Risk contributions of trading and non-trading hours: Evidence from commodity futures markets

Abstract

This paper focuses on risk contributions of trading and non-trading hours in Chinese commodity futures markets. We first examine integrated risks of Chinese copper, rubber, and soybean futures markets within the copula-VaR (value at risk) and copula-ES (expected shortfall) frameworks. Then, we evaluate the component VaR and component ES of the trading and non-trading periods to gauge their respective risk contributions to the integrated risks. We find that copula-based VaR models can appropriately measure integrated risks, as the typical VaR and ES based on close-to-close returns underestimate overall market risks. In addition, we document that the financial information accumulated during non-trading hours contributes substantially to the overall risk of futures markets, with component VaR and ES weights ranging from more than 40% to nearly 60% in these markets. In particular, the information during non-trading hours is more important than that in trading hours in explaining the total risk of copper futures in China. Moreover, the risk contribution of non-trading periods increases with their lengths, reflecting the fact that information flows constantly over time.

Keywords: risk contribution; value at risk; expected shortfall; futures markets; trading hours; non-trading hours

JEL classification: C32, G15
1. Introduction

Chinese financial markets trade only during the daytime hours from Monday to Friday, and the total trading period of any market is generally less than seven hours, which is less than one half of the non-trading hours. Consequently, the financial information accumulated during non-trading hours represents a significant source of integrated market risk, and plays an important role in price discovery in financial markets. This information arises from public announcements made during non-trading hours in China as well as trading activities in overseas markets. For risk hedging purposes, it is crucial not only to properly measure the overall risk in financial markets by accounting for all price sensitive information but also to locate sources of risk, particularly risks related to trading and non-trading hours. The purpose of this paper is to quantify integrated risks and risk contributions of trading and non-trading hours in major Chinese commodity futures markets.

To measure the integrated risk of a futures contract, we employ the value at risk (VaR) and expected shortfall (ES) concepts in our analysis, as both provide a clear interpretation in monetary terms and a direct application in risk management. VaR is defined as the maximum loss of an investment at a certain confidence level over a specified horizon; it became a widely accepted standard in the risk management industry after J.P. Morgan introduced their RiskMetrics document in 1994. However, the major drawback of this method is that it is not sub-additive, which means that the VaR of a portfolio can be larger or smaller than the sum of the VaRs of its components. Moreover, it cannot measure the expected loss resulting from extremely unlikely market factor changes. To overcome these drawbacks, Engle and Manganelli (1999) propose the concept of expected shortfall (ES),
which is the mean of losses, given that losses are greater than the VaR at the confidence level. Some research (Acerbi and Tasche, 2002; Frey and McNeil, 2002; Manganelli and Engle, 2001) illustrates that ES provides a better estimate of risk than VaR when loss distributions exhibit fat tails or empirical discreteness. In contrast with VaR, ES is a coherent risk measure in the sense of Artzner et al. (1999), and satisfies the sub-additivity property. For these reasons, ES is considered an important alternative to VaR for the purpose of risk measurement.

Integrated risk estimates depend critically on the return distribution assumed for a particular market. In this paper, we utilize copula functions based on peaks over threshold (POT) theory to link both trading and non-trading return distributions and construct the market return distribution. Traditionally, the joint distribution of multi-asset returns or risk factors is assumed to be multivariate normal with a linear correlation matrix. However, it is a stylized fact that financial asset returns typically exhibit non-normal properties and non-linear dependencies. As we know, the copula approach is more flexible in modeling multivariate distributions, as it can separate univariate marginal distributions from multivariate dependence structure. This allows us to select a rich dependence structure while preserving non-normality properties of marginal distributions. Given these advantages, the copula approach has recently received substantial attention in the finance literature. For example, Ward and Lee (2002) use a multivariate normal copula to aggregate different types of risks to create an integrated risk distribution for an insurance company. On the other hand, Rosenberg and Schuermann (2006) combine market, credit, and operational risks with copulas to obtain a total risk distribution for a financial institution. They find that the copula-based approach is more accurate than other methods
at estimating the overall risk as measured by VaR and ES. We follow this line of thought and combine the correlated trading and non-trading return distributions into a joint risk distribution using a copula function that better fits our data. Trading and non-trading returns can exhibit quite distinct distributional characteristics due to different information flow patterns in these periods. To capture the non-normality characteristics of trading and non-trading return distributions, such as skewness and fat-tails, we apply the extreme value theory (EVT) to form these distributions. The EVT is designed especially for tail estimation, and it enables us to estimate extreme probabilities and extreme quantiles without making assumptions about an unknown parent distribution. Therefore, our approach can better measure the integrated market risk than the typical method based solely on close-to-close daily returns, as it explicitly accounts for information flows during both trading and non-trading periods.

Another important objective of this paper is to quantify the risk contributions of trading and non-trading hours in the market. To this end, we decompose the total risk of the market into component VaRs and component ESs to gauge the impact of information accumulated during non-trading hours on market risk. We also distinguish among different types of non-trading periods to further investigate the role of the length of non-trading periods in risk contributions. The importance of non-trading information in price discovery and market volatility has been well documented in the literature. Tsiakas (2008) finds that the size and predictive ability of non-trading information for both European and US stock markets are substantial. Taylor (2007) also confirms the significant impact of overnight information on information flow in the regular S&P 500 futures market. On the other hand, Cliff, Cooper, and Gulen (2008) find that night returns are higher than day returns, and
conclude that the US equity premium over the last decade is solely due to overnight returns. In contrast, this paper primarily focuses on the risk aspects of trading and non-trading information in the market. This is of particular interest to both academics and practitioners, as our results help explain sources of risk and provide important implications for risk management. Additionally, it helps us better understand how the trading information in international developed markets impacts Chinese markets. Note that European and US markets are open to trade during Chinese non-trading hours.

Using data from Chinese copper and rubber futures traded on the Shanghai Futures Exchange (SHFE) and soybean futures traded on the Dalian Commodity Exchange (DCE), we show that our model can appropriately measure integrated risks, and that the typical VaR and ES calculations based on close-to-close returns significantly underestimate overall market risk. Additionally, the financial information accumulated during non-trading hours contributes substantially to the overall risk of commodity futures markets in China. The component VaR and component ES weights of non-trading hours are much higher than 40% for all markets under both 95% and 99% confidence levels. Specifically, non-trading hours contribute more to the integrated risk than trading hours in the copper futures market, indicating that non-trading hours contain more important information than do trading periods. Moreover, the risk contribution of non-trading hours increases with the length of the non-trading period. We also demonstrate that both the VaR and ES are particularly sensitive to the confidence level when the level is high.

The remainder of this paper is organized as follows. Section 2 describes the copula methodology. Section 3 discusses the data used for the analysis and provides the
descriptive statistics. Section 4 reports the empirical results, while Section 5 concludes this paper.

2. POT-based copula- VaR and ES models

2.1. Trading and non-trading returns

Let $F_t^c$ denote the daily closing price of a futures contract at date $t$, and $F_t^o$ the daily opening price. Following Tsiakas (2008), the close-to-close daily return $r_t$, the close-to-open non-trading return $r_t^o$, and the open-to-close trading return $r_t^d$ are calculated as per the following definitions:

$$r_t = 100 \times \ln(F_t^c / F_{t-1}^c),$$ \hspace{1cm} (1)

$$r_t^o = 100 \times \ln(F_t^o / F_{t-1}^c),$$ \hspace{1cm} (2)

$$r_t^d = 100 \times \ln(F_t^c / F_t^o).$$ \hspace{1cm} (3)

Thus, we can separate non-trading returns from trading returns. The typical daily return is simply a sum of the trading and non-trading returns. Separating trading from non-trading returns allows us to model their distributions differently to ensure their unique distributional characteristics are captured. Further, non-trading periods can be normal weeknights, weekends, or holidays. As a general practice in China, when a public holiday(s) happens to be on a weekday(s) from Tuesday to Thursday, a short holiday period is created by extending the nearest weekend to include those dates before or after the holiday(s). As a result, any non-trading holiday period contains at least 72 hours. Presumably, a longer non-trading period may accumulate more information, thereby making a higher contribution to integrated risks than a shorter non-trading period. We will
investigate whether a longer non-trading period indeed contains more (or more important) information about returns than does a shorter one.

