convertible debt. The price \( p = \overline{p} \) of the convertible debt as a function of \( \overline{x} \) is given by

\[
\overline{p} = \frac{(1 - \theta) - \overline{x} I + \overline{x} \theta X^H + \sqrt{(1 - \theta - \overline{x} I + \overline{x} \theta X^H)^2 + 4(1 - \theta) \overline{x} I}}{2}
\]  

(A.29)

Note that if \( I > X^L \) and \( x \) is set so low that \( x < \frac{I - (1 - \theta) X^L}{\theta X^H + (1 - \theta) X^L - I} \), convertible debt will have default at time 2 if the bad state is realized, i.e., we will have \( F = \frac{I}{p} > X^L \). Thus, in the case of risky convertible debt, the valuation of the marginal investor for the convertible security at time 1 will be given by

\[
p = \frac{\theta}{\theta + I x} X^H + (1 - \theta) \frac{X^L}{I/p}
\]  

(A.30)

which leads to

\[
\frac{x}{p} = \frac{I - (1 - \theta) X^L}{I[\theta X^H + (1 - \theta) X^L - I]}
\]  

(A.31)

From (A.31) and the conversion condition (A.15) in the good state, it follows that the conversion ratio \( x \) must be higher than \( \frac{\theta}{\theta X^H + (1 - \theta) X^L - I} \). Equivalently, the total face value of the risky convertible debt issue must be strictly less than the total face value of the risky straight debt issue, i.e., \( F < \frac{I - (1 - \theta) X^L}{\theta} \). The firm’s expected payoff from issuing risky convertible debt is

\[
EU^{Convertible} = \theta f \frac{p}{p + I x} X^H = \theta f \frac{1}{1 + I x/p} X^H
\]  

(A.32)

Plug in the value of \( \frac{x}{p} \) from above, we have

\[
EU^{Convertible} = \frac{\theta f}{\theta} (\theta X^H + (1 - \theta) X^L - I) = EU^{Debt},
\]  

(A.33)

no matter what the conversion ratio \( x \) is. Note that the firm’s expected payoff from issuing risky convertible debt
alone is equal to its expected payoff from issuing risky straight debt alone. (A.33) and (A.28) together also imply that issuing equity alone dominates issuing risky convertible as well, if the marginal investor is more optimistic than firm insiders. Q.E.D.

Proof of Proposition 1: From equations (A.7) and (A.10) we have

\[
EU^{\text{Equity}} - EU^{\text{Debt}} = \left(1 - \frac{I}{\theta X^H + (1 - \theta) X^L}\right) \left[\theta^f X^H + (1 - \theta^f) X^L\right] - \theta^f \left(X^H - \frac{I}{\theta X^H + (1 - \theta) X^L}\right).
\]

The firm will prefer equity to straight debt iff \(EU^{\text{Equity}} > EU^{\text{Debt}}\), which is equivalent to

\[
\frac{\theta X^H + (1 - \theta^f) X^L - I}{\theta X^H + (1 - \theta) X^L} > \frac{\theta^f X^H + (1 - \theta^f) X^L - I}{\theta X^H + (1 - \theta) X^L},
\]

(A.34)

when \(\theta X^H + (1 - \theta) X^L - I > 0\), it is equivalent to \(\theta^f \geq \theta\).

When \(\theta^f \geq \theta\), we have proved that convertible debt is dominated by straight debt (see equation (A.24)). When \(\theta^f < \theta\), we have proved that convertible debt is dominated by equity (see equation (A.28)). Therefore, convertible debt is never optimal. Q.E.D.

Proof of Proposition 2 First, we suppose that the firm issues \(I_D \in [0, X^L]\) units of debt, and each unit of debt has face value of 1. Since the debt is safe, the price of the debt is 1. The valuation of the equity to agent \(\theta\) is \(\theta X^H + (1 - \theta) X^L - I_D\).

