Common Factors, Information, and Portfolio Choice

Patrice Fontaine
University Pierre Mendès–France and Eurofidai

Sonia Jimenez-Garcés
University of Lyon and Eurofidai

Mark S. Seasholes
HKUST

This Version 23-Dec-2009

Abstract

We derive closed-form solutions for asset prices and portfolio holdings when agents have asset-specific information and/or information about common factors that affect many assets. The solutions are general and can be used to model a number of information structures—including international equity markets. Our model produces varying degrees of home bias (a preference for local assets), no bias (portfolio weights mirror market-capitalization weights), and reverse home bias (a preference for foreign assets).

JEL Classification: D82, G11, G12, G15

Keywords: Information Economics, REE Models

*We thank Magnus Dahlquist, Harald Hau, Soeren Hvidkjaer, Jordi Mondria, Bruno Solnik for helpful comments and suggestions as well as seminar participants at the 2009 Econometric Society Meetings (Boston), 2009 AFA Meetings (San Francisco), 2008 Cerag-Ensimag-Ilsa joint seminar, 2007 AFFI Conference (Paris), 2007 ASAP Conference (London), ESSEC Paris, HEC Paris, Tilburg University, and University of Rotterdam. Questions and comments can be emailed to “sonia.jimenez@wanadoo.fr” or to “Mark.Seasholes@emailias.com”. 
1 Introduction

This paper derives closed-form solutions for asset prices and portfolio holdings in a multi-asset, rational expectations equilibrium (REE) model. The model contains multiple agents (investors) who invest today and consume next period. The investors may receive information about a specific asset’s payoffs and/or information about common components (factors) that affect the payoffs of many assets.

The inclusion of both asset-specific and common-component information leads to rich and varied portfolio holdings. For example, an investor may have high demand for a stock for which he has valuable private information. He may also have high demand for the same stock if he has valuable private information about a stock with highly correlated payoffs. When considering common components, an investor may have high demand for a stock (even if he does not have asset-specific information) provided he has information about a common component of the stock’s payoffs. Of course, having information about a common component is not sufficient to determine if the investor will have high demands for certain stocks. Stocks must load sufficiently on the common component (factor) to outweigh private, asset-specific information that other investors’ may have. Finally, and in multi-asset settings, agents balance information about a given asset with a desire to diversify wealth across many assets.

Our model shows how an investor’s holdings of an asset is determined by his own asset-specific information, his own common-component information, and the asset’s factor loadings. The investor’s holdings are also determined by other investors’ information about common components as well as other investors’ asset-specific information. The rational expectations framework accounts for other agents simultaneously making portfolio decisions based on their common-component information, their asset specific information, asset loadings, and their desires to diversify holdings.

To highlight the range of equilibrium portfolio holdings, we analyze our results in the context of international equity markets. Investors and assets are partitioned into different nationalities and national markets around the world. As is common in studies of international finance, we assume investors from a given country (say France) have asset-specific information about assets from that country (French stocks). Such an assumption leads to the well-known “home-bias” phenomenon in which investors overweight assets from their home market (i.e., weights are higher than market capitalization-based weights).

More importantly, we assume some investors have information about market-wide factors that affect the prices of many stocks. Information about a common component can be thought of as the ability to better predict future interest rate movements, commodity
prices, or production outputs. The introduction of common-component information can lead some investors to exhibit “reverse home bias”—a situation in which they overweigh some foreign assets. Whether an investor exhibits home bias, no bias, or reverse home bias depends on the asset-specific and common-component information the investor has as well as the asset-specific and common-component information of other investors.

In solving a model with asset-specific and factor information, we provide generalized, closed-form solutions to an economic problem that has been outstanding for the past 25 years—see Admati (1985, p.653). Our solutions encompass a number of information structures. For example, we can model the information structure used in traditional home bias studies. Figure 1, Panel A depicts a market in which investors have information about local assets but not about foreign assets.

Figure 1, Panel B depicts a market with multiple factors. While factors affect payoffs of different assets, no group of investors has information about the factors. A similar structure is considered by Kodres and Pritsker (2002) and the paper is discussed below in Section 1.1. Figure 1, Panel C shows a market with a single, global factor. The factor is known by a single group of investors. A similar (but dynamic) structure is considered by Albuquerque, Bauer, and Schneider (2006) and this paper is also discussed below in Section 1.1.

The closed-form solutions allow insights into the effect of information on prices. For example, we consider the case of symmetric/complete information—i.e., all investors receive all available asset-specific and common component information. This structure leads to a form of the Capital Asset Pricing Model (or “CAPM”). The difference between the complete/symmetric case and our generalized solution can be thought of as an “information price discount”—the amount an asset’s price is below the CAPM price due to agents not having full information about future payoffs. Our solution for the information price discount can be written as a signal-to-noise difference in closed-form. The following section discusses recent and related advances in solving the factor-information problem mentioned in Admati (1985).

1.1 Literature Review

Our paper is related to theoretical work on information structures, investor holdings, and risk premia. Easley and O’Hara (2004) present a multi-asset model that focuses on the role of public and private signals in determining a firm’s cost of capital. Private signals
in their model are received only by a group of informed investors as in Grossman and Stiglitz (1980). In our model, it is possible for different groups of investors to have information about different groups of the securities. In this way, investors can be asymmetrically informed without introducing a strict information hierarchy.1 Bacchetta and van Wincoop (2006) argue in favor of structures with a “[broad] dispersion of information.”

In a paper similar in spirit to ours, Hughes, Liu, and Liu (2007) model two groups of investors. Each informed investor effectively observes a global signal “s” and these signals are perfectly correlated across investors. Unlike our paper, investors in the Hughes, Liu, and Liu (2007) paper cannot separate the asset-specific and global components. Additionally, information about the components is not differentially dispersed across investors. Similarly, Kodres and Pritsker (2002) offer a model that contains an underlying factor structure. However, there are no information asymmetries regarding the factors.

Our paper has both important differences from, and certain similarities to, a recent paper by Albuquerque, Bauer, and Schneider (2006). Their paper contains multiple stocks and multiple time periods. The payoff of a given stock is equal to the sum of three terms: a constant, a local component, and a single global factor. There are public and private signals about both the local components and the global factor. Only one group of investors (from the USA) receives private information about the global factor. Our model contains multiple, global factors, each of which may be known by a different group of investors. Moreover, their model uses a single factor loading (equal to one) for all assets. Our model is more general because it allows for different common factor loadings—both across assets and across factors. Most importantly, American investors in their model have the same informational advantage vis-a-vis each foreign stock. This assumption implies that their model does not generate cross-border holding dispersion based on common-factor information. Our model is more general and produces large differences in cross-border holdings (home bias). In our model, this dispersion is tied directly to informational differences about common factors.

The paper proceeds as follows. Section 2 presents our model, notation, and assumptions. We focus on closed-form solutions for asset prices and investor portfolios (holdings). Section 3 analyzes equilibrium asset prices and portfolios. This section focuses on analytical expressions. Section 4 provides a numerical analysis of prices and holdings. This section builds economic insights to better understand our model’s solutions. The final section concludes.

---

2 Model

The model has $I$ investors indexed $i = 1, \ldots, I$ who trade at date 0 and consume at date 1. Each agent $i$ can invest his initial wealth, $w_i^0$, in a riskless asset and $J$ risky assets indexed $j = 1, \ldots, J$. The riskless interest rate is denoted $r_f$ and we define $R \equiv (1 + r_f)$.