2.2. Modeling joint distributions with copula functions

Our focus in this paper is on a bivariate model, where the random variables are trading and non-trading returns. In general, let \((X_1, X_2)\) be a vector of two random variables with a joint distribution denoted as \(F(x_1, x_2)\), and the marginal distribution functions \(F_1\) and \(F_2\), respectively. Additionally, we assume that each marginal distribution function depends on one single parameter \(\vartheta_i\) \((i = 1, 2)\). Then, Sklar’s theorem states that there is a copula function \(C(u_1, u_2; \theta)\) such that

\[
F(x_1, x_2) = C(F_1(x_1; \vartheta_1), F_2(x_2; \vartheta_2); \theta),
\]

where \(\theta\) is the parameter vector of the copula function \(C\). If \(F_1\) and \(F_2\) are continuous, then the copula function is unique; otherwise, it is uniquely determined on \(\text{Ran}(F_1) \times \text{Ran}(F_2)\). Conversely, for any univariate risk distributions \(F_1\) and \(F_2\), Equation (4) defines a joint distribution \(F(x_1, x_2)\) with margins \(F_1\) and \(F_2\). Thus, Sklar’s theorem implies that we can combine any univariate distributions into a joint distribution together with a copula.

Differentiating \(F(x_1, x_2)\) with respect to its variables yields the joint density function

\[
f(x_1, x_2) = c(F_1(x_1; \vartheta_1), F_2(x_2; \vartheta_2); \theta) f_1(x_1; \vartheta_1) f_2(x_2; \vartheta_2),
\]

where copula density \(c(u_1, u_2; \theta) = \partial^2 C(u_1, u_2; \theta)/\partial u_1 \partial u_2\), and \(f_1(x_1; \vartheta_1)\) as well as \(f_2(x_2; \vartheta_2)\) are marginal density functions. Equation (5) shows that for any continuous joint distribution, the univariate margins and the dependence structure can be separated, where the dependence structure can be completely determined by a proper copula function. For
this reason, copula functions enable us to obtain a joint distribution with a variety of possible, but not necessarily equal, parametric univariate distributions. Consequently, we can preserve the original characteristics of marginal distributions, while allowing for a wide range of dependence relations.

For a given set of return observations \( \{(x_{1,t}, x_{2,t})\}_{t=1}^{T} \), model parameters \( \vartheta = (\vartheta_1, \vartheta_2, \vartheta) \) can be jointly estimated by maximizing the following log-likelihood function

\[
L(\vartheta | x_1, x_2) = \sum_{t=1}^{T} \ln c(F_1(x_{1,t} ; \vartheta_1), F_2(x_{2,t} ; \vartheta_2) ; \vartheta) + \prod_{t=1}^{T} \left[ \ln f_1(x_{1,t} ; \vartheta_1) + \ln f_2(x_{2,t} ; \vartheta_2) \right].
\]

(6)

This method may be computationally expensive especially for high dimensions. As we can see from Equation (6), the copula parameter is unaffected by the parameters of the marginal distributions. As a consequence, these parameters can be estimated separately. In this paper, we adopt the Inference Function for Margins (IFM) method, in which parameters are estimated in two stages. More specifically, we first estimate parameters \( \vartheta_1 \) and \( \vartheta_2 \) of marginal distributions. Then, we estimate the copula parameter vector \( \vartheta \) conditional upon the marginal distribution estimates in the first step. For the estimation at each stage, the maximum likelihood method is used. The IFM method is able to provide estimators as well as the joint distribution method in terms of the mean square errors (Xu, 1996), but is apparently computationally simpler.

2.3. Marginal distributions

2.3.1. Excess distributions

In this paper, we employ the extreme value theory (EVT) to select and specify the marginal distributions of trading and non-trading returns. As is pointed out by Longin
(2000), the major advantage of EVT is that it takes into account rare events contained in distribution tails, with no particular return distribution assumption. There are two main approaches to estimating extreme values within the EVT context: the block maxima model (BMM) and the peaks over threshold (POT) model. The BMM method considers the maximum (or minimum) observations in successive periods, which are pre-chosen as blocks. In contrast, the POT method focuses on the realizations that exceed a given high threshold. The latter method apparently uses the available data more efficiently, and therefore is adopted in this analysis. In particular, we consider the most widely used distribution in modelling excesses: the generalized Pareto distribution (GPD).

To illustrate this method, let \( F(x) = \Pr(X \leq x) \) be the distribution function of a random variable \( X \). For a given threshold \( u \), the distribution of the excess values is given by

\[
F_u(y) = \Pr(X - u \leq y \mid X > u) = \frac{F(y + u) - F(u)}{1 - F(u)}, \quad y > 0. \tag{7}
\]

In practical applications, we will have to approximate the conditional excess distribution for high threshold values, as the parent distribution \( F(x) \) is unknown.

Balkema and De Haan (1974) and Pickands (1975) prove that for a sufficiently high threshold \( u \), the excess distribution converges to the GPD, which is given as

\[
G_{\xi,\beta(u)}(y) = \begin{cases} 
1 - \left( 1 - \frac{\xi y}{\beta(u)} \right)^{-1/\xi}, & \xi \neq 0 \\
1 - e^{-\frac{y}{\beta(u)}}, & \xi = 0
\end{cases}
\] \tag{8}

where \( \beta(u) \) is a positive function of \( u \), representing a scale parameter, and \( \xi \) is a shape parameter that determines the GPD shape. When \( \xi > 0 \), \( \xi = 0 \), or \( \xi < 0 \), the
corresponding excess distribution is from the Fréchet, Gumbel, or Weibull families, respectively. The Fréchet family is particularly interesting, as it is most suitable for modelling fat-tailed return distributions.

Given that the GPD well approximates tail distributions of actual data, we use it to simulate the upper and lower tails of trading and non-trading returns. The marginal distribution is as follows:

\[
F_n(x_n) = \begin{cases} 
\frac{k_u^L}{K} \left(1 + \frac{x_n - u_n^L}{\beta_n^L} \right)^{-1/\xi_n}, & x_n < u_n^L \\
\phi_n(x_n), & u_n^L \leq x_n \leq u_n^R \\
1 - \frac{k_u^R}{K} \left(1 + \frac{x_n - u_n^R}{\beta_n^R} \right)^{-1/\xi_n}, & x_n > u_n^R 
\end{cases}
\]  

(9)

where \( u_n^R \) and \( u_n^L \) correspond to the upper and lower thresholds, respectively. \( \phi_n(x_n) \) is the empirical function of \( x_n \) in \([u_n^L, u_n^R]\). \( K \) is the number of total observations. \( k_{u_n}^L \) represents the number of returns that are larger than the upper threshold \( u_n^R \), while \( k_{u_n}^L \) represents the number of returns that are smaller than the lower threshold \( u_n^L \). Further, \( \xi_n, \beta_n^L, \beta_n^R \) are parameters to be estimated in the marginal distribution.

