Advances in Sparse Solver for Finite Element Analysis

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ABSTRACT

The focus of this paper is to describe cell-sparse storage scheme and the corresponding algorithm for the direct symmetric positive definite equation solver in finite element analysis (FEA). Based on the special shape of the stiffness matrix in finite element analysis, the contribution of each node is al-ways in cell-submatrix form. Naturally one index can be used to follow one cell or submatrix (2 by 2, 3 by 3, or 6 by 6) to decrease the number of index [1,2]. Thus the cell-index scheme requires much less disk space and is much faster than conventional direct sparse solvers for large-scale FE problems.

Well-organized storage scheme can greatly improve the efficiency, but it is not the only factor that affects the computational efficiency. Recently the loop unrolling technique of matrix operations [3], in which unnecessary data traffic between registers and RAM are minimized, has been utilized to promote the computational performance in parallel as well as sequential computing. The loop unrolling reduces the number of loops and enlarges the loop contents in matrix computation. For the full matrix factorization, the unrolling technique leads to a maximum speedup several times on variety of machines and compilers.

The cell sparse storage scheme provides an excellent data structure for two-way loop unrolling, to which we unroll two outer loops of the triply nested loop in JKI factorization. This essential change leads to a remarkable improvement in efficiency and the disk space requirement. In this paper, the cell storage scheme and two-way unrolling technique are implemented in linear and nonlinear finite element analysis. Performance in terms of elapsed time and disk space requirement of several available solvers is demonstrated by test data and some practical engineering problems. Tests indicated that the performance for either static or generalized eigenvalue problems can be improved significantly by the proposed cell sparse storage scheme and loop unrolling.

Keywords: Sparse solver; Cell index; Finite element analysis

Table 1. In-core factorization time and solution time

PROBLEM	Machine	Factorization time / Solution time (sec.)		
		MA27	MA47	This paper
HSCT16KB	CompaqXP 1000	6.57/0.11	7.06/0.09	2.91/0.10
	PIII 850	24.68/0.17	14.12/0.11	5.54/0.11
BCSSTK17	CompaqXP 1000	1.03/0.09	1.05/0.09	0.70/0.05
	PIII 850	2.86/0.16	2.03/0.16	1.15/0.11
BCSSTK18	CompaqXP 1000	0.81/0.06	0.46/0.05	0.58/0.04
	PIII 850	2.14/0.11	0.71/0.11	0.99/0.11
BCSSTK25	CompaqXP 1000	2.71/0.16	1.59/0.14	1.53/0.09
	PIII 850	9.89/0.22	3.13/0.22	3.02/0.11
BCSSTK29	CompaqXP 1000	1.77/0.14	1.56/0.13	1.32/0.08
	PIII 850	5.44/0.22	2.47/0.25	2.09/0.11
BCSSTK30	CompaqXP 1000	5.59/0.38	5.37/0.39	3.38/0.21
	PIII 850	20.65/0.55	10.99/0.55	4.84/0.27
BCSSTK31	CompaqXP 1000	5.84/0.41	5.74/0.41	4.53/0.21
	PIII 850	22.74/0.60	11.37/0.60	8.19/0.27
BCSSTK32	CompaqXP 1000	9.71/0.57	9.26/0.56	6.15/0.30
	PIII 850	38.19/0.77	22.19/0.77	8.79/0.33

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