For simplicity, we normalize the price of the riskless asset to one. Each risky asset $j$ pays a liquidating dividend $\tilde{P}_j^1$ at date 1. The vector of final payoffs $\tilde{P}^1 = (\tilde{P}_1^1, \ldots, \tilde{P}_J^1)'$ is generated by a $K$-factor linear process:

$$\tilde{P}^1 = \tilde{\theta} + B \tilde{f} + \tilde{\varepsilon}$$  \hspace{1cm} (1)

The vector $\tilde{\theta} = (\tilde{\theta}_1, \ldots, \tilde{\theta}_J)'$ is the asset-specific component of payoffs, the vector $\tilde{f} = (\tilde{f}_1, \ldots, \tilde{f}_K)'$ contains the $K$ common components (factors), and $B$ is a $J \times K$ matrix of factor loadings. The remaining part of each asset’s final payoff, $\tilde{\varepsilon} = (\tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_J)'$, is referred to as residual uncertainty. We assume that $\tilde{\theta}, B \tilde{f}$, and $\tilde{\varepsilon}$ have mean zero. Since $\tilde{\theta}$ is the asset-specific component, we assume its covariance matrix is diagonal\(^2\) and denoted $\Sigma_{\theta}$. For tractability, we assume that the covariance matrix of $\tilde{f}$ is the identity matrix. The covariance matrix of $B \tilde{f}$ is $BB'$. Finally, the covariance matrix of $\tilde{\varepsilon}$ is denoted $\Sigma_{\varepsilon}$.

The per-capita supply of risky assets is defined as the realization of a random vector $\tilde{z}$. The vector $\tilde{z}$ is independent and jointly normally distributed along with the other variables in the model and has a covariance matrix denoted $\Sigma_{z}$. The assumption of random net supply is standard in rational expectations models. As Easley and O’Hara (2004) write “one theoretical interpretation is that this approximates noise trading in the market. A more practical example of this concept is portfolio managers current switch toward using float-based indices from shares-outstanding indices.” In order to insure the existence and uniqueness of the date 0 equilibrium price vector, $\tilde{P}^0$, we assume that $\Sigma_{z}$, $\Sigma_{\theta}$, and $\Sigma_{\varepsilon}$ are regular matrices.

We assume all agents have an exponential utility function: $U(\tilde{w}_i^1) = -e^{-a\tilde{w}_i^1}$, where $\tilde{w}_i^1$ is the wealth of investor $i$ on date 1. The utility function has a constant absolute risk aversion with coefficient $a > 0$ which is the same for all agents. The choice of utility functions is also common in rational expectations equilibrium models and ensures that an investor’s demand for the risky asset is independent of his initial wealth. Let $X_i$ be investor $i$’s vector of holdings of the risky assets. Investor $i$’s final wealth is:

$$\tilde{w}_i^1 = w_i^0 R + X_i' (\tilde{P}^1 - R \tilde{P}^0)$$  \hspace{1cm} (2)

\(^2\)This assumption is not necessary to solve the model. However, it enables us to distinguish between information about single assets and information about common components that affect two or more assets.
2.1 Information Structure and Notation

We partition the $I$ investors in our model into $N$ non-overlapping groups labeled $n = 1, \ldots, N$. A group of investors can be thought of as a nationality (French investors, Japanese investors, etc.) Each group of investors represents a fraction, $\lambda_n$, of the total number of investors ($I$) in the market such that $\sum_{n=1}^{N} \lambda_n = 1$.

Asset-Specific Information: The $J$ securities are partitioned into $N$ non-overlapping groups. A group of securities can be thought of as comprising a country’s equities (French stocks, Japanese stocks, etc.). We define the set of all assets as $S$. The set of assets in group $n$ contains $J_n$ risky assets and is denoted $S_n$. Thus, $\bigcup_{n=1}^{N} S_n = S$ and $\forall (n_a, n_b), n_a \neq n_b, S_{n_a} \cap S_{n_b} = \emptyset$. A single investor $i$ in group $n$ knows the realization of the asset-specific component, $\theta_j$, of each asset $j$ in the set $S_n$. For any asset $j$ not in $S_n$, investor $i$ only knows the distribution of $\tilde{\theta}_j$ but he does not know its realization. We assume there is an equal number ($N$) of securities groups and investors groups to ensure that each security has at least one investor with asset-specific information.

Common Component Information: We assign the $K$ common factors to $N$ groups denoted $F_n$, with $n = 1, \ldots, N$. The set $F_n$ contains $K_n$ common components. An investor $i$ in group $n$ knows the realization of each common component $\tilde{f}_k$ in the set $F_n$. For any component not in $F_n$, the investor only knows the distribution of $\tilde{f}_k$ but not its realization. For tractability purposes of the model, we assume that two groups of investors do not have information about the same common component. A depiction of the generalized information structure is shown in Figure 1, Panel D.

Chen et al. (1986) document nine macroeconomic risk factors affecting stock returns. We likewise envisage the number of common components to be much less than the number of assets, $K \ll J$. If the number of common components is less than the number of investor groups, then $K < N$, some $F_n$ sets will not contain any common components ($K_n = 0$), and the corresponding investor group will not be informed about any of the common components.

Notation: The information structure of our model implies that investors belonging to the same group $n$ possess the same private information (for asset-specific components and for common factors), they face the same optimization problem, and they optimally choose identical portfolios. In this sense they can be said to be identical. We use the following terms interchangeably (and a bit loosely): “investor $i$ from group $n$”, “investor group $n$”, and “investor $n$”. Furthermore, to simplify notation, we write the payoffs of the risky assets as:
\[ \hat{P}^1 = C\hat{\eta} + \hat{\varepsilon} \]  

Where, \( \hat{\eta} = (\hat{\theta}^\top \hat{f}^\top)^\top \) is a \( J + K \) column vector and \( C \) is a \( J \times (J + K) \) block-diagonal matrix consisting of a \( J \times J \) identity matrix, \( I_J \), and the matrix \( B \). The variance-covariance matrix of \( \hat{\eta} \) is \( Q = \begin{pmatrix} \Sigma_{\theta} & 0 \\ 0 & I_K \end{pmatrix} \) where \( I_K \) is the identity matrix of order \( K \).

**Definition 2.1.** For each investor \( n \), we define the diagonal matrix \( D_n \) of order \( J + K \) with \( D_n(j,j) = 1 \) if investor \( n \) knows the realization of the \( j^{th} \) random variable in \( \hat{\eta} \) and \( D_n(j,j) = 0 \) otherwise. The \( j^{th} \) random variable represents an asset-specific component of stock \( j \)'s payoffs if \( j \leq J \), and a common component otherwise.

**Definition 2.2.** We define \( D \equiv \sum_{n=1}^N \lambda_n D_n \). The matrix \( D \) plays an important role in our model as each element on the main diagonal represents the proportion of investors who know the realization of the corresponding random variable in the vector \( \hat{\eta} \).

**Definition 2.3.** For each investor group \( n \), the matrix \( M_n \) is obtained by eliminating all the null rows of \( D_n \). Consequently, the number of rows of \( M_n \) is equal to \( J_n + K_n \), which represents the number of asset-specific and common factors about which investor \( n \) is informed. If investor \( n \) does not receive any private information, \( D_n \) becomes the null matrix and \( M_n \) cannot be defined. It is straightforward that \( M_n' M_n = D_n \) and \( M_n M_n' = I_{J_n + K_n} \), where \( I_{J_n + K_n} \) is the identity matrix of order \( J_n + K_n \).

Under these definitions, the private information received by investor \( n \) consists of the realization of the random vector \( M_n \hat{\eta} \). As in Admati (1985), equilibrium prices also reveal some information to investors beyond their own private information. Consequently, each investor \( n \) maximizes his expected utility of consumption conditional on the realization of his private information and on the observation of the public information in the form of prices at date 0.