2.3.2. Threshold selection

Before estimating the parameters in Equation (9), a proper level of threshold \( u \) needs to be selected. If we choose too high a threshold, then there may be insufficient exceedances, and this may result in high variance estimators. On the other hand, too low a threshold may not satisfy well the conditions for convergence in the distribution of threshold exceedances to the GPD, thereby yielding biased estimators. Thus, we face a
tradeoff between bias and variance in the threshold determination. The following is a review of three major approaches to threshold choice.

The first method is based on the mean excess function (MEF), which is defined as

\[ e(u) = E(X - u \mid X > u). \] (10)

It can be shown that the MEF of the GPD is a linear function of threshold \( u \). For this reason, we consider the empirical MEF

\[ \hat{e}(u) = \frac{1}{n_u} \sum_{i=1}^{n_u} (X_i - u), \] (11)

where \( X_i \) (\( i = 1, 2, \cdots, n_u \)) represents observations that exceed the threshold, and plot the MEF as a function of \( u \). The threshold we choose is the lowest \( u \) such that the empirical MEF is approximately linear.

The second method selects the optimal threshold according to the stability of parameter estimates. If the excess distribution for an initial threshold \( u_0 \) is a GPD with parameters \( \beta(u_0) \) and \( \xi \), then the new excesses over a higher threshold \( u > u_0 \) are distributed as a GPD with corresponding parameters \( \beta(u) \) and \( \xi \), where

\[ \beta(u) = \beta(u_0) + \xi(u - u_0). \] Define the modified scale parameter as \( \beta' = \beta(u) - \xi u \), then \( \beta' \) is a constant with respect to threshold \( u \). Consequently, we plot \( \beta' \) and \( \xi \) versus \( u \) together with confidence intervals for the estimated quantities, and select the smallest \( u \) such that these estimates remain nearly stable.

The third approach is based on Hill’s (1975) plot. Specifically, let \( X_1 > X_2 > \cdots > X_n \) be independent and identically distributed random variables. The tail index statistic is given by
\[ H_{k,n} = \frac{1}{k} \sum_{i=1}^{k} \ln \left( \frac{X_i}{X_k} \right). \]  \hspace{1cm} (12)

Define Hill’s plot as the dot set of \( \{(k, H_{k,n}^{-1}), 1 \leq k \leq n-1\} \). The observation \( X_k \) that corresponds to the beginning point in the tail index stable zone is selected as the threshold.

Overall, the first and third methods depend crucially on the number of exceedances, and may yield a large bias in the case of small samples. On the other hand, the second method provides a relatively objective criterion, as the threshold is usually determined by regressions in this approach. With this method, different thresholds can be selected for different precision levels. To make the threshold estimates more accurate, we first decide the range for the optimal threshold according to the first approach, and then choose the value as per the second and third methods.

2.4. Integrated VaR and ES estimation

Given the marginal distributions of trading and non-trading returns \( F_1(x_1) \) and \( F_2(x_2) \) for the selected thresholds, we then estimate copula parameters by substituting them into various types of copula functions

\[ C(u_1, u_2) = F(F_1^{-1}(u_1), F_2^{-1}(u_2)), \]  \hspace{1cm} (13)

where \( F_i^{-1}(x_i) \) is the reverse of the marginal function \( F_i(x_i) \). Next, these copula functions are simulated and compared, and the one with the smallest Bayesian Information Criteria (BIC) (or Akaike’s Information Criteria (AIC)) is chosen as the optimal copula for our analysis.\(^1\) Finally, the VaR and ES for a confidence level can be computed using the Monte Carlo method. Specifically, VaR is solved from the following definition:

\(^1\) The copula functions considered in this paper include those from ellipse copula function family (Normal copula), Archimedes copula family (Gumbel copula, Joe copula, Frank copula, BB1 copula, BB3 copula,
\( \Pr(X < -\text{VaR}_p) = p, \)  
\[ (14) \]

where \( \text{VaR}_p = w_1 \times M-\text{VaR}_1 + w_2 \times M-\text{VaR}_2 \). Namely,
\[
\int_{-\infty}^{M-\text{VaR}_1} \int_{-\infty}^{M-\text{VaR}_2} c(F_1(x_1), F_2(x_2)) \prod_{j=1}^{2} f_j(x_j) dx_1 dx_2 = p.
\]  
\[ (15) \]

The ES is defined as the expected loss that exceeds VaR. Namely,
\[
\text{ES}_p = E(X \mid X > \text{VaR}_p) = \text{VaR}_p + E(X - \text{VaR}_p \mid X > \text{VaR}_p),
\]  
\[ (16) \]

where \( E(X - \text{VaR}_p \mid X > \text{VaR}_p) \) is the mean excess function corresponding to the \( \text{VaR}_p \).

We know that the mean excess function for the GPD with parameters \( \xi < 1 \) and \( \beta \) is
\[
e(\text{VaR}_p) = E(X - \text{VaR}_p \mid X > \text{VaR}_p) = \frac{\beta + \xi(\text{VaR}_p - u)}{1 - \xi}.
\]  
\[ (17) \]

Equations (16) and (17) together imply
\[
\text{ES}_p = \frac{\text{VaR}_p}{1 - \xi} + \frac{\beta - \xi u}{1 - \xi}.
\]  
\[ (18) \]

2.5. Component VaR (C-VaR) and component ES (C-ES)

In this section, we decompose the integrated VaR and ES risk measures into the risk contributions of trading and non-trading hours. For this purpose, we adopt the concepts of marginal and component risks in the risk attribution framework.

The marginal VaRs (M-VaR) of trading and non-trading returns represent the marginal impact of a small change in the weights of trading and non-trading returns on the integrated VaR. In particular,
\[
M - \text{VaR}_i = \frac{\partial \text{VaR}_p}{\partial w_i} = -E(r_i \mid r_p = \text{VaR}_p),
\]  
\[ (19) \]
where \( w_i \) corresponds to the weight assigned to trading information \((i = 1)\) or non-trading information \((i = 2)\), and \( r_p \) is the sum of trading and non-trading returns.

Since VaR is homogeneous of degree one,\(^2\) we have the following formula, according to Ruler’s theorem (Zhang and Rachev, 2006):

\[
VaR_p = w_1 \frac{\partial VaR_p}{\partial w_1} + w_2 \frac{\partial VaR_p}{\partial w_2} = w_1 \times M - VaR_i + w_2 \times M - VaR_2. \tag{20}
\]

Thus, the first term in Equation (20) measures the risk contribution of trading returns, while the second term measures the risk contribution of non-trading returns. The sum of both terms is just equal to the integrated risk of the market under consideration. For this reason, the component VaR is simply defined as

\[
C \cdot VaR_i = w_i \times M - VaR_i, \ i = 1,2. \tag{21}
\]

Equation (21) indicates that if we obtain the estimate of M-VaR, then C-VaR follows immediately. Garman (1996; 1997) derive expressions for the M-VaR and C-VaR metrics under the assumption that returns are multivariate normally distributed. To avoid the stringent normality assumption, Hallerbach (1999) derives a distribution-free expression for the marginal contribution of an instrument to the diversified portfolio VaR. Based on this method, Huang and Yang (2007) propose a new approach that can provide more accurate estimates of M-VaRs than can Hallerbach’s procedure, especially when the observations are sparse. Therefore, in this paper we utilize this modified approach to estimate marginal VaRs.

