High-accuracy Difference and Meshless Methods for Fluid and Solid Mechanics

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ABSTRACT

In the EPMESC VIII Conference, the author presented high- and arbitrary-order finite difference techniques based on non-centered compact difference operators. They are aimed mainly on fluid dynamic problems with relatively smooth boundaries. For irregular boundaries, an idea of using meshless Radial Basis Functions (RBF) approach in a "finite difference mode" was also suggested with applications to solid mechanics.

In the present lecture, further results concerning the above methods are displayed. In the fluid dynamics area, an application of the 5th-order high resolution compact scheme for describing a fine structure of vortex flows is considered in the context of shear flow instability. However, the main emphasize is placed on applications of the above two approaches to solid mechanics problems. In both cases, direct discretizations of governing equations are used instead of their weak formulations.

In the finite-difference case, the fourth-order Pade-type compact approximations and an extension of the multioperators principle [1,2] to construct arbitrary-order schemes are considered. An application of the technique to the self-adjoint operators of the elasticity equations is outlined. It is shown that the method can be extremely accurate in the case of relatively simple geometries with regular boundaries. Illustrative examples with Kirchgoff plates with stiffeners are presented, non-linear the Karman-Fopple equations being used in several cases.

The present meshless RBF-based method is aimed at complicated geometries, irregular boundaries and large deformations. The main idea here suggested in [3] is using radial basis functions locally for constructing numerical differentiation formulas. In contrast to existing attempts to use RBF in the framework of the collocation method, it implies defining stencils (in other words, clouds,stars etc.) for each node which serve as a set of data points for the local RBF-interpolation and consequent derivatives discretizations similar to that in the finite difference procedure. However, unlike the latter, RBF approximations have the potential for providing exponential convergence when including more points in stencils. High accuracy of the method is demonstrated in the cases of the Poisson and biharmonic equations. It is shown to be considerably more accurate than high-order FEM when solving benchmark problem of bending of a very skewed plate. High accuracy of stress concentration predictions in the case of torsion of a prismatic bar with L-shaped cross section is demonstrated. Other examples concern with non-linear behavior of variously shaped plates under elastic and creep conditions.

Keywords: compact schemes, multioperators, radial basis functions, fluid and solid mechanics equations

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