### 2.2 Equilibrium Prices and Holdings

We seek closed-form solutions for prices and holdings at date 0 within the class functions that are linear in our information variable \( \hat{\eta} \) and supply variable \( \hat{\varepsilon} \). The form of the solution implies investors assume prices are a linear function of private signals and noise. In equilibrium, this hypothesis is verified. The date 0 price vector is:

\[ \hat{P}^0 = A_0 + A_1 \hat{\eta} - A_2 \hat{\varepsilon} \]
where the dimensions of the vector $A_0$ is $J \times 1$, the matrix $A_1$ is $J \times (J + K)$, and the matrix $A_2$ is $J \times J$. We suppose that $A_2$ is regular. Under these assumptions, investor $n$’s demand is:

$$\tilde{X}_n = a^{-1}V_n^{-1} \left( E_n \left[ \tilde{P}^1 \right] - R\tilde{P}^0 \right)$$  \hspace{1cm} (5)$$

Equation (5) gives an expression for agent $n$’s holdings at date 0—please see Appendix A for additional details. The expression $E_n[\tilde{P}^1] = E[\tilde{P}^1|M_n\tilde{\eta}, \tilde{P}^0]$ gives the expected prices of the risky assets at date 1 from investor $n$’s point of view (i.e. conditional on his information set). $V_n = Var[\tilde{P}^1|M_n\tilde{\eta}, \tilde{P}^0]$ represents the conditional return variance of $\tilde{P}^1$ from investors $n$’s point of view. By equating the supply and the aggregate demand of the $N$ groups of investors, $\left( \sum_{n=1}^{N} \lambda_n \tilde{X}_n = \tilde{z} \right)$, it follows:

$$\sum_{n=1}^{N} \lambda_n V_n^{-1} \left( E_n \left[ \tilde{P}^1 \right] - R\tilde{P}^0 \right) - a\tilde{z} = 0 \hspace{1cm} (6)$$

Joint normality implies that the distribution of prices, conditional on investor $n$’s private and public information, is also multi-variate normal with the following expectation:

$$E_n \left[ \tilde{P}^1 \right] = E \left[ \tilde{P}^1|M_n\tilde{\eta}, \tilde{P}^0 \right] = B_{0n} + B_{1n}M_n\tilde{\eta} + B_{2n}\tilde{P}^0 \hspace{1cm} (7)$$

where the dimension of $B_{0n}$ is $J \times 1$, $B_{1n}$ is $J \times (J_n + K_n)$, and $B_{2n}$ is $J \times J$. Equations (4), (6), and (7) imply the system to be solved is (please see Appendix B):

$$aA_2^{-1}A_0 = \sum_{n=1}^{N} \lambda_n V_n^{-1}B_{0n}$$

$$aA_2^{-1}A_1 = \sum_{n=1}^{N} \lambda_n V_n^{-1}B_{1n}M_n \hspace{1cm} (8)$$

$$aA_2^{-1} = \sum_{n=1}^{N} \lambda_n V_n^{-1} \left( RI_J - B_{2n} \right)$$

As shown in Appendix C, the matrices $B_{1n}$, $B_{2n}$ and $V_n$ can be written as functions of the matrices $A_1$ and $A_2$. The system of equations in (8) represents a fixed point problem in a $2J^2 + JK + J$ Euclidian space.

**Closed-Form Solution:** To obtain a closed-form solution for $\tilde{P}^0$, we define the matrix $U \equiv A_2^{-1}A_1$. We also introduce the function $g(G) = \sum_{n=1}^{N} D_nGD_n$, where $G$ is a matrix of order $J + K$. The function $g(\cdot)$ transforms a matrix $G$ into a $N$-block diagonal matrix whose block elements are the same as the elements of the matrix $G$. 

7
Definition 2.4. We define a “g-matrix” to be any square matrix \( G \) of order \( J + K \) which satisfies \( g(G) = G \). This means that \( G \) is an \( N \)-block diagonal matrix, the size of block \( n \) is equal to the number of specific and common factors known by investor \( n \).

Define \( \Psi \equiv \text{Var} \left[ \tilde{\eta} | \tilde{P}^0 \right] \) i.e., the variance-covariance matrix of \( \tilde{\eta} \) conditional on observing the equilibrium price vector at date 0. The matrix \( \Psi \) is endogenously defined and represents the variance of \( \tilde{\eta} \) from the point of view of an investor who does not possess any private information but only observes the equilibrium price vector. The following lemma gives an analytical solution for \( U \).

Lemma 2.1. If \( (\Psi^{-1} + C' \Sigma_{\varepsilon}^{-1} C) \) is a g-matrix, then the closed-form solution for \( U \) is:

\[
U = a^{-1} \Sigma_{\varepsilon}^{-1} CD \tag{9}
\]

Proof: See Appendix D.

For the particular case of Lemma (2.1), \( U \) is not a function of the coefficients \( B_{0n}, B_{1n}, \) and \( B_{2n} \). Therefore, to determine \( A_0, A_1, \) and \( A_2 \), we must first compute the matrix \( \Psi \) as a function of \( U \). In this way, the variance-covariance matrix of any investor group, \( V_n \), can be written as a function of \( \Psi \):

\[
V_n = \Sigma_{\varepsilon} + C \Psi C' - C \Psi M_n' \Psi^{-1} M_n \Psi C' \tag{10}
\]

Where \( \Psi_n = M_n \Psi M_n' \). Also, \( \Psi = Q - QU'M^{-1}UQ \) and \( M = UQU' + \Sigma_{\varepsilon} \). The following theorem gives a closed-form solution for the equilibrium price vector at date 0.

Theorem 2.1. Under the conditions of Lemma (2.1), there exists a closed-form solution for Equation (6) within the class of linear functions of \( \tilde{\eta} \) and \( \tilde{z} \). The solution can be written as, \( \tilde{P}^0 = A_0 + A_1 \tilde{\eta} - A_2 \tilde{z} \), where \( A_2 \) is a regular matrix and:

\[
A_0 = \frac{1}{R} \left( \left( C - RA_1 \right) E[\tilde{\eta}] + \left( RA_2 - aV_N \right) E[\tilde{z}] \right) \tag{11}
\]

\[
A_1 = \frac{1}{R} \left(CQC' + \Sigma_{\varepsilon} - V_N\right) \left(CDQC'\right)^{-1}CD \tag{12}
\]

\[
A_2 = \frac{1}{R} a \left(CQC' + \Sigma_{\varepsilon} - V_N\right) \left(CDQC'\right)^{-1} \Sigma_{\varepsilon} \tag{13}
\]

Proof: See Appendix E.
The matrix \( V_N = (\sum_{n=1}^{N} \lambda_n V_n^{-1})^{-1} \) represents the variance-covariance matrix of \( \tilde{P}_1 \) for the “average” investor in the market. The precision matrix \( V_N^{-1} \) equals the weighted mean of each group’s precisions where the weights are proportional to the number of agents in each group. From Equation (10), it is straightforward to show that \( V_N \) can be written as:

\[
V_N = (\Sigma_e + C\Psi C')(I_J + \Sigma_e^{-1}CD\Psi C')^{-1}
\]  

To conclude, we provide closed-form solutions for prices and holdings at date 0. The solution for prices takes the form shown in Equation (4) with constant values shown in (11), (12), and (13). The solution for investor group \( n \)’s holdings is given by Equation (5).

3 Analysis of Prices and Holdings

3.1 Asset Prices

We analyze the relations between model parameters \( \{r_f, a, \lambda_1, \ldots, \lambda_N, B, \Sigma_e, \Sigma_\varphi, \Sigma_z\} \) and ex-ante equilibrium prices.\(^3\) We do this by taking expectations over the three random variables in the model \( \{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\} \).