The firm needs to raise \(I - I_D\) by issuing equity. The marginal equity investor \(\tilde{\theta}\) is characterized by

\[
\int_{\tilde{\theta}}^{\theta^m + d} W \frac{d\theta}{2d} = I - I_D
\]

or equivalently

\[
\tilde{\theta} = \theta^m + d - \frac{2d(I - I_D)}{W} = \bar{\theta} + \frac{2dI_D}{W},
\]

(A.35)

where \(\bar{\theta} = \theta^m + d(1 - \frac{d}{W})\). Investors in the range \([\tilde{\theta}, \bar{\theta}]\) will purchase the debt. The marginal equity investor values the entire equity of the firm at \(\tilde{\theta} X^H + (1 - \tilde{\theta}) X^L - I_D\). Suppose the firm needs to issue \(x\) shares of new equity to raise \(I - I_D\), then we have

\[
x = \frac{I - I_D}{\tilde{\theta} X^H + (1 - \tilde{\theta}) X^L - I_D}
\]

or equivalently

\[
x = \frac{I - I_D}{\bar{\theta} X^H + (1 - \bar{\theta}) X^L - I_D},
\]

(A.36)

The expected payoff to the insiders will be

\[
EU^{\text{Comb}} = \frac{1}{1 + x} \left[\theta^f X^H + (1 - \theta^f) X^L - I\right] = \frac{\tilde{\theta} X^H + (1 - \tilde{\theta}) X^L - I}{\tilde{\theta} X^H + (1 - \tilde{\theta}) X^L - I_D} \left[\theta^f X^H + (1 - \theta^f) X^L - I\right]
\]

The objective of the insiders is to choose the optimal amount of debt, \(I_D^*\), to be issued:

\[
\begin{align*}
\max_{I_D \in [0, X^L]} & \quad \frac{\tilde{\theta} X^H + (1 - \tilde{\theta}) X^L - I}{\tilde{\theta} X^H + (1 - \tilde{\theta}) X^L - I_D} \left[\theta^f X^H + (1 - \theta^f) X^L - I\right] \\
\text{s.t.} & \quad \tilde{\theta} = \bar{\theta} + \frac{2dI_D}{W}.
\end{align*}
\]
We have the following F.O.C. to this maximization problem:

\[
\frac{\partial EU_{\text{Combi}}}{\partial I_D} = \frac{-\bar{X}^2 - 2aI_D\bar{X} + a(1-a)I_D^2 + X^f\bar{X} + I\bar{X} - (1-a)IX^f}{[\bar{X} - (1-a)I_D]^{\frac{3}{2}}} = 0,
\]

\[
= a[\bar{X} - (1-a)I_D]^2 - [\bar{X} - (1-a)X^f][\bar{X} - (1-a)I] = 0.
\]

Thus, the optimal amount of debt to be raised is

\[
I_D^* = \frac{a\bar{X} - \sqrt{a[\bar{X} - (1-a)X^f][\bar{X} - (1-a)I]}}{a}.
\]  

(A.37)

where

\[
\bar{X} = \theta X^H + (1-\theta)X^L, \quad X^f = \theta X^H + (1-\theta)X^L, \quad a = \frac{2d(X^H - X^L)}{W} > 0.
\]

Note that if \(I_D^* \leq 0\), we obtain the corner solution of issuing equity only. In particular, we find that equity alone dominates debt-equity combination when \(I_D^* \leq 0\), or equivalently,

\[
\bar{X}^2 - (I + X^f)\bar{X} + (1-a)IX^f \geq 0.
\]  

(A.38)

If we rearrange this inequality, we obtain the following equivalent restriction in terms of \(\theta\): \(\theta \geq \theta_1\), where \(\theta_1\) is given by

\[
\theta_1 = \frac{-b_1 - \sqrt{-b_1^2 - 4a_1c_1}}{2a_1},
\]  

(A.39)

where

\[
a_1 = (X^H - X^L)^2, \\
b_1 = -(X^H - X^L)(I + X^f - 2X^L), \\
c_1 = (1-a)IX^f - X^L(I + X^f - X^L).
\]

Note that if the firm issues safe debt, it holds that \(I_D \leq X_L < I\) by assumption. Thus, in the case of safe debt, we have \(I - I_D > 0\), and therefore issuing safe debt alone to raise \(I\) is infeasible. Note also that when we solve for the optimal amount of risk-free debt to be issued, we also have to consider the constraint that \(I_D^* > X_L\). When we plug in the unconstrained optimum \(I_D^*\) from (A.37) into this constraint, we obtain the following equivalent restriction in terms of \(\theta\): \(\theta \geq \theta_2\), where \(\theta_2\) is given by

\[
\theta_2 = \frac{-b_2 - \sqrt{-b_2^2 - 4a_2c_2}}{2a_2},
\]  

(A.40)

where

\[
a_2 = (X^H - X^L)^2, \\
b_2 = (X^H - X^L)(2(1 + a)X^L - X^f - I), \\
c_2 = (X^L)^2 + X^L(2aX^L - X^f - I) - (1-a)(a(X^L)^2 - IX^f).
\]

In other words, if \(\theta \geq \theta_2\), the solution to the above maximization problem is to set \(I_D = X_L\).