Huang and Yang’s (2007) modified approach starts with selecting \( N \) different observations in \( r_p \in [-VaR_p - \theta VaR_p, -VaR_p + \theta VaR_p] \) for different \( \theta \) \((0 < \theta < 1)\), where

\(^2\) A risk measure \( \rho \) is homogeneous of degree one if \( \rho(kX) = k\rho(X) \) for all \( X \) and \( k > 0 \).
each observation is a vector \((r_{i,j}, r_{2,j}, r_{p,j})\). Intuitively, the closer \(r_{p,j}\) and \(-\text{VaR}_p\) are, the better \(r_{i,j}\) can approximate \(r_i\) given that \(r_p = -\text{VaR}_p\) is true. As a result, \(r_{i,j}\) should receive a high weight. The estimate for \(M\cdot\text{VaR}_i\) is given by

\[
M\cdot\text{VaR}_i = -\sum_{j=1}^{N} m_j r_{i,j},
\]  

(22)

where \(r_{i,j}\) is the \(j\)th \((j = 1,2,\cdots,N)\) observation of returns for the \(i\)th \((i = 1,2)\) component (trading or non-trading). \(m_j\) represents the corresponding weight on \(r_{i,j}\). If \(r_{p,j} \neq -\text{VaR}_p\) for any \(j\), then

\[
m_j = \frac{1}{\sum_{n=1}^{N} 1/|r_{p,n} + \text{VaR}_p|},
\]

(23)

If there exists some \(r_{p,k_l} = -\text{VaR}_p\) for \(k_l \) \((l = 1,2,\cdots,L)\), then

\[
m_j = \begin{cases} 
\frac{1}{(L+1)} \sum_{n=1}^{N} 1/|r_{p,n} + \text{VaR}_p|, & j \neq k_1, k_2, \cdots, k_L \\
1/(L+1), & j = k_1, k_2, \cdots, k_L.
\end{cases}
\]

(24)

In addition, \(\sum_{j=1}^{N} m_j = 1 \ (m_j > 0)\), and also \(\frac{\text{VaR}_p}{\sum_{i=1}^{2} M\cdot\text{VaR}_i}\) should be close to 1.

Given that the typical daily return is simply a sum of trading and non-trading returns (i.e., \(w_1 = w_2 = 1\)), the component VaR is the same as marginal VaR in our analysis. Based on the estimates of M-VaRs and all the observations from set \(r_p \in (-\infty, -\text{VaR}_p)\), we can compute the M-ESs as follows

\[
M\cdot\text{ES}_i = \frac{1}{N} \sum_{j=1}^{N} r_{i,j}.
\]

(25)
Accordingly, the percentage risk contribution of M-VaRs and M-ESs are given by

\[ PC_{-VaR_i} = \frac{M_{-VaR_i}}{\sum_{i=1}^{2} M_{-VaR_i}}, \]  
(26)

\[ PC_{-ES_i} = \frac{M_{-ES_i}}{\sum_{i=1}^{2} M_{-ES_i}}. \]  
(27)

2.6. Backtesting

Since the late 1990’s, a variety of backtests have been proposed for gauging the adequacy of VaR models. The two well-known approaches to backtesting a risk model include Kupiec’s (1995) unconditional coverage test and Christoffersen’s (1998) conditional coverage test. To illustrate the basic idea of backtesting, we denote the actual loss for a given horizon as \( L_t \), and then define the following indicator variable as the hit function,

\[ I_t(p) = \begin{cases} 
1 & \text{if } L_t < -VaR_t(p), \\
0 & \text{if } L_t \geq -VaR_t(p).
\end{cases} \]  
(28)

Apparently, the hit function series tracks the history of whether or not a realized loss exceeds the model estimated VaR at the given confidence level \( 1 - p \). Christoffersen (1998) demonstrates that under the null hypothesis that the VaR model is correct, the resulting hit series \( \{I_t(p)\}_{t=1}^T \) must satisfy both the so called unconditional coverage property and independence property. The unconditional coverage property requires that the observed frequency of violations, which is defined as losses exceeding the VaR estimates for that period, be the same as \( p \), the expected frequency of exceedances according to the model.
On the other hand, the independence property states that any two elements of the hit series must be independent of each other.

Kupiec’s (1995) unconditional test focuses only on the unconditional coverage property. Namely, it examines whether the observed frequency of violations \( \frac{1}{T} \sum_{t=1}^{T} I_t(p) \) is statistically significantly different from \( p \). Following the binomial theory, the probability of observing \( N \) violations out of \( T \) observations is \( (1-p)^{T-N} p^N \). Thus, the likelihood ratio test statistic is given as follows:

\[
LR^{UC} = -2 \ln[(1-p)^{T-N} p^N] + 2 \ln[(1-N/T)^{T-N} (N/T)^N].
\]  

(29)

Under the null hypothesis that the expected violation frequency is \( p \), this unconditional test statistic is distributed as \( \chi^2(1) \). While this method is very intuitive and straightforward, it suffers from at least two shortcomings. The first problem is the low power of test, as pointed out by Kupiec (1995), while the second is that it fails to detect whether these VaR violations are independent of each other. A VaR model that violates the independence property may result in clustered violations, indicating that the model cannot properly capture the variability of losses under certain conditions.

In contrast, Christoffersen’s (1998) conditional coverage approach jointly tests both the unconditional coverage and independence properties. In addition, we can also test the sub-hypothesis regarding the frequency and independence of violations with this method. This procedure allows us to separate clustering effects from the distributional assumption.

The statistic for Christoffersen’s test is given by

\[
LR^{CC} = -2 \ln[(1-p)^{T-N} p^N] + 2 \ln[(1-\pi_{01})^{n_0} \pi_{01}^{n_0} (1-\pi_{11})^{n_1} \pi_{11}^{n_1}],
\]  

(30)
where \( n_{ij} (i, j = 0,1) \) is the number of times that value \( i \) is followed by \( j \) in the hit series \( \{I_i(p)\}_{i=1}^{T} \), and \( \pi_{ij} \) is the corresponding frequency, which is defined as \( \pi_{ij} = n_{ij} / \sum_{j=0}^{1} n_{ij} \).

Under the null hypothesis that the model is correct, the distribution of \( LR^{CC} \) is asymptotically \( \chi^2(2) \).

3. Data

In this paper, we focus on Chinese commodity futures markets. In particular, the data sets consist of daily opening and closing prices for copper and natural rubber futures traded on the Shanghai Futures Exchange (SHFE), as well as those prices for soybean futures on the Dalian Commodity Exchange (DCE), obtained from their respective exchanges. The sample periods extend from September 15, 1993 to July 20, 2010 for copper futures, from November 3, 1995 to July 20, 2010 for rubber futures, and from October 18, 1994 to July 20, 2010 for soybean futures, respectively. The SHFE currently trades futures on aluminum, copper, gold, zinc, natural rubber, and fuel oil, while the DCE primarily trades soybean futures. By 2009, both copper and natural rubber futures on the SHFE ranked first in the world in terms of trading volume, while the trading volume of soybean futures on the DCE is 23% of that on the Chicago Mercantile Exchange (CME), the largest soybean futures market in the world, and 13 times the trading volume of the third largest market, the Tokyo Grains Exchange.\(^3\) Therefore, the futures considered in this analysis are well representative of Chinese commodity futures markets.

Following the general practice in the literature, each futures price series is constructed by rolling over the nearby futures contract on the first trading day of the next month (for

\(^3\) Sources: [www.cnfinance.cn](http://www.cnfinance.cn) and Shanghai Securities News, August 18, 2009.
copper and rubber contracts) or the contract’s expiration month (for soybean contracts). The nearby futures contracts are used, as these are the most liquid and actively traded contracts in markets. Based on these price series, close-to-close daily returns, close-to-open, and open-to-close returns are calculated as per Equations (1), (2), and (3), respectively. The trading hours of Chinese futures markets are from 9:00 a.m. to 3:00 p.m. (Monday to Friday). Consequently, the non-trading hours on weekdays are three times as long as the trading hours.