General Model with Disperse Information: Rearranging Equation (6) gives a general expression for prices at date 0. Equation (15) below shows that asset prices at date 0 are less than the value of expected future payoffs.\(^4\) The total price discount (risk premium) is given by the expression \( aV_NE[\tilde{z}] \). The price discount depends on risk aversion \( (a) \) and the market’s “average” uncertainty about future payoffs \( (V_N) \).

\[
E \left[ \tilde{P}_0 \right] = \frac{1}{R} \left( E \left[ \tilde{P}^1 \right] - aV_NE[\tilde{z}] \right)
\]  

Model with Symmetric and Complete Information: When all investors are informed about all asset-specific components and common components, prices reduce to a form of the Capital Asset Pricing Model (CAPM), expressed in term of prices, and ad-

---

\(^3\)The approach involves taking expectations over the random variables in the model \( \{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\} \). An alternative methodology involves drawing a set of random variables \( \{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\} \) and calculating holdings at date 0. Repeating this process converges to the same expected values as the number of draws goes to infinity. Our methodology solves for holdings before agents receive private information. As such, solutions are sometimes referred to as ex-ante. Appendix F provides closed-form solutions for ex-ante prices at date 0.

\(^4\)Assuming assets are expected to be in positive net supply \( (E[\tilde{z}] > 0) \) and agents are risk averse \( (a > 0) \).
justed for supply uncertainty. Appendix F gives details of the calculations. The appendix also shows the CAPM adjusted for supply uncertainty and expressed with covariance terms—a form that is more familiar to financial economists.

\[ E \left[ \tilde{P}^0 \right] = \frac{1}{R} \left( E \left[ \tilde{P}^1 \right] - a \Sigma E \left[ \tilde{z} \right] \right) \] (16)

**Information Price Discount:** We define the “information price discount” (or “IPD”) as the difference between the price discounts shown in Equations (16) and (15). The IPD represents the amount an asset’s price at date 0 is below its expected future value due to agents not having full information about future payoffs.

\[ IPD \equiv \frac{a}{R} \nabla E \left[ \tilde{z} \right] - \frac{a}{R} \Sigma E \left[ \tilde{z} \right] \] (17)

\[ = \frac{a}{R} (\nabla - \Sigma) E \left[ \tilde{z} \right] \]

In a single-asset model with no factor structure, the information price discount is proportional to the difference between the market’s average uncertainty about future payoffs (\( \nabla \)) and residual uncertainty about the same payoffs (\( \Sigma \)). This difference is a signal-to-noise measure. When the difference is small, investors have a lot of information about future payoffs, the IPD is low, and prices are high. Note that \( IPD \geq 0 \) as the market is always bounded in its assessment of future payoffs by \( \Sigma \).

In a multi-asset model with uncorrelated residual uncertainties and no factor structure, the single-asset intuition discussed in the paragraph above continues to hold. The diagonal matrix (\( \nabla - \Sigma \)) represents a series of signal-to-noise differences.

In a multi-asset model with correlated residual uncertainties and/or a factor structure, the information price discount can be driven by both the asset-specific components of payoffs and common factors. The matrix (\( \nabla - \Sigma \)) can still be roughly interpreted as signal-to-noise differences. However, the matrix is no longer diagonal which means that covariance terms affect the IPD. Section 4 numerically analyzes asset prices in an effort to better understand the role of the covariance terms.

### 3.2 Investor Holdings (Portfolio Choice)

We analyze the relationship between model parameters \( \{r_f, a, \lambda_1, \ldots, \lambda_N, B, \Sigma_\epsilon, \Sigma_\theta, \Sigma_z\} \) and investor \( n \)'s ex-ante equilibrium holdings. We take expectations of Equations (5)
and (6). Rearranging terms, gives:

\[
E[X_n] = a^{-1}V_n^{-1}\left( E[\tilde{P}] - RE[\tilde{P}^0] \right) 
\]

\[
= V_n^{-1}V_nE[\tilde{z}] 
\]

In a single-stock world with no common components, investor \( n \)'s holdings depends on the ratio of the market’s uncertainty about the future payoff (\( V_N \)) to his own uncertainty about the same payoff (\( V_n \)). The higher the investor’s uncertainty relative to the market, the lower the ratio, and the lower the weight of the asset in his portfolio.

In a multi-asset framework with uncorrelated residual uncertainty and no common factors, the matrices (\( V_N \) and (\( V_n \)) are diagonal. The term \( V_n^{-1}V_N \) represents a series of uncertainty ratios. The same intuition described in the paragraph above holds.

In a multi-asset model with correlated residual uncertainties and/or a factor structure of payoffs, thinking about \( V_n^{-1}V_N \) as a ratio of two uncertainty measures provides rough intuition only. However, the ratio of two matrices includes covariance terms relating to uncertainty about assets’ payoffs. Investor \( n \)'s holdings of a specific asset now depends on his uncertainty about the asset’s payoffs, his uncertainty about other assets’ payoffs, and other investors' uncertainty about all assets (including the asset in question). These uncertainties can arise from information asymmetries about the asset-specific components of payoffs and/or common components. Section 4 numerically analyzes holdings in an effort to better understand the role of the covariance terms.

## 4 Numerical Analysis

This section provides a comparative static analysis of equilibrium asset prices and holdings. Our numerical analysis is carried out in the context of the global equity market. One goal is to understand the net effect of the covariance terms in Equations (17) and (18).

**Set-Up:** We consider three groups of investors in equal numbers (French, American, and Japanese people), three assets (a French stock, an American stock, and a Japanese stock), and two common factors. In terms of the model parameters from the previous section, \( J=3; \ K=2; \) and \( N=3 \). All investors have a risk aversion coefficient of \( a = 1.00 \). The payoff for a given stock follows from Equation (1). For the French (“Fr”) stock:

\[
\tilde{P}_F^1 = \tilde{\theta}_F + \beta_{1,F}f_1 + \beta_{2,F}f_2 + \tilde{\varepsilon}_F 
\]

Similar expressions hold for the American asset (subscript “Am”) and Japanese asset.

11
The expected asset-specific component of all payoffs is \( E[\tilde{\theta}_{Fr}] = E[\tilde{\theta}_{Am}] = E[\tilde{\theta}_{Jp}] = 1.00 \). We assume that the variance-covariance matrices \( \Sigma_z \) and \( \Sigma_e \) are both equal to the identity matrix. The variance-covariance matrix \( \Sigma_\theta \) is proportional to the identity matrix. We vary the degree of asset-specific information by varying the diagonal elements from 0 to 6 in the \( \Sigma_\theta \) matrix.\(^5\)

**Information Structure:** We endow each group of investors with asset-specific information about their home country’s asset. For this numerical analysis, all three groups of investors simultaneously have the same information advantage about their respective asset-specific component.

We also endow investors in the large financial centers with common-component information. The realization of the first common component (factor) is known only by the Americans. The realization of the second common component is known only by the Japanese. In the calibration, the expected factor realization is \( E[\tilde{f}_1] = E[\tilde{f}_2] = 0 \) and the variance is \( Var[\tilde{f}_1] = Var[\tilde{f}_2] = 1 \).

The degree of information advantage about the common component is proportional to the variance of \( \beta_1 \tilde{f}_1 \) and \( \beta_2 \tilde{f}_2 \). We simultaneously vary both loadings \( (\beta_{1,Fr}, \beta_{2,Fr}) \) of the French asset from 0 to 4. In order to highlight the different effects that asset-specific information and common-component information have on cross-border holdings, we set the factor loadings for the American and Japanese assets \( (\beta_{1,Am}, \beta_{2,Am}, \beta_{1,Jp}, \beta_{2,Jp}) \) to be half those of the French asset. Our choices highlight dispersion, but are not critical for the results. Similar analysis, with different parameter choices (i.e., different factor loadings) are available from the authors upon request.

### 4.1 Asset Prices

Figure 2 shows the value of all assets (referred to as the “world market portfolio”) for different levels of asset-specific and common-component information. As the asset-specific component is varied from zero to six, each of the three investor groups has an increasing advantage about its own-country’s asset. There is a symmetry in that each group enjoys an information advantage about the home country asset while simultaneously having a disadvantage about other countries’ assets.