Now we consider the case when it is optimal for the firm to issue risky debt. Suppose that the firm wants to raise amount \(I - I_D\) by issuing equity, and amount \(I_D\) by issuing debt. The debt is risky so that \(I_D > X_L\). The
face value of debt $D$ (number of units of debt the firm has to issue) can be determined by the following equation

$$I_D = \bar{\theta} D + (1 - \bar{\theta}) X^L \quad (A.41)$$

The face value of debt is therefore

$$D = \frac{I_D - (1 - \bar{\theta}) X^L}{\bar{\theta}} \quad (A.42)$$

The marginal equity investor is characterized by his belief $\theta = \bar{\theta} + \frac{2d I_D}{W}$. Investors in the range $[\bar{\theta}, \bar{\theta}]$ will purchase the debt, and the price of debt will be determined by the marginal debt investor with belief $\bar{\theta}$. The total market value of equity in this case is $\bar{\theta}[X^H - D]$. Suppose the firm needs to issue $x$ shares of new equity to raise $I - I_D$, then we have

$$x = \frac{I - I_D}{\bar{\theta}[X^H - D]} \quad (A.43)$$

or equivalently

$$x = \frac{I - I_D}{\bar{\theta}[X^H - D] - (1 - I_D)} \quad (A.44)$$

Therefore, the expected payoff of the insiders is equal to

$$EU^{Combi} = \left(1 - \frac{I - I_D}{\bar{\theta}(X^H - D)}\right) \bar{\theta}(X^H - D) \quad (A.45)$$

The firm has to choose the optimal split between debt and equity, i.e.,

$$\max_{I_D \in [X^L, I]} \left(1 - \frac{I - I_D}{\bar{\theta}(X^H - D)}\right) \bar{\theta}(X^H - D) \quad (A.46)$$

subject to $\bar{\theta} = \bar{\theta} + \frac{2d I_D}{W}$ and $D = \frac{I_D - (1 - \bar{\theta}) X^L}{\bar{\theta}}$.

The solution to this maximization problem is

$$I^*_D = \sqrt{\bar{\theta} W (\bar{\theta} W + 2d I) - \bar{\theta} W} \quad (A.47)$$

Note that it is never true that $I^*_D \geq I$ where $I^*_D$ is given by (A.46). Therefore, issuing risky debt alone is always dominated by issuing a combination of debt and equity. Note also that an interior optimal solution to this problem is obtained only if the constraint $I^*_D \geq X^L$ doesn’t bind. Given (A.46), this leads to to the following necessary condition for the optimality of an interior solution:

$$2d I^2 - [(\theta + d) W + 4d X^L] I + 2X^L ((\theta + d) W + d X^L) > 0. \quad (A.48)$$

Equivalently,

$$I > I_{2b} = \frac{(\theta + d) W + 4d X^L - \sqrt{[(\theta + d) W + 4d X^L]^2 - 16d X^L ((\theta + d) W + d X^L)}}{4d} \quad (A.49)$$

Thus, if $I \leq I_{2b}$, it is optimal to set $I_D = X^L$ in the above maximization problem. We assume that $I$ is sufficiently large relative to $X^L$ so that the condition in (A.48) is always satisfied.