Figure 1 plots the trading and non-trading returns for each futures contract, whereas Table 1 reports the descriptive statistics of these return series. From the results, we can see that there are indeed some big differences in the distributional characteristics between trading and non-trading returns. On average, trading returns are substantially lower than the average non-trading returns for copper and soybean markets, while the opposite is true for the rubber market. This is primarily due to how good/bad the news released during non-trading hours is relative to that released during trading hours in a particular market. The returns for all the three futures are negatively or positively skewed with excess kurtosis, indicating that they are not normally distributed. Further, in terms of standard deviations, non-trading returns are more volatile than trading returns for both copper and soybean futures, while non-trading returns are slightly less volatile than trading returns for rubber futures. As a result, the information accumulated during non-trading hours is significant in Chinese commodity futures markets, given the fact that volatilities are directly related to information flows.

4. Empirical results

4.1. Marginal distribution estimation
Figures 2, 3, and 4 display the mean residual life, stabilities of GPD parameters, and Hill plots, respectively. Based on these plots, upper and lower thresholds for trading and non-trading returns for these futures are selected, and are reported in Table 2.

For the selected thresholds, scale parameter $\beta$ and shape parameter $\xi$ in the GPD are then estimated using the maximum likelihood method. These estimates are presented in Table 2 as well. To evaluate the goodness of fit of the data series to the model, Figure 5 depicts the QQ-plot of residuals from GPD fit to the actual loss data over the threshold and the estimated tail for trading and non-trading returns in each futures market. As we can see from these plots, all the points on the graphs lie very nearly along the solid line, indicating that the estimated GPDs fit the data satisfactorily.

For the estimated marginal distributions $F_1(\hat{x}_1)$ and $F_2(\hat{x}_2)$ of trading and non-trading returns, we have a copula function such that $C(u_1, u_2) = F\left(F_1^{-1}(u_1), F_2^{-1}(u_2)\right)$, according to Sklar’s theorem. To identify the copula function that best describe the binary characteristics of the marginal functions, we simulate 10 different copula functions and compute their respective log-likelihood, AIC, and BIC using the maximum likelihood method. The results are reported in Table 3.

Apparently, Tawn function generates the lowest BIC and AIC, and the highest log-likelihood among these copulas considered, indicating that it fits the data the best. Thus,

---

4 A QQ-plot (quantile-quantile plot) plots the quantiles of an empirical distribution against the quantiles of a hypothesized distribution. It is usually used in statistics to examine whether a sample comes from a specific distribution.

5 Tawn copula (Tawn, 1988) belongs to the extreme value copula class, which is represented in the form of $C(u, v) = \exp\left\{ \ln(uv)A\left(\frac{\ln u}{\ln(uv)}\right) \right\}$, where $A(z)$ is called the dependence function. Tawn copula has
we estimate parameters \( \alpha \), \( \beta \), and \( \gamma \) in Tawn function for different futures markets, which are presented in Table 4. We note that these estimates are all significant at the 1% level. In addition, Figure 6 plots the contours of empirical copula and the Tawn function for copper, rubber, and soybean futures markets. Clearly, the contour of the empirical copula inosculates with that of the Tawn function very well for all three futures, which further demonstrates that using Tawn function is most appropriate to incorporate \( F_1(\hat{x}_i) \) and \( F_2(\hat{x}_j) \) into their joint distribution.

4.2. Integrated VaR and ES estimation and backtesting results

The integrated VaRs and ESs with confidence levels of 95% and 99% for copper, rubber, and soybean futures are presented in Table 5. At the 95% confidence level, the integrated VaRs are 1.8100, 2.1438, and 1.4726 for copper, rubber, and soybean futures, respectively, while the corresponding integrated ESs are 2.6274, 3.0756, and 2.1316, respectively. Therefore, measured by the integrated VaR and ES, the Chinese rubber futures market exhibits the highest overall risk, followed by the copper market, with the soybean market least risky. As expected, this ordering of market risks is consistent with that implied by volatilities observed in Table 1. Not surprisingly, the results at the 99% confidence level are higher than those at the 95% level, but indicate a similar pattern.

To examine the robustness of the above results, backtesting is conducted, and the results are presented in Table 6. We find that the spillover ratios are all lower than the

\[
A(z) = 1 - \beta + (\beta - \alpha) + \left[ \alpha' z' + \beta' (1 - z) \right]^{\frac{1}{\gamma}}, \quad \text{where } \alpha \geq 0, \beta \leq 1, \text{ and } 1 \leq \gamma < \infty.
\]
target violation rate, and both unconditional and conditional backtesting statistics are
significant at the 1% or 5% level. This further demonstrates the adequacy of our model.

To evaluate the accuracy of our risk measures relative to the measures computed by
the typical method, the VaRs and ESs are computed based on close-to-close market returns
for each futures, and the results are reported in Table 7. The backtesting results are also
reported in Table 7, which clearly indicate that the VaR model can accurately predict both
the frequency and the size of expected losses. Comparing these VaR and ESs with those in
Tables 5 shows that they are lower than the corresponding integrated values obtained by
the copula method. This implies that risk estimation based on close-to-close returns
understates market risk relative to the integrated measures. This is because the calculation
of close-to-close returns only uses closing prices, and it cannot fully capture individual risk
components in trading and non-trading hours. On the contrary, the integrated VaR and ES
employ both closing and opening prices and contain more information in a futures market.
Consequently, these measures explicitly take into consideration the risk characteristics of
trading and non-trading returns and their dependence structure.

Note that the integrated VaR or ES greatly depends on the confidence level. To gauge
the sensitivity of our integrated risk measures to changes in confidence levels, we simulate
VaRs and ESs for various confidence levels ranging from 10% to 100% by iterating 10,000
times of samples. Figure 7 plots these VaRs/ESs against $p$ for copper, rubber, and
soybean markets, respectively. This figure shows that both the integrated VaR and ES
increase with the confidence level, $1-p$, which is expected given the definitions of these
measures. Moreover, the plots become steeper when the confidence level is higher,
indicating that the integrated VaR and ES are more sensitive to the confidence level when
the level is higher than when it is lower. This finding is in line with those in the literature (Fu and Xing, 2009), and is true regardless of the futures market under consideration.

4.3. Component VaR (C-VaR) and component ES (C-ES)

In this section, we focus on the risk contributions of trading and non-trading hours, which are measured by their respective C-VaR and C-ES. The results in Table 8 show that under the 95% confidence level, the C-VaRs of trading and non-trading returns are 1.2814 and 2.3334 for copper, 2.2694 and 1.9276 for rubber, and 1.5452 and 1.1626 for soybean futures, respectively. Accordingly, the non-trading hours contribute 64.55%, 45.94%, and 42.94% to the overall risk measured by integrated VaRs for copper, rubber, and soybean futures, respectively. Under the 99% confidence level, the respective C-VaR weights are 58.80%, 47.16%, and 48.83%. Overall, the non-trading C-VaR weights are substantial for all markets. In particular, in the case of copper, the risk contribution of non-trading hours is even larger than that of trading hours. These results highlight the huge amount of information accumulated during non-trading hours and its impact on the overall risk of futures markets in China. This information includes not only announcements made concerning these commodities in China, but also the trading activity in corresponding international futures markets. News released during non-trading hours can be more negative relative to that released during trading hours (Patell and Wolfson, 1982; Bagnoli, Clement, and Watts, 2005); it could also be relatively positive or neutral (Doyle and Magilke, 2008; Cliff, Copper, and Gulen, 2008). The risk associated with these announcements, particularly negative announcements made during non-trading hours, is captured by the C-VaR weights, as VaR focuses on the downward tail of return distributions. Our results also reflect the fact that Chinese futures markets have become
more and more integrated with international futures markets, especially US and European markets. Given that these overseas futures markets trade during the non-trading hours in Chinese markets, their trading information apparently contributes greatly to the high risk component of non-trading hours in China. This is consistent with findings in Liu and An (2010), who document the leading role of US copper and soybean futures markets in information transmission and the price discovery process between Chinese and US markets. Finally, we believe that the high liquidity risk during non-trading hours is another factor that can partially explain our findings.