\(^5\)Fundamentally, the information asymmetry/advantage about an asset should be measured by the corresponding element of the matrix \( \Psi \). However, as seen in the model section, \( \Psi \) is an endogenous matrix. All else being equal, an increase in the matrix \( \Sigma_\theta \) corresponds to an increase in the matrix \( \Psi \), and vice-versa. This is due to the fact that an increase in the variance of the asset-specific component corresponds to an increase in the asymmetric information surrounding this asset.
As the degree of information advantage about common components increases, the French asset becomes relatively less valuable to French investors and more valuable to the American and Japanese investors. The net effect is a decrease in asset values. The bottom line (with “O” markings) is approximately 26% less than the top (solid) line.

Figure 3 shows an asset’s weight in the world market portfolio. The top-left panel focuses on the French asset. When there are no asymmetries about common components (top solid line), the French asset has a constant 33.3% weight in the world market portfolio. As information asymmetries about the common components increase, the weight of the French asset falls to approximately 27% (bottom line).

For the American (or Japanese) asset, the weight is constant at 33.3% as shown in the bottom line. As the weight of the French asset falls, the weight of the American asset rises to approximately 36.3% (top line with “O” markings).

To summarize, an asset is less risky from the perspective of a single agent if he has precise information about its future payoffs. If all agents enjoy asset-specific information advantages in their home countries, then they are at an informational disadvantage vis-a-vis other countries’ assets. Effects offset each other and may leave asset prices unaffected.

When common components of payoffs are considered, a single asset may no longer be viewed as low-risk even if the agent has information about the asset-specific component of the stock’s payoff. As the common components become more prominent in payoffs, asset-specific information becomes less valuable (see the French asset in Figure 3). Thus, the price of a given asset is sensitive to how many agents have information about common components, how sensitive the asset’s payoffs are to the common components, and what other information the agents have.

4.2 Holdings, Home Bias, and Reverse Home Bias

We calculated home bias measures for a given investor group with respect to a set of assets. A home bias measure of zero (0) indicates the weight of a foreign asset (or group of assets) in the investor’s portfolio is the same as the asset(s) weight in the world market portfolio. A home bias measure of one (1) indicates the investor holds hold zero shares of the foreign asset.

We start by analyzing American investors’ holdings of the French asset. We compare the
weight of the French asset in the Americans’ portfolios to the weight of the French asset
in the world market portfolio:

\[
HB_{\text{(American Investors)}} = 1 - \frac{Wgt_{F,F} (\text{American Portfolio})}{Wgt_{F,F} (\text{World Portfolio})}
\]

Figure 4 plots a measure of home bias for different levels of asset-specific information
advantages and different levels of information advantage about the common components.
The top (red) line shows cross-border holdings when the French asset does not load on
either of the common factors. Focusing only on the top line, when asset-specific informa-
tion advantages are zero, the home bias measure is zero. As asset-specific information
advantages increase (along the x-axis), the home bias measure rises and eventually reaches
54.6% in the upper right-hand corner of the graph.

[ Insert Figure 4 About Here ]

As the French stock loads more and more heavily on the common components, home
bias decreases. Consider the bottom line (marked with circles) which represents \( \beta_{1,F} = 4 \)
and \( \beta_{2,F} = 4 \). The home bias measure ranges from -8.0% in the lower-left of the figure to
34.0% in the far right. Home bias for the American investor is low because the French
stock loads (heavily) on \( \{f_1, f_2\} \) and American investors have information about the \( f_1 \)
component.

Figure 4 has two noteworthy aspects. First, we see home bias can be negative (called
“reverse home bias”) indicating the weight of the French asset is higher in the American
investors’ portfolios than it is in the world market portfolio. Second, we see large dis-
\( WB_{\text{(French Investors)}} = 1 - \frac{Wgt_{Am+Jp,F} (\text{French Portfolio})}{Wgt_{Am+Jp,F} (\text{World Portfolio})}
\]

When there are no information advantages about the common factor (the solid red line),
the home bias measure ranges from 0.0% to 54.6%. This result mirrors the same graph line for home bias in the American investors’ portfolios due to symmetry in our numerical analysis.

Figure 5 allows us to see the joint effects related to information advantages about common factors and asset-specific components. The French investor is assumed to have asset-specific information about the French asset. Therefore, as asset-specific advantages increase, the French investors’ home bias increases. The increase is seen in the upward sloping lines in Figure 5.

The level of French home bias depends on factor loadings. When asset-specific advantages are low, the thin circle-line is below the solid red line. As asset-specific advantages get larger, the two lines first cross before the thin circle line goes above the solid red line. Due to covariance terms, the exact shape of a graph line is difficult to predict without numerical analysis. Equation (18) shows investor n’s holdings are the ratio of two matrices. These matrices are given in Equations (14) and (10).

We conclude by repeating that our model generates significant dispersion in cross-border holdings and home bias measures. As discussed in Section 1.1 most other papers do not allow investors to have information about common factors. The one paper that does—Alburquerque, Bauer, and Schneider (2006)—endows the American investors with the same informational advantage vis-a-vis each foreign stock. In our model, ownership dispersion stems from differences in asset-specific information advantages, differential factors loadings, or both. Home bias may vary from asset to asset depending on factor loadings, whether some investors have information about a given factor, and what other information these investors have.

5 Conclusion

This paper proposes a multi-asset, rational expectations equilibrium model in which agents are asymmetrically informed about asset-specific and common components of payoffs. Our model allows agents to have asset-specific information and/or common component information. The model produces closed-form solutions for asset prices as well as the holdings of individual agents.

Our solution for equilibrium prices is general and can be applied to numerous information structures. We also solve the model for the case when all investors have symmetric
and complete information. Our analysis leads to closed-form solution for an information price discount—the amount equilibrium prices are reduced due to agents not having full information about assets’ future payoffs.

We also provide equilibrium solutions for an agent’s holding of assets in the market. When all variables in our model are uncorrelated, an investor’s holdings of a given stock is straightforward to analyze—the holdings are related to the ratio of the market’s uncertainty about the stock’s future payoff to the investor’s uncertainty. When asset payoffs are correlated (possibly through exposure to common components) or when supply shocks are correlated, analysis of holdings is much more complicated. An investor’s position is related to the ratio of two matrices (the market’s uncertainty and his own uncertainty).

An investor may have high demand for a stock for which he has valuable private information. He may also have high demand for the same stock if he has valuable private information about a stock with highly correlated payoffs. When considering common components, the investor may have high demand for a stock (even if he does not have stock-specific information) provided he has information about a common component of the stock’s payoffs. Of course, having information about a common component is not sufficient to determine if the investor will have high demands for certain stocks. Stocks must load heavily on the factor and the net effect of the loading must be sufficient to outweigh private, stock-specific information that other investors’ may have.

To better understand prices and holdings when asset payoffs are correlated, we conduct a numerical analysis of our model in the context of international equity markets. We consider three stocks from three countries along with two factors. We focus on prices and cross-border holdings. Our model generates large dispersion in home bias measures. This dispersion can result from different levels of asset-specific information advantages. The dispersion can also result from differential loadings on the model’s common components.

Our model can generate reverse (or negative) home bias indicating an investor holds more of an asset than he would if he invested in the world market portfolio. A strength of our model is its ability to produce large variations in home bias as well as its ability to produce reverse home bias.

