Instabilities in Deformation Processes of Solids

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ABSTRACT

The stability of deformation processes of solids is examined for various types of material including elastic-plastic [1, 2, 3] viscous [4, 5, 6] and quasi-brittle resp. brittle-damaging [7]. The analytical and computational background is exposed, applications are presented.

The issue of stability is investigated with respect to a perturbed deformation path of the system [8]. Uniqueness of solution and global stability [9] as discussed for the discretized representation of the deforming solid [10] base on the momentary tangent response operator of the system. The transition to local conditions for bifurcation and instability focuses on the constitutive level, intrinsic to the material. The incremental nonlinearity arising from the essential difference between momentary loading and unloading, inherent to elastic-plastic as well as brittle-damaging material behaviour, requires careful interpretation toward assessment of states marked critical during the course of the evolving deformation process because of a singular tangent operator. Depending on the direction in deformation space of the associated eigenvector the state can be unstable, or it allows for a multitude of solutions, resp. it is not critical in conjunction with the applied loading sequence. If the material is viscous, stability is studied by the rate of amplification of disturbances from the deformation path. The evolution of disturbances depends on both the momentary viscosity matrix of the system and the geometrical stiffness. The former describes the sensitivity of the stress resultants with respect to variations in velocity, the latter that with respect to variations in geometry. Non-isothermal processes imply interaction of the material motion with a transient temperature field. This motivates reference to thermodynamics which provides an appropriate basis for the previous isothermal statements as well.

Possible interference and distinction between physical and numerical instability is outlined, and the increasing sensitivity to random fluctuations of process parameters is pointed out. Aspects of instability are elucidated by a number of examples. The conventional case of elastic-plastic necking serves for an illustration of computational strategies at the critical state, and compares bifurcation analysis with results based on initial imperfection. The effect of non-isothermal conditions is also presented. The influence of inertial terms is discussed in connection with elastic-plastic buckling of a column. The phenomenon of localization is studied for a quasi-brittle solid with damaging microstructure [11]. Thereby it is seen how initially diffuse microcracking patterns localize to form rather coherent bands. The stability of viscous deformation is investigated for the extrusion to a circular rod of ceramic paste subject to gravity [12]. The analysis is employed for the interpretation of the onset of pronounced reduction in cross-section, and of surface irregularities observed in the laboratory.

Keywords: Inelastic deformation, stability, elastic-plastic-, quasi-brittle-, viscous solids

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