Finally, from the above discussion, it is clear that if $\bar{\theta} < \theta_2$, it is optimal for the firm to issue a combination of risky debt and equity rather than a combination of risk-free debt and equity. Moreover, by comparing the indirect utility functions of the above optimization problems, it is easy to verify that there exists a threshold value of $\theta_2 > \bar{\theta}$, such that it is optimal for the firm to issue a combination of risky debt and equity if $\theta_2 \leq \bar{\theta} < \theta_2$ as well. If, however, $\theta_2 \leq \bar{\theta} < \theta_1$, it is optimal for the firm to issue a combination of risk-free debt and equity.
Proof of Proposition 3: When the firm issues convertible debt, let us assume that the firm chooses the conversion ratio optimally so that the face value of the convertible debt is $F^*$. The price of the convertible debt (both the equity component and the straight debt component) is determined by the valuation of the marginal investor $\tilde{\theta}$. Now assume that the firm issues $F^*$ units of straight debt and issues equity to raise the remaining amount, $I - PD_1 \times F^*$. The price of straight debt is determined by the valuation of the marginal investor $\tilde{\theta}$, while the price of equity is determined by the marginal investor $\hat{\theta} = \tilde{\theta} + \frac{2dPD_1 + F^*}{W}$.

If $\tilde{\theta} \leq \theta^f$, we showed in Lemma 3 that $F^*$ can take any value in the interval $[X^L, \frac{I - (1 - \tilde{\theta})X^L}{\tilde{\theta}}]$. Without loss of generality, set $F^* = X^L$. Thus, $PD_1 = 1$. Then, we know from Lemma 3 that the expected payoff to the current shareholders from issuing convertible debt only is equal to

$$EU^{\text{convertible}} = \frac{\theta^f}{\tilde{\theta}} (\tilde{\theta}X^H + (1 - \tilde{\theta})X^L - I),$$

$$= \theta^f (X^H - X^L) - \frac{\theta^f}{\tilde{\theta}} (I - X^L).$$

If the firm issues a combination of risk free debt and equity, with the amount of debt issued equal to $I_D = X^L$, the expected payoff to the current shareholders (from Proposition 2) is given by

$$EU^{\text{combined}} = \left(1 - \frac{I - X^L}{\theta^f} \frac{1 - (1 - \tilde{\theta})X^L}{X^L} \right) \theta^f (X^H - X^L),$$

$$= \theta^f (X^H - X^L) - \frac{\theta^f}{\theta^f + 2dX^L}(I - X^L).$$

Thus, if $\tilde{\theta} \leq \theta^f$, $EU^{\text{combined}} > EU^{\text{convertible}}$. Thus, the firm can sell the equity component at a higher price to outside investors if it issues a debt-equity combination rather than convertible debt only. If $\tilde{\theta} > \theta^f$, we know from Lemma 3 that issuing equity alone dominates issuing convertible debt alone. Moreover, in this case, it follows from Proposition 2 that, if $I > I_1$, issuing a combination of debt and equity dominates issuing equity alone. Thus, $EU^{\text{combined}} > EU^{\text{convertible}}$, if $\tilde{\theta} > \theta^f$ as well. Q.E.D.

Proof of Proposition 4: When outsiders’ average belief is optimistic, and dispersion is high, so that $\theta^f \leq \tilde{\theta}$, we have shown in the proof of Lemma 3 that $E^{\text{Convertible}} = X^f - I \frac{X^f}{X} = E^{\text{Equity}}$. Therefore, equity will dominate convertible debt. We have already shown that equity dominates debt in Proposition 1. Hence, it is optimal for the firm to issue equity in this case.

When $\theta^f > \tilde{\theta}$, the choice is between convertible debt and straight debt and which one of these is optimal depends on the required investment financing $I$. If the firm can issue risk-free straight debt, the costs of financial distress will be 0 for both securities. Lemma 3 shows that risk-free convertible debt will be more undervalued than risk-free straight debt. Therefore, the firm will choose to issue risk-free straight debt alone in order to minimize the undervaluation cost. If the firm’s straight debt issue has to be risky since $I > X^L$, we know from Lemma 3 that the undervaluation of the risky straight debt is the same as the undervaluation of the risk-free convertible debt with total face value $F = X^L$. Since the firm can minimize the cost of financial distress to 0 by setting $F = X^L$, it will optimally choose to issue risk-free convertible debt with face value $F = X^L$ rather than risky straight debt. Q.E.D.

Proof of Proposition 5: The proof of part (i) is similar to that of Proposition 2. One difference relates to the issue cost $C^f$. Note that the firm now has to incur the issue cost $C^f$ for each tranche of security when it issues a combination of debt and equity. Since $C^f$ is nonzero, the belief threshold $\theta_{1,b}$ above which the firm finds it optimal to issue equity alone rather than a combination of risk-free debt and equity will be higher compared to the value of $\theta_1$ in Proposition 2.