There are a number of potential avenues for future research. First, one could try to extend our model to multiple periods. This would provide expressions for net trading as in Brennan and Cao (1997) as well as suggest empirical tests based on trading (as opposed to holdings) data. Second, one could work to devise methods of empirically identifying different information structures. While no small task, structures could then be used to test relative asset prices using expressions in this paper. Third, our model may be adapted
to better understanding partially segmented markets. In such cases, information is the “friction” that segments markets. One may be able to model groups of investors who face low frictions only when trading securities from their home country, groups of investors who face low frictions when trading securities in a contiguous block of countries (a geographic region), or groups of investors who face low frictions when investing in any global security. None of the three extensions is likely to be easy—all are potentially interesting.
References


Appendix A

The information set of investor \( n \) (formally, investor \( i \) in group \( n \)) consists of the realization of private signals \( M_n \tilde{\eta} \) and of equilibrium prices \( \tilde{P}^0 \). The equilibrium price vector \( \tilde{P}^0 \) is a linear function of the information \( \tilde{\eta} \) and the supply \( \tilde{z} \) with \( \tilde{P}^0 = A_0 + A_1 \tilde{\eta} - A_2 \tilde{z} \).

Since, \( \tilde{w}^1_n = w^0_n R + X'_n (\tilde{P}^1 - R \tilde{P}^0) \) and \( \tilde{P}^1 \) is a linear function of \( \tilde{\eta} \) and \( \tilde{z} \), it follows that \( \tilde{w}^1_n \) joins the multivariate normal distribution of \( (\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}) \). Consequently, \( \tilde{w}^1_n \) is a normal random variable conditional on \( M_n \tilde{\eta} \) and \( \tilde{P}^0 \). Properties of normal distributions imply that investor \( n \)'s expected utility can be written as:

\[
E \left[ U(\tilde{w}^1_n)|M_n \tilde{\eta}, \tilde{P}^0 \right] = U \left\{ E \left[ \tilde{w}^1_n|M_n \tilde{\eta}, \tilde{P}^0 \right] - \frac{a}{2} Var \left[ \tilde{w}^1_n|M_n \tilde{\eta}, \tilde{P}^0 \right] \right\}
\]

Since the utility function is exponential, maximizing this expected utility is identical to maximizing:

\[
\max_{X_n} \left\{ E \left[ w^0_n R + X'_n (\tilde{P}^1 - R \tilde{P}^0)|M_n \tilde{\eta}, \tilde{P}^0 \right] - \frac{a}{2} Var \left[ w^0_n R + X'_n (\tilde{P}^1 - R \tilde{P}^0)|M_n \tilde{\eta}, \tilde{P}^0 \right] \right\}
\]

The equation to be solved is:

\[
0 = E \left[ (\tilde{P}^1 - R \tilde{P}^0)|M_n \tilde{\eta}, \tilde{P}^0 \right] - a Var \left[ (\tilde{P}^1 - R \tilde{P}^0)|M_n \tilde{\eta}, \tilde{P}^0 \right] X_n \tag{19}
\]

This implies that investor \( n \)'s demand vector is:

\[
X_n = a^{-1} Var^{-1} \left[ \tilde{P}^1|M_n \tilde{\eta}, \tilde{P}^0 \right] \times \left( E \left[ \tilde{P}^1|M_n \tilde{\eta}, \tilde{P}^0 \right] - R \tilde{P}^0 \right) \tag{20}
\]
Appendix B

From Equations (4), (6), and (7), we have:

\[ 0 = \sum_{n=1}^{N} \lambda_n V_n^{-1} (B_{0n} + B_{1n} M_n \tilde{\eta} + (B_{2n} - R I_J) (A_0 + A_1 \tilde{\eta} - A_2 \tilde{z}) - a \tilde{z} ) \]

By canceling the \( \tilde{z}, \tilde{\eta}, \) and constant terms, it is straightforward to show that:

\[
\begin{align*}
    a A_2^{-1} A_0 &= \sum_{n=1}^{N} \lambda_n V_n^{-1} B_{0n} \\
    a A_2^{-1} A_1 &= \sum_{n=1}^{N} \lambda_n V_n^{-1} B_{1n} M_n \\
    a A_2^{-1} &= \sum_{n=1}^{N} \lambda_n V_n^{-1} (R I_J - B_{2n}) 
\end{align*}
\]
Appendix C

The vector \(( \tilde{P}_1' \quad M_n \tilde{\eta}' \quad \tilde{P}_0' )'\) is normally distributed and its var-cov matrix is:

\[
\text{Var} \left[ ( \tilde{P}_1' \quad M_n \tilde{\eta}' \quad \tilde{P}_0' )' \right] = \begin{pmatrix}
    CQC' + \Sigma_{\varepsilon} & CQM_n' & CQA_1'
    M_n QC' & M_n QM_n' & M_n QA_1'
    A_1 QC' & A_1 QM_n' & A_1 QA_1' + A_2 \Sigma_{\varepsilon} A_2'
\end{pmatrix}
\] (22)

The conditional expectation is:

\[
E_n \left[ \tilde{P}_1 \right] = E \left[ \tilde{P}_1 | M_n \tilde{\eta}, \tilde{P}_0 \right] = E[\tilde{P}_1] + \text{Cov} \left[ \tilde{P}_1; \begin{pmatrix} M_n \tilde{\eta} \\ \tilde{P}_0 \end{pmatrix} \right] \times \text{Var}^{-1} \left[ \begin{pmatrix} M_n \tilde{\eta} \\ \tilde{P}_0 \end{pmatrix} \right] \times \left( \begin{pmatrix} M_n \tilde{\eta} \\ \tilde{P}_0 \end{pmatrix} - E \left[ \begin{pmatrix} M_n \tilde{\eta} \\ \tilde{P}_0 \end{pmatrix} \right] \right) 
\] (23)

Normal distributions give \( E \left[ \tilde{P}_1 | M_n \tilde{\eta}, \tilde{P}_0 \right] = B_{0n} + B_{1n} M_n \tilde{\eta} + B_{2n} \tilde{P}_0 \). Hence,

\[
\begin{pmatrix}
    B_{1n} & B_{2n}
\end{pmatrix}
\begin{pmatrix}
    CQM_n' & CQA_1'
\end{pmatrix}
= \begin{pmatrix}
    B_{1n} & B_{2n}
\end{pmatrix}
\begin{pmatrix}
    M_n QM_n' & M_n QA_1'
    A_1 QM_n' & A_1 QA_1' + A_2 \Sigma_{\varepsilon} A_2'
\end{pmatrix}
\] (24)

The variance of returns conditional on \( n \)'s information is:

\[
V_n = \text{Var} \left[ \tilde{P}_1 | M_n \tilde{\eta}, \tilde{P}_0 \right] = \text{Var} \left[ \tilde{P}_1 \right] - \text{Cov} \left[ \tilde{P}_1; \begin{pmatrix} M_n \tilde{\eta} \\ \tilde{P}_0 \end{pmatrix} \right] \times \text{Var}^{-1} \left[ \begin{pmatrix} M_n \tilde{\eta} \\ \tilde{P}_0 \end{pmatrix} \right] \times \text{Cov} \left[ \begin{pmatrix} M_n \tilde{\eta} \\ \tilde{P}_0 \end{pmatrix} ; \tilde{P}_1 \right]
\] (25)

We use Equation (23) to get:

\[
V_n = \text{Var} \left[ \tilde{P}_1 \right] - \begin{pmatrix}
    B_{1n} & B_{2n}
\end{pmatrix}
\text{Cov} \left[ \begin{pmatrix} M_n \tilde{\eta} \\ \tilde{P}_0 \end{pmatrix} ; \tilde{P}_1 \right]
\]

Because \( \text{Cov} \left[ \begin{pmatrix} M_n \tilde{\eta} \\ \tilde{P}_0 \end{pmatrix} ; \tilde{P}_1 \right] = \begin{pmatrix} M_n QC' \\ A_1 QC' \end{pmatrix} \), we get:

\[
V_n = CQC' + \Sigma_{\varepsilon} - B_{1n} M_n QC' - B_{2n} A_1 QC'
\] (26)
Appendix D