The results of the C-ESs for different markets under the 95% and 99% confidence levels are also reported in Table 8. Measured by the C-ES under the 99% level, it seems that the non-trading returns have a slightly higher contribution to the overall risk for both copper and rubber markets. All other major conclusions are re-confirmed in this case.

4.4. A further analysis of VaRs and ESs for non-trading returns

To further understand the risk of non-trading hours, we analyze how the VaR and ES are related to the length of non-trading periods. For this purpose, we estimate the VaRs and ESs of weeknights, weekends, and holidays based on the individual distribution of each type of non-trading returns. The results are presented in Table 9. We find that the VaRs of weeknights, weekends, and holidays under the 95% confidence level are 1.8501, 2.0212, and 3.1180 for copper, 1.5140, 2.1726, and 3.0184 for rubber, and 1.2254, 1.4799, and 2.3058 for soybean futures markets, respectively. This suggests that the longer the non-trading period, the higher the risk of the non-trading period for all three markets. However, in the case of the 99% confidence level, this effect is less pronounced for the rubber market, and the weekend VaR is even greater than the holiday VaR for the soybean market. We
observe a similar pattern when ES is considered. Nevertheless, weeknight VaR/ES is generally lower than weekend/holiday VaR/ES, regardless of the market. Intuitively, the information continues to accumulate as time goes by, and therefore, weekends or holidays contain more price sensitive information than weeknights.

5. Conclusions

This paper investigates the risk contributions of trading and non-trading hours in Chinese copper, rubber, and soybean futures markets. Using the copula method, we incorporate the distributions of trading and non-trading returns into a joint distribution, and examine the integrated VaRs and ESs for these markets to gauge overall market risk. Then we decompose these risk measures into component VaRs and ESs to evaluate the risk contributions of trading and non-trading returns to overall risk. Backtests are also performed to assess the adequacy of the models.

Our empirical results show that the copula method provides appropriate measures for integrated risks, and the typical VaR and ES based on close-to-close returns underestimate overall market risks. Measured by both integrated VaRs and ESs, Chinese rubber futures are most risky, followed by copper futures, and soybean futures are least risky. Our results also demonstrate that the risk contributions of non-trading hours are substantial for all futures considered, with C-VaR and C-ES weights being more than 40% in any market. In the case of copper futures in particular, non-trading hours seem to contribute to the total risk more than do the trading hours. This sheds lights on the important role of non-trading information in predicting market returns and explaining market risks. Finally, we find that weeknight VaRs and ESs are lower than weekend/holiday VaRs and ESs for all markets,
indicating that the risk of non-trading periods is positively related to the length of non-trading periods.
References


This figure plots the trading (daytime) and non-trading (overnight) returns for copper, rubber, and soybean futures markets (from top to bottom) in China. The sample periods extend from September 15, 1993 to July 20, 2010 for copper futures, from November 3, 1995 to July 20, 2010 for rubber futures, and from October 18, 1994 to July 20, 2010 for soybean futures, respectively.
Table 1. Descriptive statistics for trading and non-trading returns

<table>
<thead>
<tr>
<th>Markets</th>
<th>Returns</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>Trading returns</td>
<td>7.7327</td>
<td>-6.0414</td>
<td>0.0085</td>
<td>0.9268</td>
<td>0.1063</td>
<td>9.2185</td>
</tr>
<tr>
<td></td>
<td>Non-trading returns</td>
<td>9.1343</td>
<td>-6.2046</td>
<td>0.0168</td>
<td>1.2458</td>
<td>-0.2510</td>
<td>7.1876</td>
</tr>
<tr>
<td></td>
<td>Traditional returns</td>
<td>8.4412</td>
<td>-3.3853</td>
<td>0.2284</td>
<td>0.8557</td>
<td>1.3906</td>
<td>10.6016</td>
</tr>
<tr>
<td>Rubber</td>
<td>Trading returns</td>
<td>8.2040</td>
<td>-6.8602</td>
<td>0.0586</td>
<td>1.4382</td>
<td>0.1028</td>
<td>5.7606</td>
</tr>
<tr>
<td></td>
<td>Non-trading returns</td>
<td>9.3584</td>
<td>-9.2019</td>
<td>-0.0251</td>
<td>1.3231</td>
<td>-0.5875</td>
<td>10.6238</td>
</tr>
<tr>
<td></td>
<td>Traditional returns</td>
<td>8.6654</td>
<td>-5.0649</td>
<td>0.3336</td>
<td>1.0943</td>
<td>1.2336</td>
<td>8.9815</td>
</tr>
<tr>
<td>Soybeans</td>
<td>Trading returns</td>
<td>5.6587</td>
<td>-6.8726</td>
<td>0.0003</td>
<td>0.9220</td>
<td>-0.3099</td>
<td>7.9391</td>
</tr>
<tr>
<td></td>
<td>Non-trading returns</td>
<td>8.8235</td>
<td>-8.1469</td>
<td>0.0191</td>
<td>1.0038</td>
<td>0.5549</td>
<td>19.8457</td>
</tr>
<tr>
<td></td>
<td>Traditional returns</td>
<td>8.1309</td>
<td>-3.1360</td>
<td>0.1783</td>
<td>0.7802</td>
<td>2.7020</td>
<td>20.8706</td>
</tr>
</tbody>
</table>

This table reports the descriptive statistics of various return series for Chinese copper, rubber, and soybean futures markets. Trading, non-trading, and traditional returns are defined in Equations (3), (2), and (1), respectively. The sample periods extend from September 15, 1993 to July 20, 2010 for copper futures, from November 3, 1995 to July 20, 2010 for rubber futures, and from October 18, 1994 to July 20, 2010 for soybean futures, respectively.
Figure 2. Plots of mean residual life, stabilities of GPD parameters, and Hill of trading and non-trading returns for copper futures.
**Figure 3.** Plots of mean residual life, stabilities of GPD parameters, and Hill of trading and non-trading returns for rubber futures.

This figure plots the mean residual life (top), stabilities of GPD parameters (middle), and Hill (bottom) of trading returns (left) and non-trading returns (right) for rubber futures.
Figure 4. Plots of mean residual life, stabilities of GPD parameters, and Hill of trading and non-trading returns for soybean futures.

This figure plots the mean residual life (top), stabilities of GPD parameters (middle), and Hill (bottom) of trading returns (left) and non-trading returns (right) for soybean futures.
### Table 2. Estimated parameters of the marginal distribution functions

<table>
<thead>
<tr>
<th>Markets</th>
<th>Returns</th>
<th>$u^R$</th>
<th>$\xi^R$</th>
<th>$\beta^R$</th>
<th>Log-likelihood of upper tail</th>
<th>$u^L$</th>
<th>$\xi^L$</th>
<th>$\beta^L$</th>
<th>Log-likelihood of lower tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>Trading</td>
<td>1.200</td>
<td>0.0824</td>
<td>0.7438</td>
<td>-205.2</td>
<td>-1.200</td>
<td>0.1042</td>
<td>0.6675</td>
<td>-187.6</td>
</tr>
<tr>
<td></td>
<td>non-trading</td>
<td>1.600</td>
<td>0.0209</td>
<td>0.9306</td>
<td>-246.7</td>
<td>-0.850</td>
<td>0.0262</td>
<td>0.9473</td>
<td>-567.7</td>
</tr>
<tr>
<td>Rubber</td>
<td>Trading</td>
<td>1.600</td>
<td>0.0487</td>
<td>0.972</td>
<td>-272.4</td>
<td>-1.500</td>
<td>-0.1013</td>
<td>1.0983</td>
<td>-271.9</td>
</tr>
<tr>
<td></td>
<td>non-trading</td>
<td>1.200</td>
<td>0.2154</td>
<td>0.7419</td>
<td>-229.2</td>
<td>-0.700</td>
<td>0.2033</td>
<td>0.8545</td>
<td>-446.6</td>
</tr>
<tr>
<td>Soybeans</td>
<td>Trading</td>
<td>1.400</td>
<td>0.0363</td>
<td>0.685</td>
<td>-107.3</td>
<td>-1.400</td>
<td>0.0574</td>
<td>0.7499</td>
<td>-144.7</td>
</tr>
<tr>
<td></td>
<td>non-trading</td>
<td>0.730</td>
<td>0.3608</td>
<td>0.6244</td>
<td>-323.0</td>
<td>-1.190</td>
<td>0.1356</td>
<td>0.8832</td>
<td>-162.8</td>
</tr>
</tbody>
</table>