The proof of part (ii) is as follows. If the outsiders’ average belief about future cash flows is not too high, and the dispersion in outsider beliefs is not too large so that $\tilde{\theta} < \theta_{1,b}$ (i.e., $X^2 - (I + X^f)X + (1 - a)IX^f < 0$, and issue cost $C^f$ is not too high), it is not optimal for the firm to issue equity alone because equity will be too much undervalued by outsiders. In this case, the choice is between issuing an equity-debt combination and issuing debt alone.
First, regardless of the issue cost, the firm has to determine whether it is preferable to issue a combination of risk-free debt and equity or a combination of risky debt and equity. From Proposition 2, we know that there exists a threshold belief level $\theta_2$ below which the firm finds it optimal to issue a combination of risky debt and equity, since selling a combination of a large amount of equity and risk-free debt involves high undervaluation costs as the marginal investor becomes less optimistic. But now in the full-fledged model, selling risky debt will also be costly, since the firm expects a positive cost of financial distress $C^H$ in the case of a default when the low state cash flow $X^L < I$ is realized. Therefore, the belief threshold below which the firm finds it optimal to issue a combination of risky debt and equity decreases from $\theta_2$ to $\theta_{2,b}$.

Second, once the firm insiders determine which combination of debt and equity is most preferable, they compare it to issuing debt only. We have proved in Proposition 2 that issuing a combination of debt and equity dominates issuing debt alone without issue costs. However, with positive issue cost $C^I$, the firm faces a trade-off. It is optimal for the firm to issue a combination of equity and debt, if the issue cost is low ($C^I \leq C^I_1$ if $\theta_{2,b} \leq \bar{\theta} < \theta_{1,b}$, and $C^I \leq C^I_2$ if $\bar{\theta} < \theta_{2,b}$). Otherwise, the firm will choose to issue straight debt alone when the issue cost is too high, since the additional issue cost associated with the equity-debt combination will outweigh its benefits. Q.E.D.

**Proof of Proposition 6:** The proof is very similar to the proof of part (ii) of Proposition 5. The firm faces a choice between issuing a combination of debt and equity and issuing convertible debt if and only if the outsiders’ average belief about future cash flows is not too high, and the dispersion in outsider beliefs is not too large so that $\bar{\theta} < \theta_{1,b}$ (i.e., $\bar{X}^2 - (I + X^H)X + (1-\alpha)IX^L < 0$, and issue cost $C^I$ is not too high). In the absence of issue costs and costs of financial distress, we showed in Proposition 3 that a combination of equity and straight debt will lead to a higher market valuation of the firm’s security issue than convertible debt alone. This is because the firm can sell equity and debt to different groups of investors while the firm is forced to sell the convertible debt to the same group of investors. However, issuing equity-debt combination instead of convertible debt leads to additional issue costs. The firm will choose the optimal security based on this trade-off. As in the proof of Proposition 5, we first determine whether it is preferable to issue a combination of risk-free debt and equity or a combination of risky debt and equity (when the issue costs are zero) based on the trade-off between selling undervalued equity and costs of financial distress $C^H$. Then, the firm compares the preferred debt-equity combination to issuing convertible debt alone. The firm will choose a combination of debt and equity, if the issue cost $C^I$ is low ($C^I \leq C^I_3$ if $\theta_{2,b} \leq \bar{\theta} < \theta_{1,b}$, and $C^I \leq C^I_4$ if $\bar{\theta} < \theta_{2,b}$). Otherwise, the firm will choose to issue convertible debt. Q.E.D.

**Proof of Proposition 7:** The marginal investor’s belief on the firm value is $\theta^m + d$ at time 0. Investors anticipate that the firm will issue $E_1$ shares of new equity at time 1 to raise the amount $I$. The price of stock before issuance of equity is therefore

$$PE_0^{Equity} = \frac{1}{1 + E_1}[\theta^m + d]X^H + (1-\theta^m - d)X^L]$$  \hspace{1cm} (A.53)

$$= \left(1 - \frac{I}{\theta X^H + (1-\theta)X^L}\right)\left[(\theta^m + d)X^H + (1-\theta^m - d)X^L\right]$$  \hspace{1cm} (A.54)