In order to determine a closed-form solution for $U$, we solve the second equation from the system shown in Equation (8):

$$aA_2^{-1}A_1 = aU = \sum_{n=1}^{N} \lambda_n V_n^{-1} B_{1n} M_n$$

(27)

The following properties apply to matrices $D_n$ and $M_n$. Below we use $n_a$ and $n_b$ to denote two different groups of investors such that $n = \{1, 2, 3, \ldots, n_a, \ldots, n_b, \ldots, N\}$:

P1: $\sum_{n=1}^{N} D_n = I_J$

P2: $\forall n_a \neq n_b$: $D_{na} D_{nb} = 0_{J+K}$ where $0_{J+K}$ is the null matrix of order $J + K$;

P3: $M_{na} M_{nb}' = 0_{Jn_a+Kn_a,Jn_b+Kn_b}$ where $0_{Jn_a+Kn_a,Jn_b+Kn_b}$ is the null matrix;

P4: $D_n D_n = D_n$ and $M_n M_n^{-1} = I_{Jn+Kn}$

P5: $\forall G_1, G_2$: $g(G_1)g(G_2) = \sum_{n=1}^{N} D_n G_1 D_n G_2 D_n$

P6: $\forall G$: $g(GD) = g(G)D = Dg(G)$

There are three matrices key to obtaining a closed-form solution for $U$:

$$M = UQU' + \Sigma_z$$

$$\Psi = Var[\bar{y}_i|\bar{F}_0] = Q - QU'M^{-1}UQ$$

$$\Psi_n = M_n \Psi M_n'$$

We first solve Equation (24) for $B_{1n}$ and $B_{2n}$. Note, we do not assume $A_1$ is invertable to get the following results (in fact, $A_1$ is not square). The two equations to be solved are:

$$B_{1n} (M_n QM_n') + B_{2n} (A_1 QM_n') = CQM_n'$$

(28)

$$B_{1n} (M_n QA_1') + B_{2n} (A_1 QA_1' + A_2 \Sigma_z A_2') = CQA_1'$$

(29)

Using $M = UQU' + \Sigma_z$, we obtain $A_1 QA_1' + A_2 \Sigma_z A_2' = A_2 MA_2'$. Starting with Equation (29), we get:
B_{1n} (M_n Q A_1') + B_{2n} (A_1 Q A_1' + A_2 \Sigma_2 A_2') = C Q A_1'
B_{1n} (M_n Q A_1') + B_{2n} (A_2 M A_2') = C Q A_1'
(C Q A_1' - B_{1n} M_n Q A_1')(A_2 M A_2')^{-1} = B_{2n}
(C - B_{1n} M_n) Q (A_2^{-1} A_1)'^{1} M^{-1} A_2^{-1} = B_{2n}
(C - B_{1n} M_n) Q U' M^{-1} A_2^{-1} = B_{2n}

In a second step, we solve Equation (28):

B_{1n} (M_n Q M_n') + (C - B_{1n} M_n) Q U' M^{-1} A_2^{-1} (A_1 Q M_n') = C Q M_n'
B_{1n} M_n (Q M_n' - Q U' M^{-1} U Q M_n') = (C - C Q U' M^{-1} U Q) M_n'
B_{1n} M_n (Q - Q U' M^{-1} U Q) M_n' = C (Q - Q U' M^{-1} U Q) M_n'
B_{1n} M_n \Psi M_n' = C \Psi M_n'
B_{1n} \Psi_n = C \Psi M_n'
B_{1n} = C \Psi M_n' \Psi_n^{-1}

We have thus demonstrated that:

B_{1n} = C \Psi M_n' \Psi_n^{-1} \quad (30)
B_{2n} = (C - B_{1n} M_n) Q U' M^{-1} A_2^{-1}

By substituting \(B_{1n}\) and \(B_{2n}\) into Equation (26) we obtain the variance-covariance matrix \(V_n\) as a function of \(\Psi\)

\[
V_n = \Sigma \xi - B_{1n} M_n Q C' - B_{2n} A_1 Q C'
\]

\[
V_n = \Sigma C Q C' + \Sigma \xi - C \Psi M_n' \Psi_n^{-1} M_n Q C' - (C - C \Psi M_n' \Psi_n^{-1} M_n) Q U' M^{-1} A_2^{-1} A_1 Q C'
\]

\[
V_n = \Sigma \xi + C (Q - Q U' M^{-1} U Q) C' - C \Psi M_n' \Psi_n^{-1} M_n Q C' + C \Psi M_n' \Psi_n^{-1} M_n Q U' M^{-1} U Q C'
\]

\[
V_n = \Sigma \xi + C \Psi C' - C \Psi M_n' \Psi_n^{-1} M_n (Q - Q U' M^{-1} U Q) C'
\]

\[
V_n = \Sigma \xi + C \Psi C' - C \Psi M_n' \Psi_n^{-1} M_n Q C'
\quad (31)
\]

We use Equation (27) to determine \(U\) by right-multiplying by \(M_n'\). Note that \(M_n'\) concerns investor group \(n\). Property P3, from the start of this appendix, shows we need only carry out the multiplication with terms from the same investor group.

Thus, we obtain \(\lambda_n V_n^{-1} B_{1n} = a U M_n'\). Also from P3, \(M_{n_k} M_{n_k} = 0\). We then multiply this last expression by \(V_n\) on the left and we replace \(B_{1n}\) with its value from (30):

24
\[ \lambda_n C \Psi M_n \Psi_n^{-1} = a V_n U M_n' \]  

(32)

If we multiply (32) by \( M_n \) on the right and if we sum for \( n = 1, \ldots, N \), we obtain Equation (27). We conclude that Equation (27) is equivalent to Equation (32) for all \( n = 1, \ldots, N \). If we multiply Equation (32) by \( \Psi_n \) and \( M_n \) on the right and if we replace \( V_n \) with its value in Equation (31) we then obtain:

\[ \lambda_n C \Psi D_n = a \left( \Sigma_c + C \Psi C' - C \Psi M_n' \Psi_n^{-1} M_n \Psi C' \right) U M_n' \Psi_n M_n \]

If we now sum for \( n = 1, \ldots, N \) we obtain:

\[ \sum_{n=1}^{N} \lambda_n C \Psi D_n = a \left( \Sigma_c \sum_{n=1}^{N} U M_n' \Psi_n M_n + C \Psi C' \sum_{n=1}^{N} U M_n' \Psi_n M_n - \sum_{n=1}^{N} C \Psi M_n' \Psi_n^{-1} M_n \Psi C' U M_n' \Psi_n M_n \right) \]

which is equivalent to:

\[ C \Psi D = a \left( \Sigma_c U \sum_{n=1}^{N} D_n \Psi D_n + C \Psi C' U \sum_{n=1}^{N} D_n \Psi D_n - C \Psi \sum_{n=1}^{N} M_n' \Psi_n^{-1} M_n \Psi C' U M_n' \Psi_n M_n \right) \]

By introducing the function \( g(\cdot) \), we obtain the expression below. The reader can easily check that (33) is equivalent to (27).

\[ C \Psi D = a \Sigma_c U \sum_{n=1}^{N} D_n \Psi D_n + a C \Psi C' U g(\Psi) - a C \Psi \left( \sum_{n=1}^{N} M_n' \Psi_n^{-1} M_n \Psi C' U M_n' \Psi_n M_n \right) \]

(33)

To prove Lemma 2.1, we start by assuming \( U = a^{-1} \Sigma_c^{-1} C D \). We then substitute this expression for \( U \) into (33) and check that the following equality holds:

\[ C \Psi D = C D g(\Psi) + C \Psi C' \Sigma_c^{-1} C D g(\Psi) - C \Psi \left( \sum_{n=1}^{N} M_n' \Psi_n^{-1} M_n \Psi C' \Sigma_c^{-1} C D M_n' \Psi_n M_n \right) \]