This table reports the estimated parameters of the marginal distribution functions of trading and non-trading returns, as well as the log-likelihood values for copper futures, rubber, and soybean futures markets.
**Figure 5.** Simulation plots of GPD of both trading and non-trading returns

<table>
<thead>
<tr>
<th></th>
<th>trading returns</th>
<th>non-trading returns</th>
<th>trading returns</th>
<th>non-trading returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper tails</td>
<td><img src="image1" alt="Upper tails Plot" /></td>
<td><img src="image2" alt="Upper tails Plot" /></td>
<td><img src="image3" alt="Upper tails Plot" /></td>
<td><img src="image4" alt="Upper tails Plot" /></td>
</tr>
<tr>
<td>Lower tails</td>
<td><img src="image5" alt="Lower tails Plot" /></td>
<td><img src="image6" alt="Lower tails Plot" /></td>
<td><img src="image7" alt="Lower tails Plot" /></td>
<td><img src="image8" alt="Lower tails Plot" /></td>
</tr>
</tbody>
</table>

This figure depicts the QQ-plots of residuals from GPD fit to the loss data (the first and third from top to bottom) and the tail estimates (the second and fourth from top to bottom) for copper futures (the first two plots from left to right), rubber futures (the third and fourth plots from left to right), and soybean futures (the last two plots from left to right).
### Table 3. Testing results for various copula functions

<table>
<thead>
<tr>
<th></th>
<th>Copula functions</th>
<th>Gumbel</th>
<th>Joe</th>
<th>Frank</th>
<th>Tawn</th>
<th>BB1</th>
<th>BB3</th>
<th>BB4</th>
<th>BB6</th>
<th>BB7</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AIC</td>
<td>-27.06</td>
<td>-30.90</td>
<td>-1.47</td>
<td>-48.47</td>
<td>-25.06</td>
<td>-24.75</td>
<td>-11.82</td>
<td>-28.90</td>
<td>-29.99</td>
<td>-1.15</td>
</tr>
<tr>
<td></td>
<td>BIC</td>
<td>-20.87</td>
<td>-24.72</td>
<td>4.72</td>
<td>-29.93</td>
<td>-12.69</td>
<td>-12.39</td>
<td>0.54</td>
<td>-16.54</td>
<td>-17.62</td>
<td>5.03</td>
</tr>
<tr>
<td>Rubber</td>
<td>Log-likelihood</td>
<td>6.37</td>
<td>6.67</td>
<td>0.61</td>
<td>12.19</td>
<td>6.74</td>
<td>6.81</td>
<td>3.66</td>
<td>6.67</td>
<td>7.80</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>BIC</td>
<td>-5.01</td>
<td>-5.61</td>
<td>6.52</td>
<td>-1.20</td>
<td>1.97</td>
<td>1.84</td>
<td>8.14</td>
<td>2.12</td>
<td>-0.14</td>
<td>7.03</td>
</tr>
<tr>
<td>Soybeans</td>
<td>Log-likelihood</td>
<td>6.89</td>
<td>8.26</td>
<td>0.08</td>
<td>21.42</td>
<td>4.87</td>
<td>6.90</td>
<td>3.13</td>
<td>8.26</td>
<td>8.49</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>BIC</td>
<td>-5.74</td>
<td>-8.47</td>
<td>7.88</td>
<td>-18.71</td>
<td>6.33</td>
<td>2.28</td>
<td>9.83</td>
<td>-0.43</td>
<td>-0.89</td>
<td>8.03</td>
</tr>
</tbody>
</table>

This table reports the testing results of log-likelihood, BIC and AIC for 10 different copula functions for copper, rubber, and soybean futures markets, respectively.
Table 4. Estimation results for the Tawn function

<table>
<thead>
<tr>
<th>Markets</th>
<th>Parameters</th>
<th>Value</th>
<th>Std. Error</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>(\alpha)</td>
<td>0.1374**</td>
<td>0.0284</td>
<td>4.8316</td>
</tr>
<tr>
<td></td>
<td>(\beta)</td>
<td>0.1215**</td>
<td>0.0265</td>
<td>4.5811</td>
</tr>
<tr>
<td></td>
<td>(\gamma)</td>
<td>2.4913**</td>
<td>0.4715</td>
<td>5.2837</td>
</tr>
<tr>
<td>Rubber</td>
<td>(\alpha)</td>
<td>0.1573**</td>
<td>0.0507</td>
<td>3.1043</td>
</tr>
<tr>
<td></td>
<td>(\beta)</td>
<td>0.0978**</td>
<td>0.0324</td>
<td>3.0148</td>
</tr>
<tr>
<td></td>
<td>(\gamma)</td>
<td>2.1296**</td>
<td>0.5675</td>
<td>3.7528</td>
</tr>
<tr>
<td>Soybeans</td>
<td>(\alpha)</td>
<td>0.0727**</td>
<td>0.0160</td>
<td>4.5456</td>
</tr>
<tr>
<td></td>
<td>(\beta)</td>
<td>0.0690**</td>
<td>0.0148</td>
<td>4.6660</td>
</tr>
<tr>
<td></td>
<td>(\gamma)</td>
<td>7.0272**</td>
<td>2.1279</td>
<td>3.3024</td>
</tr>
</tbody>
</table>

This table presents the estimated \(\alpha\), \(\beta\), and \(\gamma\) values in the Tawn function for copper, rubber, and soybean futures markets, respectively. \*\* indicates significance at the 1% level.
Figure 6. The contours of the empirical copula and fitted Tawn functions

This figure plots the empirical copula and fitted Tawn functions using the data for copper, rubber, and soybean futures markets (from top to bottom).
Table 5. Integrated VaR and ES estimates for different futures markets

<table>
<thead>
<tr>
<th>Markets</th>
<th>Risk measures</th>
<th>Confidence level</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>integrated VaR</td>
<td>95%</td>
<td>1.8100</td>
</tr>
<tr>
<td></td>
<td>integrated ES</td>
<td>95%</td>
<td>2.6274</td>
</tr>
<tr>
<td></td>
<td>integrated ES</td>
<td>99%</td>
<td>3.9704</td>
</tr>
<tr>
<td>Rubber</td>
<td>integrated VaR</td>
<td>95%</td>
<td>2.1438</td>
</tr>
<tr>
<td></td>
<td>integrated ES</td>
<td>95%</td>
<td>3.0756</td>
</tr>
<tr>
<td></td>
<td>integrated ES</td>
<td>99%</td>
<td>4.5870</td>
</tr>
<tr>
<td>Soybeans</td>
<td>integrated VaR</td>
<td>95%</td>
<td>1.4726</td>
</tr>
<tr>
<td></td>
<td>integrated ES</td>
<td>95%</td>
<td>1.4726</td>
</tr>
<tr>
<td></td>
<td>integrated ES</td>
<td>99%</td>
<td>2.5290</td>
</tr>
</tbody>
</table>