After the issuance of equity, the total number of shares outstanding is $1 + E_1$, and the marginal investor is now $\bar{\theta}$ and the equity price per share is

$$PE_1^{Equity} = \frac{1}{1 + E_1}[\bar{\theta} X^H + (1-\bar{\theta})X^L] = \bar{\theta}X^H + (1-\bar{\theta})X^L - I$$  \hspace{1cm} (A.55)

Therefore, the stock price will go down immediately after more shares come to the market. The price impact is

$$\Delta PE^{Equity} = PE_1^{Equity} - PE_0^{Equity} = -\left(1 - \frac{I}{\theta X^H + (1-\theta)X^L}\right)\frac{2dI}{W}(X^H - X^L) < 0. \hspace{1cm} (A.56)$$
If the firm issues debt, the face value of the debt is \( F = \frac{I - (1 - \bar{\theta})X_L}{\bar{\theta}} \) and the equity price will be

\[
P_{E_0}^{\text{Debt}} = P_{E_1}^{\text{Debt}} = (\theta^m + d) \left( X^H - I - (1 - \bar{\theta})X_L \right)
\]  

(A.57)

at both time 0 and time 1. Therefore, the price impact of debt issuance is zero.

Note that

\[
\partial | \Delta P_{E_{\text{Equity}}} | / \partial d > 0
\]  

(A.59)

which means the price impact of equity increases with the dispersion in outsiders’ beliefs. Q.E.D.

Proof of Proposition 8: We showed in the proof of Proposition 7 that the stock price at issuance is

\[
P_{E_0}^{\text{Equity}} = \frac{\bar{\theta}X^H + (1 - \bar{\theta})X_L}{1 + \frac{I}{\theta X^H + (1 - \bar{\theta})X_L - I}}
\]  

(A.60)

where \( \bar{\theta} = \theta^m + d \left( 1 - \frac{2I}{W} \right) \). The stock price at time 2 is

\[
P_{E_2}^{\text{Equity}} = \frac{\bar{\theta}_2X^H + (1 - \bar{\theta}_2)X_L}{1 + \frac{I}{\theta X^H + (1 - \bar{\theta}_2)X_L - I}}
\]  

(A.61)

where \( \bar{\theta}_2 = \theta^m + d(1 - \delta) \left( 1 - \frac{2I}{W} \right) \). The long-run stock return is therefore

\[
LR_{E_{\text{Equity}}} = \frac{P_{E_2}^{\text{Equity}} - P_{E_1}^{\text{Equity}}}{P_{E_1}^{\text{Equity}}} = \frac{-\delta d \left( 1 - \frac{2I}{W} \right) (X^H - X_L)}{\theta X^H + (1 - \bar{\theta})X_L} < 0.
\]  

(A.62)

Q.E.D.

Proof of Proposition 9: Partially differentiating \( LR_{E_{\text{Equity}}} \) from (A.62), we obtain:

\[
\frac{\partial LR_{E_{\text{Equity}}}}{\partial \delta} = \frac{-d \left( 1 - \frac{2I}{W} \right) (X^H - X_L)}{\theta X^H + (1 - \bar{\theta})X_L} < 0,
\]  

(A.63)

\[
\frac{\partial LR_{E_{\text{Equity}}}}{\partial d} = \frac{-\delta \left( 1 - \frac{2I}{W} \right) (X^H - X_L) (\theta^m X^H + (1 - \theta^m)X_L)}{(\theta X^H + (1 - \bar{\theta})X_L)^2} < 0.
\]  

(A.64)

Q.E.D.

Proof of Proposition 10: The equity price at the time of straight debt issue (time 1) is

\[
P_{E_1}^{\text{Debt}} = (\theta^m + d) \left( X^H - I - (1 - \bar{\theta})X_L \right)
\]  

(A.65)

and the equity price at time 2 is

\[
P_{E_2}^{\text{Debt}} = (\theta^m + d(1 - \delta)) \left( X^H - I - (1 - \bar{\theta})X_L \right)
\]  

(A.66)
The long-run stock return subsequent to the straight debt issue is therefore

\[ LR_{\text{Debt}} = \frac{PE^{2}_{\text{Debt}} - PE^{1}_{\text{Debt}}}{PE^{1}_{\text{Debt}}} = -\frac{\delta d}{\theta m + d} < 0. \]  
(A.67)