(34)

Under the assumption that \( \Psi^{-1} + C' \Sigma_c^{-1} C \) is a \( g \)-matrix, we can write \( g(\Psi^{-1} + C' \Sigma_c^{-1} C) = \Psi^{-1} + C' \Sigma_c^{-1} C \). We then replace \( C' \Sigma_c^{-1} C \) by \( g(\Psi^{-1} + C' \Sigma_c^{-1} C) - \Psi^{-1} \) in the right hand side of Equation (34) and confirm the equality. We conclude \( U = a^{-1} \Sigma_c^{-1} C D \) represents a solution.
Appendix E

We replace $B_{2n}$ in the third equation of (8) with its value given in (30). We then obtain $A_2$. We obtain $A_1$ directly from the expression for $U$. In order to determine $A_0$, we substitute the following expression for $B_{0n}$ into the first equation of (8).

$$B_{0n} = (C - B_{1n}M_n - B_{2n}A_1) E [\tilde{\eta}] - B_{2n} (A_0 - A_2 E [\tilde{z}])$$

To check that $A_2$ is a regular matrix, we start with Equation (13). Note that matrices $C$, $D$, $Q$ and $\Sigma_\epsilon$ are, by definition, regular matrices. Moreover, $CQC' + \Sigma_\epsilon - V_N$ is a positive definite matrix, thus regular. The positive-definiteness can be seeing by noting that $CQC' + \Sigma_\epsilon$ is the variance-covariance of a totally uninformed investor who does not even observe equilibrium prices. Matrix $V_N$, on the other hand, is the variance-covariance matrix of the “average investor” who has some private information and observes prices.
Appendix F

To get the CAPM in terms of prices, first subtract $R$ times Equation (4) from Equation (3) and take expectations to get:

$$E \left[ \tilde{P}_1 \right] - RE \left[ \tilde{P}_0 \right] = (C - RA_1)E[\tilde{\eta}] + RA_2E[\tilde{z}] - RA_0$$

Equations (11), (12), and (13) enable us to write:

$$RA_1 = (CQC') + \Sigma_{\epsilon} - V_N)(CDQC')^{-1}CD = C$$
$$RA_2 = a(CQC') + \Sigma_{\epsilon} - V_N)(CDQC')^{-1}\Sigma_{\epsilon} = a\Sigma_{\epsilon}$$
$$RA_0 = (C - RA_1)E[\tilde{\eta}] + (RA_2 - aV_N)E[\tilde{z}] = 0$$

Therefore:

$$E \left[ \tilde{P}_1 \right] - RE \left[ \tilde{P}_0 \right] = a\Sigma_{\epsilon}E[\tilde{z}]$$

We can express the CAPM in terms of covariance which makes it more familiar to financial economists. When all investors are informed, they know the realization of $\tilde{\eta}$ is $\eta$. Therefore, $\tilde{P}^1 = C\eta + \tilde{\epsilon}$ and $Var \left[ \tilde{P}^1 \right] = \Sigma_{\epsilon}$:

$$a\Sigma_{\epsilon}E[\tilde{z}] = aVar \left[ \tilde{P}^1 \right] E[\tilde{z}] = aCov \left[ \tilde{P}^1, \tilde{P}^1 \right] E[\tilde{z}] = aCov \left[ \tilde{P}^1, \left( \tilde{P}^1 \right)'E[\tilde{z}] \right]$$

$$= aCov \left[ \tilde{P}^1, \tilde{P}^1_m \right]$$

Above, $\tilde{P}^1_m$ is the payoff of the market portfolio (the one that contains all the assets) divided by the number of investors (since $\tilde{z}$ has been defined as the supply per investor). As the supply is unknown by the agents in the market, we consider the expectations of the supply, rather than the supply itself. Using Equation (16) and the above result gives the following expression for a single asset $j$:

$$E \left[ \tilde{P}^1_j \right] - RE \left[ \tilde{P}^0_j \right] = aCov \left[ \tilde{P}^1_j, \tilde{P}^1_m \right]$$
Figure 1
Information Structures

Panel A: Traditional Home Bias

The figure depicts a specific information structure. Traditional home bias studies assume investors receive asset-specific information about stocks in their local market. There are no common factors and, thus, no information about common factors. The information advantage about local assets is shown with dashed lines between an investor group and assets in the home market. Assets are shown with the “$\mathcal{A}$” symbol and investors are shown with the “$\mathcal{I}$” symbols.
The figure depicts a specific information structure. Kodres and Pritsker (2002) assume stock payoffs are affected by an underlying factor structure. No group of investors, however, has information about these underlying factors. Dashed lines indicate an investor group has information about an asset or factor. Solid lines indicate the global factor affects an asset’s payoffs. Assets are shown with the “𓁛” symbol, investors are shown with the “𓁛𓁛𓁛𓁛” symbols, common components (factors) are shown as “F1” and “F2”.

Panel B: Factors without Information
The figure depicts a specific information structure. Albuquerque, Bauer, and Schneider (2006) assume stock payoffs are affected by a single, global factor. Only one group of investors (USA investors) receives private information about the global factor. The USA investors are the third group on the top row. Dashed lines indicate an investor group has information about an asset or factor. Solid lines indicate the global factor affects an asset’s payoffs. Assets are shown with the “●” symbol, investors are shown with the “intree” symbols, and the global factor is shown with “G”.
The figure depicts a generalized information structure. The generalized structure studied in this paper assumes multiple assets, multiple investors, and multiple factors. Dashed lines indicate an investor group has information about an asset or factor. Solid lines indicate the factor affects an asset’s payoffs. Assets are shown with the “$\mathcal{A}$” symbol, investors are shown with the “$\mathbb{I}$” symbols, and common components (factors) are shown as “F1”, “F2”, and “F3”.
Figure 2

World Market Capitalization

The figure shows world market capitalization in dollars. The X-axis shows the degree of asset-specific information advantages. The four curves (labeled 0, 1, 2, 4) represent four levels of information advantage about the common components of payoffs. The thin, purple graph line (with “O” markings) represents the highest level of advantage about the common components. Details of the numerical analysis are given in the text.
Figure 3
Weight of Assets in the World Market Portfolio

The figure shows the weight of assets in the world market portfolio. The Y-Axis shows the fraction of the world market portfolio. The top-left panel shows the weight of the French asset. The bottom-right panel shows the weight of the American asset (a similar graph applies to the Japanese asset). The X-axis shows the degree of asset-specific information advantages. The four curves (labeled 0, 1, 2, 4) represent four levels of information advantage about the common components of payoffs. The thin, purple graph line (with “O” markings) represents the highest level of advantage about the common components. Details of the numerical analysis are given in the text.
Figure 4
American Investors’ Home Bias With Respect to the French Asset

The figure shows American investors’ holdings of the French asset. The Y-Axis shows the degree of home bias. A home bias value of zero (0) indicates no home bias and negative numbers indicates reverse home bias. The X-axis shows the degree of asset-specific information advantages. The four curves (labeled 0, 1, 2, 4) represent four levels of information advantage about the common components of payoffs. The thin, purple graph line (with “O” markings) represents the highest level of advantage about the common components. Details of the numerical analysis are given in the text.
The figure shows French investors’ holdings of American and Japanese assets (i.e., cross-border holdings). The Y-Axis shows the degree of home bias. A home bias value of zero (0) indicates no home bias and negative numbers indicates reverse home bias. The X-axis shows the degree of asset-specific information advantages. The four curves (labeled 0, 1, 2, 4) represent four levels of information advantage about the common components of payoffs. The thin, purple graph line (with “O” markings) represents the highest level of advantage about the common components. Details of the numerical analysis are given in the text.