This table reports the estimated integrated VaR and the corresponding integrated ES under the 95% and 99% confidence levels in copper, rubber, and soybean futures markets.
### Table 6. Backtesting results

<table>
<thead>
<tr>
<th>Markets</th>
<th>Confidence level</th>
<th>Spillover number</th>
<th>Spillover ratio</th>
<th>LR^UC</th>
<th>LR^CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>95%</td>
<td>172</td>
<td>4.32%</td>
<td>1.3289*</td>
<td>3.9910**</td>
</tr>
<tr>
<td></td>
<td>99%</td>
<td>53</td>
<td>1.33%</td>
<td>1.0483*</td>
<td>4.0255**</td>
</tr>
<tr>
<td>Rubber</td>
<td>95%</td>
<td>118</td>
<td>4.62%</td>
<td>0.8110*</td>
<td>1.4035*</td>
</tr>
<tr>
<td></td>
<td>99%</td>
<td>37</td>
<td>1.45%</td>
<td>1.0483*</td>
<td>4.5436**</td>
</tr>
<tr>
<td>Soybeans</td>
<td>95%</td>
<td>142</td>
<td>4.07%</td>
<td>0.0750*</td>
<td>6.6886**</td>
</tr>
<tr>
<td></td>
<td>99%</td>
<td>40</td>
<td>1.15%</td>
<td>0.7338*</td>
<td>13.1017**</td>
</tr>
</tbody>
</table>

This table reports the backtesting results for integrated VaRs under the 95% and 99% confidence levels in copper, rubber, and soybean futures markets. The sample is iterated 10,000 times. * and ** indicate significance at the 5% and 1% levels, respectively.
Table 7. Estimates of VaR and ES, and backtesting results based on close-to-close returns

<table>
<thead>
<tr>
<th>Markets</th>
<th>Confidence level</th>
<th>VaR</th>
<th>ES</th>
<th>Spillover times</th>
<th>Spillover ratio</th>
<th>LR^UC</th>
<th>LR^CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>95%</td>
<td>1.2768</td>
<td>1.8261</td>
<td>196</td>
<td>4.93%</td>
<td>0.0432*</td>
<td>50.1874**</td>
</tr>
<tr>
<td></td>
<td>99%</td>
<td>2.1664</td>
<td>2.6385</td>
<td>38</td>
<td>0.96%</td>
<td>0.0799*</td>
<td>18.2474**</td>
</tr>
<tr>
<td>Rubber</td>
<td>95%</td>
<td>1.6177</td>
<td>2.2505</td>
<td>109</td>
<td>4.26%</td>
<td>3.0572**</td>
<td>19.7498**</td>
</tr>
<tr>
<td></td>
<td>99%</td>
<td>2.6423</td>
<td>3.0992</td>
<td>19</td>
<td>0.74%</td>
<td>1.8662**</td>
<td>9.1448**</td>
</tr>
<tr>
<td>Soybeans</td>
<td>95%</td>
<td>0.9272</td>
<td>1.5187</td>
<td>163</td>
<td>4.68%</td>
<td>0.7806**</td>
<td>19.1006**</td>
</tr>
<tr>
<td></td>
<td>99%</td>
<td>1.8908</td>
<td>2.5331</td>
<td>31</td>
<td>0.89%</td>
<td>0.4462*</td>
<td>9.8354**</td>
</tr>
</tbody>
</table>

This table presents the VaR and ES estimates and corresponding backtesting results under the 95% and 99% confidence levels in copper, rubber, and soybean futures markets. The sample is iterated 10,000 times. * and ** indicate significance at the 5% and 1% levels, respectively.
Figure 7. Sensitivities of VaR and ES with respect to confidence levels

This figure plots the integrated VaR and ES against the violation probability $p$ for copper, rubber, and soybean futures markets (from top to bottom), respectively.
Table 8. Component VaR and ES, as well as risk contributions of trading and non-trading returns

<table>
<thead>
<tr>
<th>Markets</th>
<th>Risk measures</th>
<th>95% confidence level</th>
<th>99% confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Trading returns</td>
<td>Non-trading returns</td>
</tr>
<tr>
<td></td>
<td>Component VaR</td>
<td>1.2814</td>
<td>2.5906</td>
</tr>
<tr>
<td>Copper</td>
<td>Risk contribution (PC-VaR)</td>
<td>35.45%</td>
<td>41.20%</td>
</tr>
<tr>
<td></td>
<td>Component ES</td>
<td>2.7724</td>
<td>3.6992</td>
</tr>
<tr>
<td></td>
<td>Risk contribution (PC-ES)</td>
<td>42.84%</td>
<td>44.19%</td>
</tr>
<tr>
<td>Rubber</td>
<td>Component VaR</td>
<td>2.2694</td>
<td>4.1470</td>
</tr>
<tr>
<td></td>
<td>Risk contribution (PC-VaR)</td>
<td>54.07%</td>
<td>52.84%</td>
</tr>
<tr>
<td></td>
<td>Component ES</td>
<td>4.0404</td>
<td>6.2212</td>
</tr>
<tr>
<td></td>
<td>Risk contribution (PC-ES)</td>
<td>51.91%</td>
<td>49.59%</td>
</tr>
<tr>
<td>Soybeans</td>
<td>Component VaR</td>
<td>1.5452</td>
<td>2.6694</td>
</tr>
<tr>
<td></td>
<td>Risk contribution (PC-VaR)</td>
<td>57.06%</td>
<td>51.17%</td>
</tr>
<tr>
<td></td>
<td>Component ES</td>
<td>2.8654</td>
<td>4.6768</td>
</tr>
<tr>
<td></td>
<td>Risk contribution (PC-ES)</td>
<td>51.97%</td>
<td>53.12%</td>
</tr>
</tbody>
</table>

This table reports the component VaRs and ESs under the 95% and 99% confidence levels, as well as the risk contributions of trading and non-trading hours in copper, rubber, and soybean futures markets.
Table 9. VaRs and ESs of weeknights, weekends, and holidays for different futures markets

<table>
<thead>
<tr>
<th>Contracts</th>
<th>Non-trading periods</th>
<th>95% confidence level</th>
<th>99% confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>VaR</td>
<td>ES</td>
</tr>
<tr>
<td>Copper</td>
<td>weeknights</td>
<td>1.8501</td>
<td>2.9627</td>
</tr>
<tr>
<td></td>
<td>weekends</td>
<td>2.0212</td>
<td>3.2973</td>
</tr>
<tr>
<td></td>
<td>holidays</td>
<td>3.1180</td>
<td>3.7495</td>
</tr>
<tr>
<td>Rubber</td>
<td>weeknights</td>
<td>1.5140</td>
<td>3.2839</td>
</tr>
<tr>
<td></td>
<td>weekends</td>
<td>2.1726</td>
<td>3.6832</td>
</tr>
<tr>
<td></td>
<td>holidays</td>
<td>3.0184</td>
<td>4.0486</td>
</tr>
<tr>
<td>Soybeans</td>
<td>weeknights</td>
<td>1.2254</td>
<td>2.4567</td>
</tr>
<tr>
<td></td>
<td>weekends</td>
<td>1.4799</td>
<td>3.2755</td>
</tr>
<tr>
<td></td>
<td>holidays</td>
<td>2.3058</td>
<td>2.9288</td>
</tr>
</tbody>
</table>

This table reports the estimated VaRs and ESs under the 95% and 99% confidence levels based on weeknight, weekend, and holiday returns for copper, rubber, and soybean futures markets.