For convertible debt, at the time of convertible debt issue (time 1), we have

\[ PE^{1}_{\text{Convertible}} = (\theta m + d)\frac{1}{1 + xF}X^{H}, \]  
(A.68)

and the equity price at time 2 is

\[ PE^{2}_{\text{Convertible}} = (\theta m + d(1 - \delta))\frac{1}{1 + xF}X^{H}. \]  
(A.69)

The long-run stock return subsequent to the convertible debt issue is therefore

\[ LR_{\text{Convertible}} = \frac{PE^{2}_{\text{Convertible}} - PE^{1}_{\text{Convertible}}}{PE^{1}_{\text{Convertible}}} = -\frac{\delta d}{\theta m + d} < 0. \]  
(A.70)

Note that \( \frac{\partial LR_{\text{Debt}}}{\partial \delta} = -\frac{-d}{\theta m + d} < 0 \) and \( \frac{\partial LR_{\text{Debt}}}{\partial \theta} = -\frac{-\theta m}{(\theta m + d)^2} < 0 \). Q.E.D.

**Proof of Proposition 11:** We assume that \( d \) is uniformly distributed in the interval \([0, 1 - \theta m]\). The firm will issue equity at time 1 only when \( \theta^l < \tilde{\theta} \), i.e., when \( d > \frac{\theta^l - \theta m}{1 - \theta^l} \). Therefore, the expected long-run stock return of equity issuance is

\[
LR_{\text{Equity}} = \int_{\frac{\theta^l - \theta m}{1 - \theta^l}}^{1 - \theta m} dx \frac{1 - 2l}{\theta m - \frac{\theta^l - \theta m}{1 - \theta^l} dx} \left( X^{H} - X^{L} \right) \frac{1}{1 - \theta m + \frac{\theta^l - \theta m}{1 - \theta^l}} dx,
\]

\[
= -\delta \left( X^{H} - X^{L} \right) \frac{1 - \theta m - \frac{\theta^l - \theta m}{1 - \theta^l}}{1 - \theta m + \frac{\theta^l - \theta m}{1 - \theta^l}} \left( X^{H} - X^{L} \right) \ln \left( \frac{X^{m} + \frac{\theta^l - \theta m}{1 - \theta^l}(X^{H} - X^{L})}{X^{m} + \frac{\theta^l - \theta m}{1 - \theta^l}(X^{H} - X^{L})} \right). \]  
(A.71)

The expected long-run stock return following debt issues are

\[
LR_{\text{Debt}} = \frac{1 - 2l}{\theta m - \theta^l} \int_{0}^{\frac{\theta^l - \theta m}{1 - \theta^l}} \frac{-\delta x}{\theta m + x} dx = \left( \frac{1 - 2l}{\theta m - \theta^l} \delta \left( x - \theta m \ln(\theta m + x) \right) \right)_{0}^{\frac{\theta^l - \theta m}{1 - \theta^l}},
\]

\[
= -\left( \frac{1 - 2l}{\theta m - \theta^l} \delta \left( \frac{\theta^l - \theta m}{1 - \theta^l} - \theta m \ln \left( 1 + \frac{\theta^l - \theta m}{\theta m (1 - 2l)} \right) \right) \right),
\]

\[
= -\delta \left( 1 - \frac{\theta^m (1 - 2l)}{\theta^l - \theta m} \ln \left( 1 + \frac{\theta^l - \theta m}{\theta m (1 - 2l)} \right) \right). \]  
(A.72)
Note that $\frac{LR_{Equity}}{LR_{Debt}} < 1$ if and only if the following parameter condition is satisfied:

$$
1 - \frac{\theta_m(1 - \frac{2I}{W})}{\theta_f - \theta_m} \ln \left( 1 + \frac{\theta_f - \theta_m}{\theta_m(1 - \frac{2I}{W})} \right) - \frac{X^m}{(1 - \frac{2I}{W})(1 - \theta_f - \frac{2I}{W}(1 - \theta_m))(X^H - X^L)} \ln \left( \frac{X^m + (1 - \frac{2I}{W})(X^H - X^L)(1 - \theta_m)}{X^m + (1 - \frac{2I}{W})(X^H - X^L) \frac{\theta_f - \theta_m}{1 - \frac{2I}{W}}} \right) > 0. \quad (A.73)
$$

Q.E.D.